Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.7-Miscellaneous/140-4.7.6-f^-a+b-x+c-x^2-trig-d+e-x+f-x^2-^n

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [142]. This is test number [140].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (142)	0.00 (0)
Rubi	98.59 (140)	1.41 (2)
Fricas	80.99 (115)	19.01 (27)
Maple	80.28 (114)	19.72 (28)
Maxima	80.28 (114)	19.72 (28)
Giac	44.37 (63)	55.63 (79)
Mupad	35.21 (50)	64.79 (92)
Sympy	30.28 (43)	69.72 (99)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

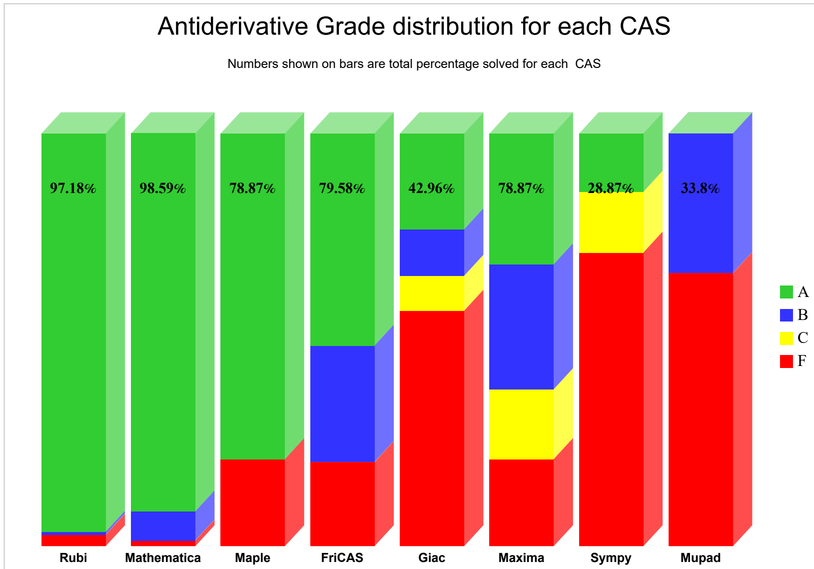
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

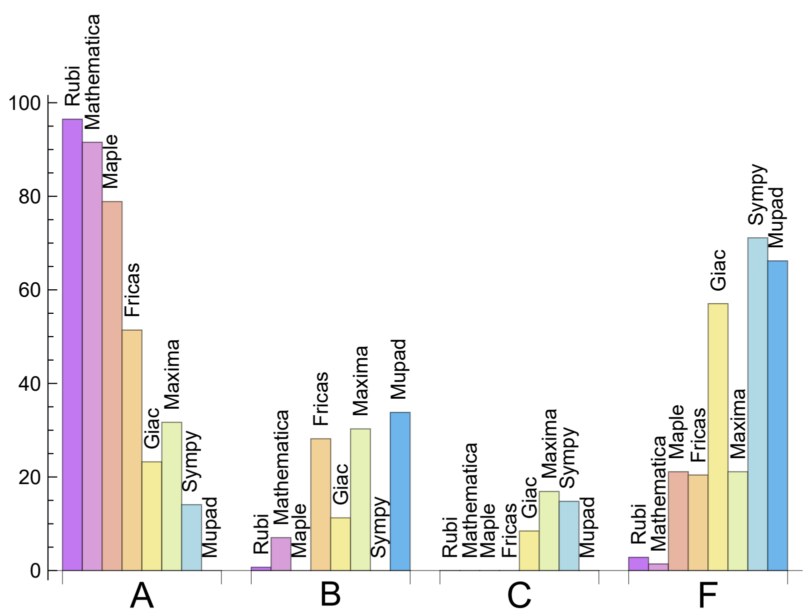
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.479	0.704	0.000	2.817
Mathematica	91.549	7.042	0.000	1.408
Maple	78.873	0.000	0.000	21.127
Fricas	51.408	28.169	0.000	20.423
Maxima	31.690	30.282	16.901	21.127
Giac	23.239	11.268	8.451	57.042
Sympy	14.085	0.000	14.789	71.127
Mupad	0.000	33.803	0.000	66.197

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	2	100.00	0.00	0.00
Fricas	27	100.00	0.00	0.00
Maple	28	100.00	0.00	0.00
Maxima	28	100.00	0.00	0.00
Giac	79	100.00	0.00	0.00
Mupad	92	0.00	100.00	0.00
Sympy	99	96.97	3.03	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.25
Maxima	0.27
Giac	0.31
Rubi	0.34
Maple	0.86
Mathematica	1.25
Sympy	4.29
Mupad	20.67

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	68.50	1.12	25.50	0.90
Maple	125.89	0.86	108.00	0.84
Rubi	140.69	1.13	129.00	1.00
Fricas	201.50	1.26	156.00	1.05
Mathematica	246.56	1.32	117.50	1.00
Maxima	499.46	3.97	238.00	1.66
Giac	522.17	12.25	127.00	1.17
Sympy	598.74	4.57	70.00	2.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

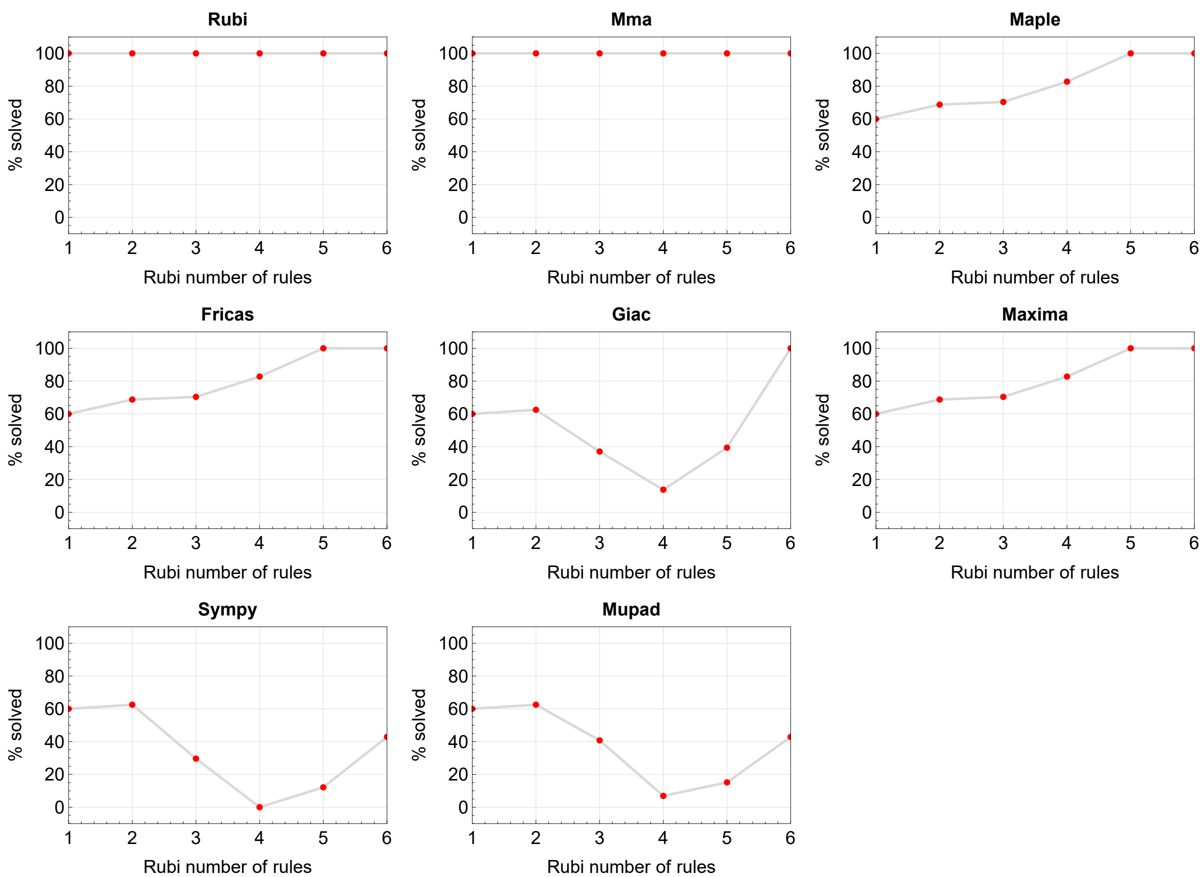


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

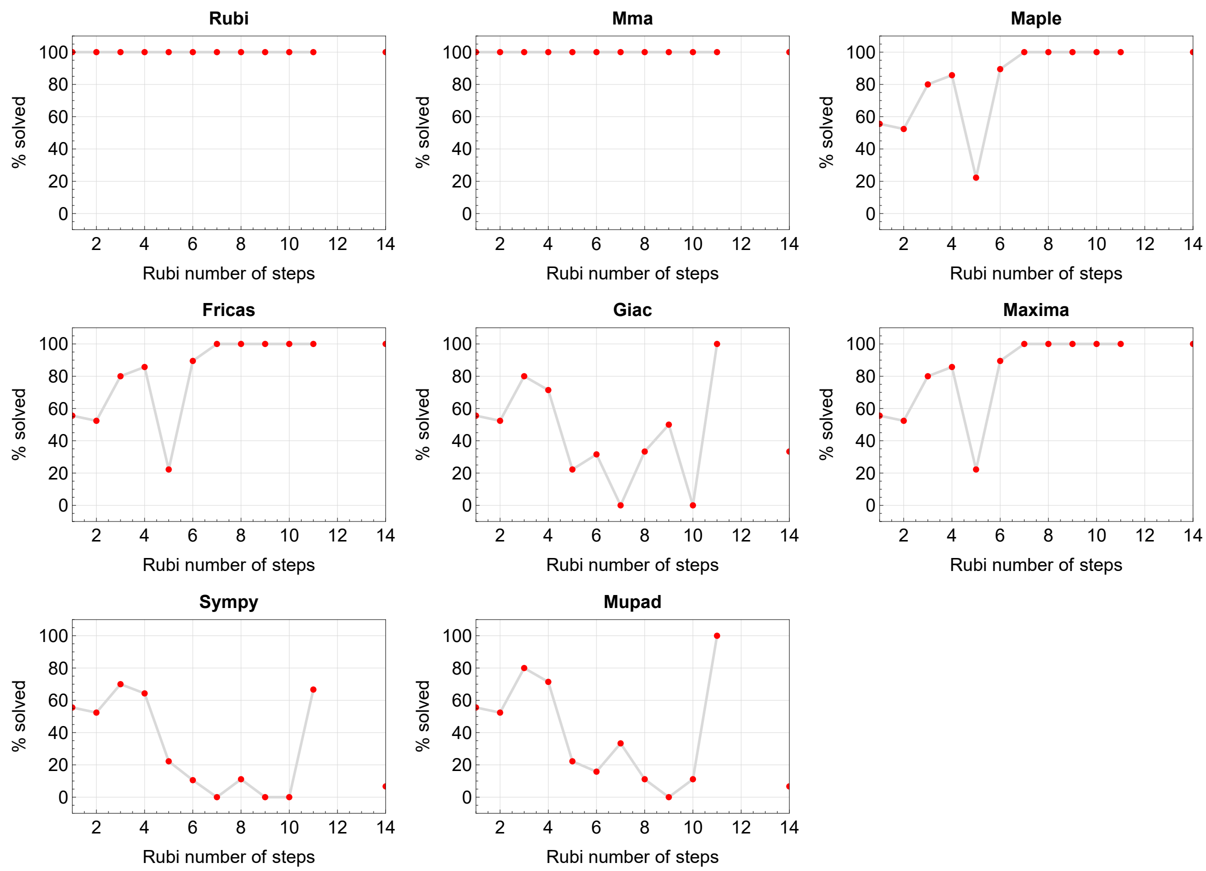


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

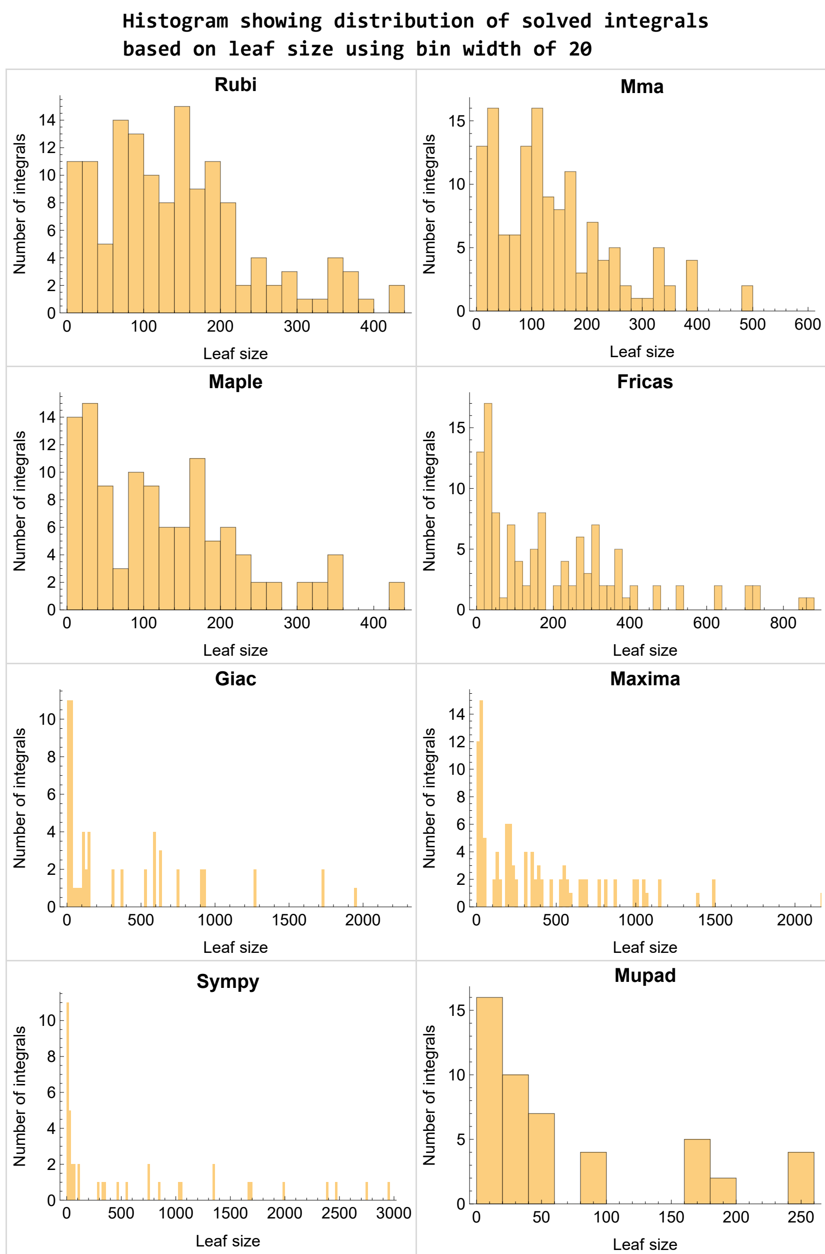


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

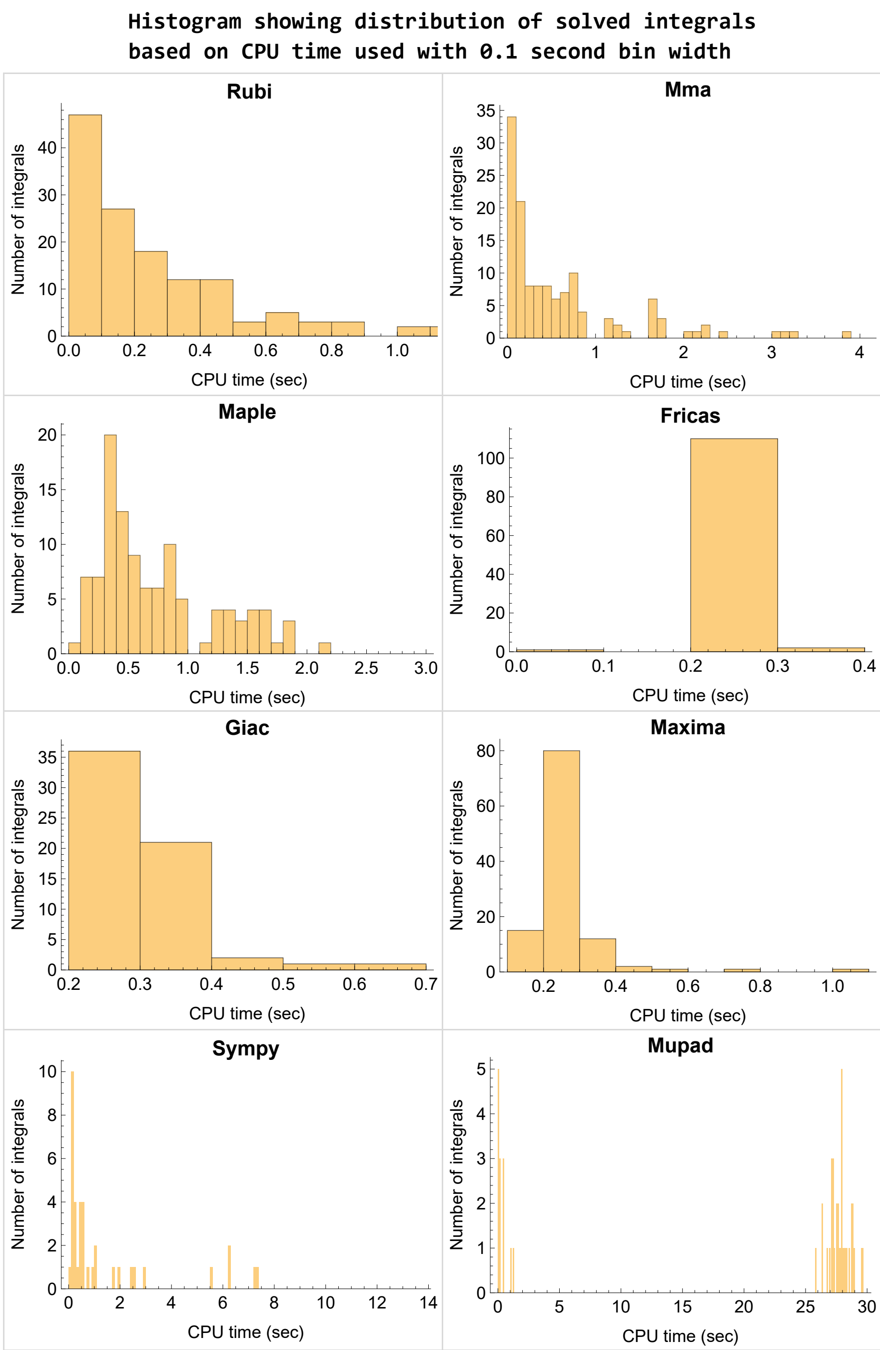


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

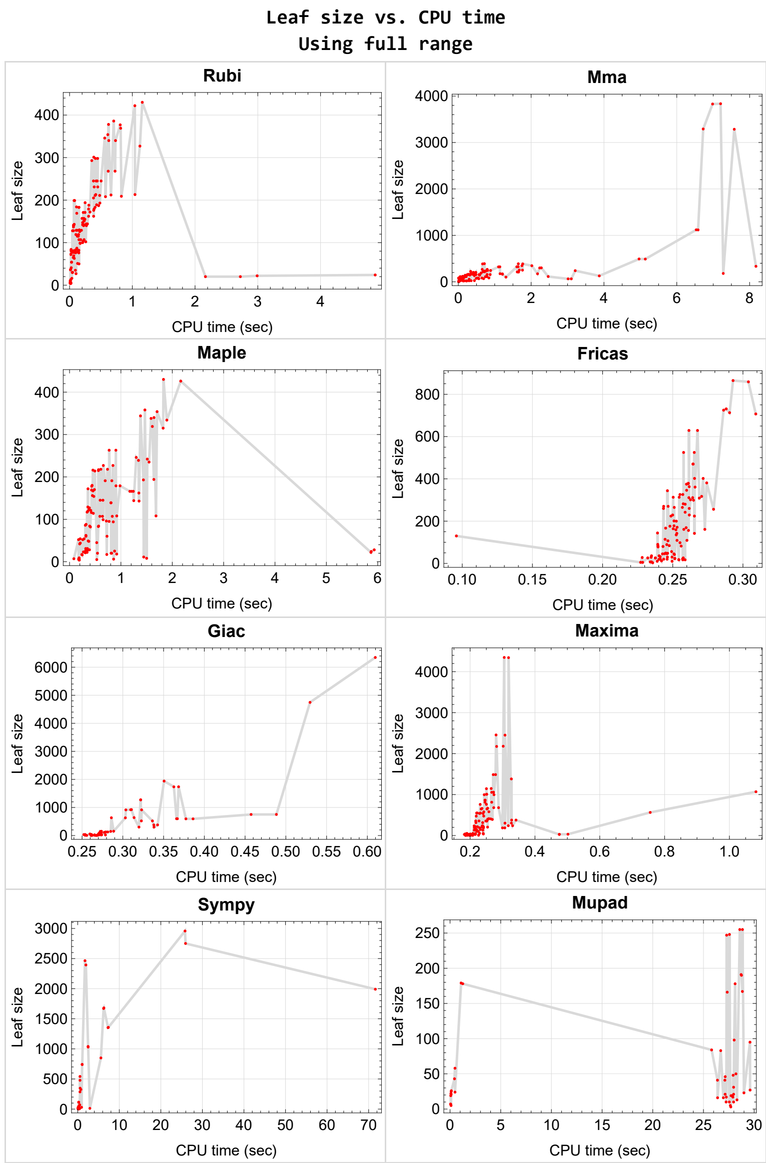


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{29, 30\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{98, 100, 101, 102, 103, 129, 131, 132, 133, 134\}$

Maple $\{\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

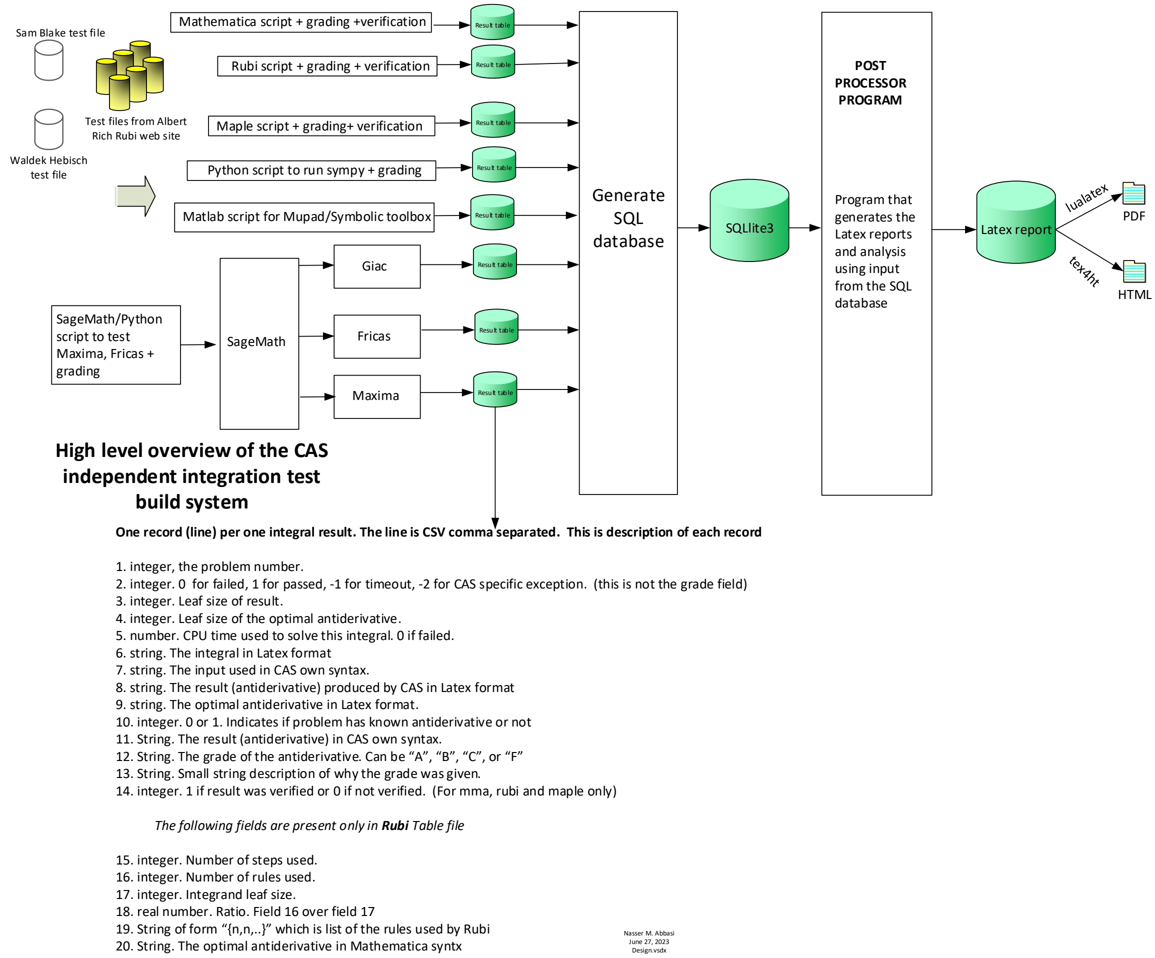
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142 }

B grade { 34 }

C grade { }

F normal fail { 28, 32 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142 }

B grade { 7, 21, 22, 63, 99, 101, 102, 130, 132, 133 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 2, 3, 4, 9, 11, 12, 13, 18, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140 }

B grade { }

C grade { }

F normal fail { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 53, 54, 55, 56, 137, 138, 141, 142 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 3, 4, 9, 11, 12, 13, 18, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 71, 72, 73, 76, 77, 78, 85, 86, 87, 88, 89, 94, 95, 96, 104, 107, 108, 109, 116, 117, 118, 119, 120, 125, 126, 127, 135, 136, 139, 140 }

B grade { 63, 70, 74, 75, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 105, 106, 110, 111, 112, 113, 114, 115, 121, 122, 123, 124, 128, 129, 130, 131, 132, 133, 134 }

C grade { }

F normal fail { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 53, 54, 55, 56, 137, 138, 141, 142 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 9, 18, 31, 32, 33, 38, 47, 48, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 84, 104, 105, 106, 107, 110, 111, 112, 113, 115 }

B grade { 2, 3, 4, 11, 12, 13, 34, 35, 39, 40, 41, 42, 43, 44, 45, 46, 63, 77, 78, 83, 88, 90, 91, 93, 97, 99, 100, 102, 108, 109, 114, 119, 121, 122, 124, 128, 130, 131, 133, 135, 136, 139, 140 }

C grade { 36, 37, 85, 86, 87, 89, 92, 94, 95, 96, 98, 101, 103, 116, 117, 118, 120, 123, 125, 126, 127, 129, 132, 134 }

F normal fail { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 53, 54, 55, 56, 137, 138, 141, 142 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 9, 18, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 61, 62, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 104, 105, 106 }

B grade { 31, 32, 63, 70, 79, 80, 81, 82, 83, 84, 110, 111, 112, 113, 114, 115 }

C grade { 2, 3, 4, 11, 12, 13, 34, 35, 135, 136, 139, 140 }

F normal fail { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 33, 36, 37, 53, 54, 55, 56, 57, 58, 59, 60, 76, 77, 78, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 141, 142 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 2, 3, 4, 9, 11, 12, 13, 18, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 104, 135, 136, 139, 140 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 53, 54, 55, 56, 57, 58, 59, 60, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 141, 142 }

F(-2) exception fail { }

Sympy

A grade { 9, 18, 34, 35, 47, 48, 49, 50, 51, 52, 61, 62, 64, 65, 66, 67, 69, 70, 71, 72 }

B grade { }

C grade { 2, 3, 4, 11, 12, 13, 38, 39, 40, 41, 42, 43, 44, 45, 46, 73, 104, 135, 136, 139, 140 }

F normal fail { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 32, 33, 36, 37, 53, 54, 55, 56, 57, 58, 59, 60, 63, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 137, 138, 141, 142 }

F(-1) timedout fail { 68, 102, 133 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	110	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	154	143	813	171	1681	1275	190
N.S.	1	1.00	0.77	0.72	4.09	0.86	8.45	6.41	0.95
time (sec)	N/A	0.081	0.569	1.353	0.267	0.253	6.297	0.322	28.751

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	86	95	356	89	741	915	95
N.S.	1	1.00	0.67	0.74	2.78	0.70	5.79	7.15	0.74
time (sec)	N/A	0.062	0.145	0.770	0.217	0.246	1.065	0.323	29.591

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	194	49	347	634	50
N.S.	1	1.00	0.66	0.67	2.66	0.67	4.75	8.68	0.68
time (sec)	N/A	0.021	0.084	0.363	0.215	0.245	0.581	0.286	28.202

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	114	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	2.469	0.000	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	101	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	1.308	0.000	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	334	0	0	0	0	0	0
N.S.	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	8.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	173	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	2.176	0.000	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	33	31	37	36	70	35	41
N.S.	1	1.00	0.61	0.57	0.69	0.67	1.30	0.65	0.76
time (sec)	N/A	0.031	0.041	0.315	0.241	0.240	0.482	0.261	26.381

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	110	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	0.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	155	144	813	142	1671	1271	191
N.S.	1	1.00	0.78	0.72	4.09	0.71	8.40	6.39	0.96
time (sec)	N/A	0.072	0.486	1.250	0.246	0.265	6.231	0.322	28.714

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	85	94	356	78	741	915	98
N.S.	1	1.00	0.66	0.73	2.78	0.61	5.79	7.15	0.77
time (sec)	N/A	0.047	0.149	0.851	0.227	0.240	1.082	0.304	28.030

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	47	48	192	48	286	631	48
N.S.	1	1.00	0.65	0.67	2.67	0.67	3.97	8.76	0.67
time (sec)	N/A	0.021	0.076	0.375	0.213	0.246	0.545	0.303	27.928

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	0.018	0.000	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	0.219	0.000	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	111	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	0.195	0.000	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	33	31	37	36	70	35	41
N.S.	1	1.00	0.61	0.57	0.69	0.67	1.30	0.65	0.76
time (sec)	N/A	0.032	0.057	0.348	0.217	0.235	0.481	0.262	27.113

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	212	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	1.619	0.000	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	175	0	0	0	0	0	0
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	1.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	166	0	0	0	0	0	0
N.S.	1	1.00	2.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	0.333	0.000	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	163	0	0	0	0	0	0
N.S.	1	1.00	2.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	1.212	0.000	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	126	171	0	0	0	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.162	1.211	0.000	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	188	210	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	1.655	0.000	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	133	0	0	0	0	0	0
N.S.	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.147	0.441	0.000	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	102	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.102	0.000	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	102	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	0.109	0.000	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	0	143	0	0	130	0	0	0
N.S.	1	0.00	1.03	0.00	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.000	0.588	0.000	0.000	0.096	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	23	25	26	20	25	25
N.S.	1	1.00	1.10	1.10	1.19	1.24	0.95	1.19	1.19
time (sec)	N/A	0.879	16.773	0.162	0.511	0.239	2.979	0.288	27.463

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	31	22	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.35	0.96	1.09	1.09
time (sec)	N/A	1.033	14.701	0.214	0.634	0.240	2.834	0.295	28.569

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	26	28	32	26	0	6346	27
N.S.	1	1.00	1.08	1.17	1.33	1.08	0.00	264.42	1.12
time (sec)	N/A	4.871	0.443	5.932	0.502	0.257	0.000	0.610	29.600

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	0	24	24	30	24	0	4746	23
N.S.	1	0.00	1.04	1.04	1.30	1.04	0.00	206.35	1.00
time (sec)	N/A	0.000	0.352	5.873	0.475	0.251	0.000	0.530	28.992

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	22	27	22	0	0	21
N.S.	1	1.00	1.00	1.00	1.23	1.00	0.00	0.00	0.95
time (sec)	N/A	2.990	0.296	5.868	0.475	0.251	0.000	0.000	27.977

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	327	18	18	1382	18	17	1941	17
N.S.	1	19.24	1.06	1.06	81.29	1.06	1.00	114.18	1.00
time (sec)	N/A	1.121	0.137	0.918	0.327	0.254	0.317	0.351	27.257

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	392	17	15	639	16
N.S.	1	1.00	1.00	1.06	24.50	1.06	0.94	39.94	1.00
time (sec)	N/A	0.036	0.026	0.725	0.269	0.254	0.147	0.314	26.961

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	20	564	20	0	0	19
N.S.	1	1.00	1.00	1.00	28.20	1.00	0.00	0.00	0.95
time (sec)	N/A	2.165	0.234	0.820	0.756	0.244	0.000	0.000	27.720

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	20	1069	20	0	0	19
N.S.	1	1.00	1.00	1.00	53.45	1.00	0.00	0.00	0.95
time (sec)	N/A	2.722	0.219	0.809	1.082	0.246	0.000	0.000	27.838

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	45	44	56	325	55	46
N.S.	1	1.00	0.70	0.71	0.70	0.89	5.16	0.87	0.73
time (sec)	N/A	0.075	0.116	0.529	0.215	0.249	0.768	0.260	27.145

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	74	96	538	109	1030	98	166
N.S.	1	1.00	0.62	0.81	4.52	0.92	8.66	0.82	1.39
time (sec)	N/A	0.144	0.548	0.716	0.240	0.253	2.510	0.271	27.344

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	82	108	550	135	1353	111	178
N.S.	1	1.00	0.64	0.84	4.26	1.05	10.49	0.86	1.38
time (sec)	N/A	0.189	0.649	0.919	0.222	0.252	7.290	0.272	28.122

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	74	99	538	98	1040	100	167
N.S.	1	1.00	0.62	0.83	4.52	0.82	8.74	0.84	1.40
time (sec)	N/A	0.101	0.494	0.653	0.229	0.255	2.494	0.274	28.840

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	60	236	90	850	66	58
N.S.	1	1.00	0.72	0.76	2.99	1.14	10.76	0.84	0.73
time (sec)	N/A	0.092	0.298	0.729	0.212	0.239	5.577	0.270	0.488

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	110	166	1148	201	2958	155	255
N.S.	1	1.00	0.60	0.91	6.27	1.10	16.16	0.85	1.39
time (sec)	N/A	0.149	0.754	1.207	0.265	0.251	25.823	0.289	28.581

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	81	107	550	114	1357	111	179
N.S.	1	1.00	0.63	0.83	4.26	0.88	10.52	0.86	1.39
time (sec)	N/A	0.106	0.552	0.832	0.232	0.245	7.378	0.273	1.092

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	110	166	1144	200	2751	152	255
N.S.	1	1.00	0.60	0.91	6.25	1.09	15.03	0.83	1.39
time (sec)	N/A	0.148	0.744	1.174	0.250	0.253	25.944	0.274	28.849

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	111	108	550	156	1991	111	178
N.S.	1	1.00	0.86	0.84	4.26	1.21	15.43	0.86	1.38
time (sec)	N/A	0.122	0.786	1.683	0.220	0.261	71.619	0.277	1.255

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	19	17	17	17	27	16	16
N.S.	1	1.00	0.63	0.57	0.57	0.57	0.90	0.53	0.53
time (sec)	N/A	0.049	0.031	0.239	0.193	0.259	0.145	0.262	26.392

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	25	23	26	26	48	25	21
N.S.	1	1.00	0.50	0.46	0.52	0.52	0.96	0.50	0.42
time (sec)	N/A	0.150	0.035	0.270	0.182	0.232	0.260	0.265	27.137

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	18	16	17	17	27	15	17
N.S.	1	1.00	0.60	0.53	0.57	0.57	0.90	0.50	0.57
time (sec)	N/A	0.046	0.024	0.246	0.196	0.257	0.144	0.269	27.901

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	23	21	26	26	48	24	22
N.S.	1	1.00	0.45	0.41	0.51	0.51	0.94	0.47	0.43
time (sec)	N/A	0.128	0.027	0.306	0.198	0.238	0.274	0.274	0.085

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	39	21	27	39	19
N.S.	1	1.00	0.81	0.74	1.44	0.78	1.00	1.44	0.70
time (sec)	N/A	0.103	0.167	0.544	0.186	0.241	0.096	0.273	0.073

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	33	28	23	26	32	23	31
N.S.	1	1.00	0.80	0.68	0.56	0.63	0.78	0.56	0.76
time (sec)	N/A	0.027	0.056	0.387	0.201	0.238	0.175	0.271	27.973

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	64	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.148	3.103	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	64	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	3.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	68	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.143	0.695	0.000	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	66	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	0.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	43	42	37	30	0	0	0
N.S.	1	1.00	0.62	0.61	0.54	0.43	0.00	0.00	0.00
time (sec)	N/A	0.067	0.025	0.197	0.184	0.246	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	47	44	38	32	0	0	0
N.S.	1	1.00	0.72	0.68	0.58	0.49	0.00	0.00	0.00
time (sec)	N/A	0.059	0.023	0.209	0.201	0.241	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	52	51	45	0	0	0
N.S.	1	1.00	1.00	0.64	0.63	0.56	0.00	0.00	0.00
time (sec)	N/A	0.080	0.072	0.257	0.240	0.251	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	82	54	52	46	0	0	0
N.S.	1	1.00	1.06	0.70	0.68	0.60	0.00	0.00	0.00
time (sec)	N/A	0.059	0.074	0.283	0.190	0.244	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	22	29	29	29	21	21
N.S.	1	1.00	0.69	0.63	0.83	0.83	0.83	0.60	0.60
time (sec)	N/A	0.093	0.050	0.341	0.197	0.235	0.928	0.279	0.110

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.010	0.013	0.176	0.198	0.227	0.119	0.261	27.651

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	21	5	19	21	0	17	43
N.S.	1	1.00	4.20	1.00	3.80	4.20	0.00	3.40	8.60
time (sec)	N/A	0.021	0.022	0.530	0.190	0.257	0.000	0.269	0.439

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	3	3	3	3
N.S.	1	1.00	1.00	1.00	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.010	0.012	0.184	0.186	0.236	0.102	0.267	27.695

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	7	7	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	0.70	0.70	0.70
time (sec)	N/A	0.011	0.012	0.188	0.186	0.235	0.102	0.268	0.066

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	10	7	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	1.00	0.70	0.70
time (sec)	N/A	0.011	0.011	1.501	0.192	0.234	0.187	0.266	0.048

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	11	10	10	12	10	10
N.S.	1	1.00	1.00	0.85	0.77	0.77	0.92	0.77	0.77
time (sec)	N/A	0.019	0.016	1.444	0.190	0.239	2.924	0.254	27.234

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	42	0	24	24
N.S.	1	1.00	1.00	0.83	0.80	1.40	0.00	0.80	0.80
time (sec)	N/A	0.042	0.108	0.877	0.186	0.251	0.000	0.269	0.491

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	4	12	10	7	10
N.S.	1	1.00	1.00	1.00	0.57	1.71	1.43	1.00	1.43
time (sec)	N/A	0.012	0.012	0.084	0.200	0.242	0.101	0.261	27.551

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	9	8	19	10	29	10
N.S.	1	1.00	1.00	1.80	1.60	3.80	2.00	5.80	2.00
time (sec)	N/A	0.011	0.006	0.185	0.205	0.240	0.478	0.272	27.972

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	5	5	3	5	5
N.S.	1	1.00	1.00	1.00	1.25	1.25	0.75	1.25	1.25
time (sec)	N/A	0.021	0.010	0.191	0.208	0.229	0.167	0.256	0.100

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	10	5	7	13
N.S.	1	1.00	1.00	1.00	1.17	1.67	0.83	1.17	2.17
time (sec)	N/A	0.020	0.017	0.858	0.226	0.250	0.400	0.263	28.306

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	27	27	28	30	114	35	26
N.S.	1	1.00	0.73	0.73	0.76	0.81	3.08	0.95	0.70
time (sec)	N/A	0.016	0.053	0.349	0.207	0.249	0.246	0.254	0.128

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	108	88	100	260	0	127	0
N.S.	1	1.00	0.94	0.77	0.87	2.26	0.00	1.10	0.00
time (sec)	N/A	0.139	0.147	0.322	0.225	0.243	0.000	0.279	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	134	119	131	229	0	147	0
N.S.	1	1.00	0.93	0.83	0.91	1.59	0.00	1.02	0.00
time (sec)	N/A	0.285	0.187	0.355	0.213	0.250	0.000	0.272	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	52	51	45	0	0	0
N.S.	1	1.00	1.00	0.64	0.63	0.56	0.00	0.00	0.00
time (sec)	N/A	0.064	0.007	0.175	0.208	0.243	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	129	62	137	82	0	0	0
N.S.	1	1.00	1.48	0.71	1.57	0.94	0.00	0.00	0.00
time (sec)	N/A	0.116	0.197	0.319	0.218	0.256	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	165	129	475	161	0	0	0
N.S.	1	1.00	1.06	0.83	3.06	1.04	0.00	0.00	0.00
time (sec)	N/A	0.244	0.444	0.356	0.242	0.258	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	132	116	147	265	0	300	0
N.S.	1	1.00	0.93	0.82	1.04	1.87	0.00	2.11	0.00
time (sec)	N/A	0.236	0.187	0.425	0.232	0.251	0.000	0.338	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	156	139	186	270	0	521	0
N.S.	1	1.00	0.99	0.89	1.18	1.72	0.00	3.32	0.00
time (sec)	N/A	0.227	0.875	0.808	0.300	0.247	0.000	0.323	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	268	239	302	525	0	595	0
N.S.	1	1.00	0.90	0.80	1.01	1.76	0.00	2.00	0.00
time (sec)	N/A	0.447	0.794	1.342	0.308	0.258	0.000	0.386	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	162	156	190	313	0	378	0
N.S.	1	1.00	1.00	0.96	1.17	1.93	0.00	2.33	0.00
time (sec)	N/A	0.383	0.394	0.445	0.241	0.250	0.000	0.338	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	244	179	240	326	0	599	0
N.S.	1	1.00	1.36	1.00	1.34	1.82	0.00	3.35	0.00
time (sec)	N/A	0.390	0.887	0.904	0.331	0.255	0.000	0.366	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	323	319	377	629	0	751	0
N.S.	1	1.00	0.95	0.94	1.11	1.85	0.00	2.21	0.00
time (sec)	N/A	0.732	1.141	1.612	0.328	0.262	0.000	0.457	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	119	123	206	144	0	0	0
N.S.	1	1.00	0.79	0.81	1.36	0.95	0.00	0.00	0.00
time (sec)	N/A	0.234	0.120	0.378	0.237	0.239	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	132	145	236	161	0	0	0
N.S.	1	1.00	0.77	0.85	1.38	0.94	0.00	0.00	0.00
time (sec)	N/A	0.246	0.195	0.596	0.239	0.273	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	224	246	412	282	0	0	0
N.S.	1	1.00	0.74	0.82	1.37	0.94	0.00	0.00	0.00
time (sec)	N/A	0.386	0.338	1.298	0.258	0.257	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	170	84	209	107	0	0	0
N.S.	1	1.00	1.59	0.79	1.95	1.00	0.00	0.00	0.00
time (sec)	N/A	0.231	0.373	0.392	0.241	0.247	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	188	107	315	169	0	0	0
N.S.	1	1.00	1.34	0.76	2.25	1.21	0.00	0.00	0.00
time (sec)	N/A	0.268	0.626	0.590	0.218	0.260	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	386	166	661	317	0	0	0
N.S.	1	1.00	1.81	0.78	3.10	1.49	0.00	0.00	0.00
time (sec)	N/A	0.418	1.770	1.241	0.237	0.271	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	216	169	760	299	0	0	0
N.S.	1	1.00	1.16	0.90	4.06	1.60	0.00	0.00	0.00
time (sec)	N/A	0.463	0.720	0.428	0.231	0.265	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	251	191	863	363	0	0	0
N.S.	1	1.00	1.19	0.91	4.09	1.72	0.00	0.00	0.00
time (sec)	N/A	0.479	1.763	0.694	0.246	0.262	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	490	338	2175	713	0	0	0
N.S.	1	1.00	1.30	0.90	5.77	1.89	0.00	0.00	0.00
time (sec)	N/A	0.807	5.136	1.587	0.282	0.290	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	155	172	354	178	0	0	0
N.S.	1	1.00	0.88	0.98	2.01	1.01	0.00	0.00	0.00
time (sec)	N/A	0.385	0.271	0.359	0.248	0.251	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	204	218	399	224	0	0	0
N.S.	1	1.00	0.88	0.94	1.73	0.97	0.00	0.00	0.00
time (sec)	N/A	0.449	0.482	0.609	0.254	0.265	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	391	344	680	346	0	0	0
N.S.	1	1.00	1.10	0.97	1.92	0.98	0.00	0.00	0.00
time (sec)	N/A	0.609	0.716	1.381	0.273	0.260	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	230	180	647	309	0	0	0
N.S.	1	1.00	1.19	0.93	3.35	1.60	0.00	0.00	0.00
time (sec)	N/A	0.483	0.755	0.439	0.234	0.269	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	299	227	997	402	0	0	0
N.S.	1	1.00	1.22	0.93	4.07	1.64	0.00	0.00	0.00
time (sec)	N/A	0.503	2.235	0.658	0.244	0.265	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	3291	358	2451	731	0	0	0
N.S.	1	1.00	8.53	0.93	6.35	1.89	0.00	0.00	0.00
time (sec)	N/A	0.704	6.726	1.467	0.308	0.288	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	347	216	1007	375	0	0	0
N.S.	1	1.00	1.64	1.02	4.75	1.77	0.00	0.00	0.00
time (sec)	N/A	0.660	2.019	0.455	0.251	0.260	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	1120	263	1487	470	0	0	0
N.S.	1	1.00	4.18	0.98	5.55	1.75	0.00	0.00	0.00
time (sec)	N/A	0.725	6.535	0.771	0.271	0.265	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	430	430	3835	430	4343	865	0	0	0
N.S.	1	1.00	8.92	1.00	10.10	2.01	0.00	0.00	0.00
time (sec)	N/A	1.159	7.203	1.829	0.319	0.293	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	324	217	1054	379	0	0	0
N.S.	1	1.00	1.52	1.02	4.95	1.78	0.00	0.00	0.00
time (sec)	N/A	1.043	1.607	0.564	0.267	0.261	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	26	26	25	28	114	33	25
N.S.	1	1.00	0.72	0.72	0.69	0.78	3.17	0.92	0.69
time (sec)	N/A	0.014	0.060	0.313	0.216	0.228	0.235	0.252	0.113

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	109	86	100	260	0	127	0
N.S.	1	1.00	0.95	0.75	0.87	2.26	0.00	1.10	0.00
time (sec)	N/A	0.124	0.153	0.307	0.220	0.262	0.000	0.281	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	135	117	131	229	0	147	0
N.S.	1	1.00	0.94	0.81	0.91	1.59	0.00	1.02	0.00
time (sec)	N/A	0.267	0.184	0.342	0.239	0.262	0.000	0.285	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	82	54	52	46	0	0	0
N.S.	1	1.00	1.06	0.70	0.68	0.60	0.00	0.00	0.00
time (sec)	N/A	0.078	0.057	0.207	0.229	0.246	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	107	60	133	84	0	0	0
N.S.	1	1.00	1.29	0.72	1.60	1.01	0.00	0.00	0.00
time (sec)	N/A	0.098	0.177	0.319	0.220	0.248	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	166	127	474	164	0	0	0
N.S.	1	1.00	1.10	0.84	3.14	1.09	0.00	0.00	0.00
time (sec)	N/A	0.228	0.423	0.386	0.219	0.250	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	133	114	147	265	0	300	0
N.S.	1	1.00	0.94	0.80	1.04	1.87	0.00	2.11	0.00
time (sec)	N/A	0.208	0.186	0.463	0.220	0.257	0.000	0.320	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	158	139	186	270	0	521	0
N.S.	1	1.00	1.01	0.89	1.18	1.72	0.00	3.32	0.00
time (sec)	N/A	0.199	0.790	0.804	0.307	0.244	0.000	0.336	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	267	235	302	525	0	595	0
N.S.	1	1.00	0.90	0.79	1.01	1.76	0.00	2.00	0.00
time (sec)	N/A	0.411	0.657	1.548	0.327	0.265	0.000	0.377	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	163	154	190	313	0	378	0
N.S.	1	1.00	1.01	0.95	1.17	1.93	0.00	2.33	0.00
time (sec)	N/A	0.303	0.273	0.473	0.229	0.255	0.000	0.343	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	245	179	240	326	0	599	0
N.S.	1	1.00	1.37	1.00	1.34	1.82	0.00	3.35	0.00
time (sec)	N/A	0.302	0.807	0.983	0.318	0.257	0.000	0.367	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	322	315	377	629	0	751	0
N.S.	1	1.00	0.95	0.93	1.11	1.85	0.00	2.21	0.00
time (sec)	N/A	0.625	1.105	1.822	0.342	0.268	0.000	0.488	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	116	121	204	142	0	0	0
N.S.	1	1.00	0.79	0.82	1.39	0.97	0.00	0.00	0.00
time (sec)	N/A	0.208	0.122	0.413	0.238	0.253	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	131	145	236	159	0	0	0
N.S.	1	1.00	0.77	0.85	1.38	0.93	0.00	0.00	0.00
time (sec)	N/A	0.218	0.193	0.650	0.237	0.252	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	218	242	406	280	0	0	0
N.S.	1	1.00	0.74	0.83	1.39	0.96	0.00	0.00	0.00
time (sec)	N/A	0.354	0.343	1.514	0.264	0.259	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	170	82	205	109	0	0	0
N.S.	1	1.00	1.65	0.80	1.99	1.06	0.00	0.00	0.00
time (sec)	N/A	0.213	0.363	0.406	0.254	0.260	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	189	107	315	167	0	0	0
N.S.	1	1.00	1.35	0.76	2.25	1.19	0.00	0.00	0.00
time (sec)	N/A	0.224	0.624	0.641	0.239	0.245	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	389	162	667	311	0	0	0
N.S.	1	1.00	1.90	0.79	3.25	1.52	0.00	0.00	0.00
time (sec)	N/A	0.381	1.648	1.349	0.254	0.261	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	217	167	761	301	0	0	0
N.S.	1	1.00	1.19	0.91	4.16	1.64	0.00	0.00	0.00
time (sec)	N/A	0.407	0.682	0.457	0.250	0.263	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	252	191	863	361	0	0	0
N.S.	1	1.00	1.19	0.91	4.09	1.71	0.00	0.00	0.00
time (sec)	N/A	0.407	1.674	0.823	0.247	0.266	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	492	334	2180	707	0	0	0
N.S.	1	1.00	1.33	0.91	5.91	1.92	0.00	0.00	0.00
time (sec)	N/A	0.814	4.965	1.894	0.302	0.309	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	151	170	354	176	0	0	0
N.S.	1	1.00	0.88	0.99	2.06	1.02	0.00	0.00	0.00
time (sec)	N/A	0.326	0.236	0.484	0.252	0.248	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	204	218	399	224	0	0	0
N.S.	1	1.00	0.88	0.94	1.73	0.97	0.00	0.00	0.00
time (sec)	N/A	0.393	0.417	0.739	0.256	0.249	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	386	340	680	344	0	0	0
N.S.	1	1.00	1.12	0.98	1.97	0.99	0.00	0.00	0.00
time (sec)	N/A	0.560	0.666	1.644	0.288	0.246	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	231	178	648	311	0	0	0
N.S.	1	1.00	1.22	0.94	3.43	1.65	0.00	0.00	0.00
time (sec)	N/A	0.431	0.733	0.431	0.252	0.264	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	301	227	997	402	0	0	0
N.S.	1	1.00	1.23	0.93	4.07	1.64	0.00	0.00	0.00
time (sec)	N/A	0.441	2.280	0.845	0.274	0.272	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	3285	354	2456	725	0	0	0
N.S.	1	1.00	8.69	0.94	6.50	1.92	0.00	0.00	0.00
time (sec)	N/A	0.621	7.579	1.704	0.280	0.286	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	348	214	1008	377	0	0	0
N.S.	1	1.00	1.67	1.03	4.85	1.81	0.00	0.00	0.00
time (sec)	N/A	0.572	1.749	0.492	0.251	0.261	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	1118	263	1487	470	0	0	0
N.S.	1	1.00	4.17	0.98	5.55	1.75	0.00	0.00	0.00
time (sec)	N/A	0.616	6.585	0.902	0.278	0.264	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	3829	426	4348	859	0	0	0
N.S.	1	1.00	9.07	1.01	10.30	2.04	0.00	0.00	0.00
time (sec)	N/A	1.040	6.983	2.165	0.305	0.304	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	325	215	1058	381	0	0	0
N.S.	1	1.00	1.56	1.03	5.06	1.82	0.00	0.00	0.00
time (sec)	N/A	0.826	1.643	0.562	0.272	0.274	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	180	194	581	256	2465	1738	248
N.S.	1	1.00	0.73	0.79	2.37	1.04	10.06	7.09	1.01
time (sec)	N/A	0.423	7.276	1.641	0.258	0.279	1.749	0.368	27.566

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	83	84	218	83	542	923	84
N.S.	1	1.00	0.84	0.85	2.20	0.84	5.47	9.32	0.85
time (sec)	N/A	0.167	0.840	0.566	0.216	0.239	0.548	0.311	25.804

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	128	0	0	0	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	3.872	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	184	184	240	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	3.211	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	228	193	578	242	2394	1736	247
N.S.	1	1.00	0.93	0.79	2.36	0.99	9.77	7.09	1.01
time (sec)	N/A	0.381	0.706	1.438	0.242	0.259	1.970	0.363	27.284

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	82	83	216	80	478	920	83
N.S.	1	1.00	0.84	0.85	2.20	0.82	4.88	9.39	0.85
time (sec)	N/A	0.169	0.517	0.560	0.223	0.243	0.553	0.310	26.702

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	80	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	145	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.115	0.317	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [48] had the largest ratio of [.5555999999999999983]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	18	0.111
2	A	2	2	1.00	18	0.111
3	A	2	2	1.00	18	0.111
4	A	1	1	1.00	16	0.062
5	A	1	1	1.00	16	0.062
6	A	1	1	1.00	18	0.056
7	A	2	2	1.00	18	0.111
8	A	2	2	1.00	18	0.111
9	A	3	2	1.00	8	0.250
10	A	2	2	1.00	18	0.111
11	A	2	2	1.00	18	0.111
12	A	2	2	1.00	18	0.111
13	A	1	1	1.00	16	0.062
14	A	1	1	1.00	16	0.062
15	A	1	1	1.00	18	0.056
16	A	2	2	1.00	18	0.111
17	A	2	2	1.00	18	0.111
18	A	3	2	1.00	8	0.250
19	A	6	3	1.00	18	0.167
20	A	5	3	1.00	18	0.167
21	A	4	3	1.00	16	0.188
22	A	4	3	1.00	16	0.188
23	A	5	3	1.00	18	0.167
24	A	6	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	5	4	1.00	27	0.148
26	A	2	2	1.00	18	0.111
27	A	2	2	1.00	18	0.111
28	F	0	0	N/A	0.000	N/A
29	N/A	0	0	1.00	21	0.000
30	N/A	0	0	1.00	23	0.000
31	A	11	5	1.00	44	0.114
32	F	0	0	N/A	0.000	N/A
33	A	7	4	1.00	43	0.093
34	B	14	6	19.24	35	0.171
35	A	1	1	1.00	30	0.033
36	A	6	3	1.00	38	0.079
37	A	10	4	1.00	38	0.105
38	A	3	3	1.00	20	0.150
39	A	4	2	1.00	22	0.091
40	A	4	2	1.00	22	0.091
41	A	4	2	1.00	22	0.091
42	A	4	3	1.00	24	0.125
43	A	5	2	1.00	24	0.083
44	A	4	2	1.00	22	0.091
45	A	5	2	1.00	24	0.083
46	A	4	2	1.00	24	0.083
47	A	4	3	1.00	7	0.429
48	A	11	5	1.00	9	0.556
49	A	4	3	1.00	7	0.429
50	A	11	5	1.00	9	0.556
51	A	4	3	1.00	19	0.158
52	A	3	1	1.00	15	0.067
53	A	5	4	1.00	26	0.154
54	A	5	4	1.00	27	0.148
55	A	5	4	1.00	26	0.154
56	A	5	4	1.00	27	0.148
57	A	6	3	1.00	10	0.300
58	A	6	3	1.00	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	A	6	3	1.00	12	0.250
60	A	6	3	1.00	12	0.250
61	A	2	2	1.00	15	0.133
62	A	2	2	1.00	8	0.250
63	A	3	3	1.00	12	0.250
64	A	2	2	1.00	8	0.250
65	A	2	2	1.00	12	0.167
66	A	2	2	1.00	12	0.167
67	A	3	3	1.00	10	0.300
68	A	4	3	1.00	26	0.115
69	A	2	2	1.00	8	0.250
70	A	2	2	1.00	8	0.250
71	A	3	3	1.00	12	0.250
72	A	3	3	1.00	10	0.300
73	A	1	1	1.00	10	0.100
74	A	6	4	1.00	12	0.333
75	A	6	4	1.00	15	0.267
76	A	6	3	1.00	12	0.250
77	A	4	2	1.00	14	0.143
78	A	6	3	1.00	17	0.176
79	A	8	5	1.00	16	0.312
80	A	9	6	1.00	18	0.333
81	A	14	5	1.00	18	0.278
82	A	8	5	1.00	19	0.263
83	A	9	6	1.00	21	0.286
84	A	14	5	1.00	21	0.238
85	A	8	4	1.00	16	0.250
86	A	9	4	1.00	18	0.222
87	A	14	4	1.00	18	0.222
88	A	6	4	1.00	18	0.222
89	A	7	4	1.00	20	0.200
90	A	10	4	1.00	20	0.200
91	A	8	5	1.00	21	0.238
92	A	9	5	1.00	23	0.217
93	A	14	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	8	4	1.00	19	0.210
95	A	10	4	1.00	21	0.190
96	A	14	4	1.00	21	0.190
97	A	8	5	1.00	21	0.238
98	A	10	5	1.00	23	0.217
99	A	14	5	1.00	23	0.217
100	A	8	5	1.00	24	0.208
101	A	10	5	1.00	26	0.192
102	A	14	5	1.00	26	0.192
103	A	8	5	1.00	24	0.208
104	A	1	1	1.00	10	0.100
105	A	6	4	1.00	12	0.333
106	A	6	4	1.00	15	0.267
107	A	6	3	1.00	12	0.250
108	A	4	2	1.00	14	0.143
109	A	6	3	1.00	17	0.176
110	A	8	5	1.00	16	0.312
111	A	9	6	1.00	18	0.333
112	A	14	5	1.00	18	0.278
113	A	8	5	1.00	19	0.263
114	A	9	6	1.00	21	0.286
115	A	14	5	1.00	21	0.238
116	A	8	4	1.00	16	0.250
117	A	9	4	1.00	18	0.222
118	A	14	4	1.00	18	0.222
119	A	6	4	1.00	18	0.222
120	A	7	4	1.00	20	0.200
121	A	10	4	1.00	20	0.200
122	A	8	5	1.00	21	0.238
123	A	9	5	1.00	23	0.217
124	A	14	5	1.00	23	0.217
125	A	8	4	1.00	19	0.210
126	A	10	4	1.00	21	0.190
127	A	14	4	1.00	21	0.190
128	A	8	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	10	5	1.00	23	0.217
130	A	14	5	1.00	23	0.217
131	A	8	5	1.00	24	0.208
132	A	10	5	1.00	26	0.192
133	A	14	5	1.00	26	0.192
134	A	8	5	1.00	24	0.208
135	A	8	6	1.00	22	0.273
136	A	6	5	1.00	20	0.250
137	A	2	2	1.00	22	0.091
138	A	3	3	1.00	22	0.136
139	A	8	6	1.00	22	0.273
140	A	6	5	1.00	20	0.250
141	A	2	2	1.00	22	0.091
142	A	3	3	1.00	22	0.136

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int F^{c(a+bx)} \sin^n(d+ex)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int F^{c(a+bx)} \sin^n(d+ex)\, dx = -\frac{\left(1-e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)}\, \text{Hypergeometric2F1}\left(-n, -\frac{en+ibc\log(F)}{2e}, \frac{1}{2}\left(2-n-\frac{ibc\log(F)}{e}\right), e^{2i(d+ex)}\right) \sin^n(\cdots}{ien-bc\log(F)}$$

```
[Out] -F^(c*(b*x+a))*hypergeom([-n, 1/2*(-e*n-I*b*c*ln(F))/e],[1-1/2*n-1/2*I*b*c*
ln(F)/e],exp(2*I*(e*x+d)))*sin(e*x+d)^n/((1-exp(2*I*(e*x+d)))^n)/(I*e*n-b*c
*ln(F))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4525, 2291}

$$\int F^{c(a+bx)} \sin^n(d+ex)\, dx = -\frac{\left(1-e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)} \sin^n(d+ex)\, \text{Hypergeometric2F1}\left(-n, -\frac{en+ibc\log(F)}{2e}, \frac{1}{2}\left(-n-\frac{ibc\log(F)}{e}+2\right), \cdots\right.}{-bc\log(F)+ien}$$

```
[In] Int[F^(c*(a + b*x))*Sin[d + e*x]^n,x]
```

```
[Out] -((F^(c*(a + b*x))*Hypergeometric2F1[-n, -1/2*(e*n + I*b*c*Log[F])/e, (2 -
n - (I*b*c*Log[F])/e)/2, E^((2*I)*(d + e*x))]*Sin[d + e*x]^n)/((1 - E^((2*I
)*(d + e*x)))^n*(I*e*n - b*c*Log[F])))
```

Rule 2291

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 4525

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Dist[E^(I*n*(d + e*x))*(Sin[d + e*x]^n/(-1 + E^(2*I*(d + e*x)))^n), I
nt[F^(c*(a + b*x))*((-1 + E^(2*I*(d + e*x)))^n/E^(I*n*(d + e*x))), x], x] /
; FreeQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(e^{in(d+ex)}\left(-1+e^{2i(d+ex)}\right)^{-n}\sin^n(d+ex)\right)\int e^{-in(d+ex)}\left(-1+e^{2i(d+ex)}\right)^n F^{c(a+bx)}\,dx$$
$$= -\frac{\left(1-e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(-n,-\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(2-n-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{ien-bc\log(F)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}\sin^n(d+ex)\,dx$$
$$= \frac{\left(1-e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(-n,-\frac{i(-ien+bc\log(F))}{2e},1-\frac{i(-ien+bc\log(F))}{2e},e^{2i(d+ex)}\right)\sin^n(d+}{-ien+bc\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Sin[d + e*x]^n,x]
[Out] (F^(c*(a + b*x))*Hypergeometric2F1[-n, ((-1/2*I)*((-I)*e*n + b*c*Log[F]))/e
, 1 - ((I/2)*((-I)*e*n + b*c*Log[F]))/e, E^((2*I)*(d + e*x))]*Sin[d + e*x]^
n)/((1 - E^((2*I)*(d + e*x)))^n*((-I)*e*n + b*c*Log[F]))
```

Maple [F]

$$\int F^{c(xb+a)} \sin(ex+d)^n \, dx$$

```
[In] int(F^(c*(b*x+a))*sin(e*x+d)^n,x)
[Out] int(F^(c*(b*x+a))*sin(e*x+d)^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \sin^n(d+ex) \, dx = \int F^{(bx+a)c} \sin(ex+d)^n \, dx$$

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^n,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*sin(e*x + d)^n, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \sin^n(d+ex) \, dx = \int F^{c(a+bx)} \sin^n(d+ex) \, dx$$

```
[In] integrate(F**(c*(b*x+a))*sin(e*x+d)**n,x)
[Out] Integral(F**(c*(a + b*x))*sin(d + e*x)**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \sin^n(d+ex) \, dx = \int F^{(bx+a)c} \sin(ex+d)^n \, dx$$

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^n,x, algorithm="maxima")
[Out] integrate(F^((b*x + a)*c)*sin(e*x + d)^n, x)
```

Giac [F]

$$\int F^{c(a+bx)} \sin^n(d+ex)\,dx = \int F^{(bx+a)c} \sin{(ex+d)}^n \;dx$$

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sin(e*x + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \sin^n(d+ex)\,dx = \int F^{c\,(a+b\,x)} \sin{(d+e\,x)}^n\,dx$$

```
[In] int(F^(c*(a + b*x))*sin(d + e*x)^n,x)
```

```
[Out] int(F^(c*(a + b*x))*sin(d + e*x)^n, x)
```

3.2 $\int F^{c(a+bx)} \sin^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 199

$$\int F^{c(a+bx)} \sin^3(d+ex)\,dx = -\frac{6e^3F^{c(a+bx)}\cos(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{6bce^2F^{c(a+bx)}\log(F)\sin(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} - \frac{3eF^{c(a+bx)}\cos(d+ex)\sin^2(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{bcF^{c(a+bx)}\log(F)\sin^3(d+ex)}{9e^2+b^2c^2\log^2(F)}$$

```
[Out] -6*e^3*F^(c*(b*x+a))*cos(e*x+d)/(9*e^4+10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)
^4)+6*b*c*e^2*F^(c*(b*x+a))*ln(F)*sin(e*x+d)/(9*e^4+10*b^2*c^2*e^2*ln(F)^2+
b^4*c^4*ln(F)^4)-3*e*F^(c*(b*x+a))*cos(e*x+d)*sin(e*x+d)^2/(9*e^2+b^2*c^2*l
n(F)^2)+b*c*F^(c*(b*x+a))*ln(F)*sin(e*x+d)^3/(9*e^2+b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used

$= \{4519, 4517\}$

$$\int F^{c(a+bx)}\sin^3(d+ex)\,dx = \frac{bc\log(F)\sin^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2} - \frac{3e\sin^2(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2} + \frac{6bce^2\log(F)\sin(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)+10b^2c^2e^2\log^2(F)+9e^4} - \frac{6e^3\cos(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)+10b^2c^2e^2\log^2(F)+9e^4}$$

```
[In] Int[F^(c*(a + b*x))*Sin[d + e*x]^3,x]
```

```
[Out] (-6*e^3*F^(c*(a + b*x))*Cos[d + e*x])/(9*e^4 + 10*b^2*c^2*e^2*Log[F]^2 + b^
4*c^4*Log[F]^4) + (6*b*c*e^2*F^(c*(a + b*x))*Log[F]*Sin[d + e*x])/(9*e^4 +
10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) - (3*e*F^(c*(a + b*x))*Cos[d +
e*x]*Sin[d + e*x]^2)/(9*e^2 + b^2*c^2*Log[F]^2) + (b*c*F^(c*(a + b*x))*Log[
F]*Sin[d + e*x]^3)/(9*e^2 + b^2*c^2*Log[F]^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4519

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d +
 e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{3eF^{c(a+bx)}\cos(d+ex)\sin^2(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{bcF^{c(a+bx)}\log(F)\sin^3(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{(6e^2)\int F^{c(a+bx)}\sin(d+ex)\,dx}{9e^2+b^2c^2\log^2(F)}\\ &= -\frac{6e^3F^{c(a+bx)}\cos(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{6bce^2F^{c(a+bx)}\log(F)\sin(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} - \frac{3eF^{c(a+bx)}\cos(d+ex)\sin^2(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{bcF^{c(a+bx)}\log(F)\sin^3(d+ex)}{9e^2+b^2c^2\log^2(F)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.77

$$\int F^{c(a+bx)} \sin^3(d+ex)\,dx$$
$$= \frac{F^{c(a+bx)}\left(-3e\cos(d+ex)\left(9e^2+b^2c^2\log^2(F)\right)+3\cos(3(d+ex))\left(e^3+b^2c^2e\log^2(F)\right)-2bc\log(F)\left(-13e\right.\right.}{4\left(9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F\right.}$$

```
[In] Integrate[F^(c*(a + b*x))*Sin[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(-3*e*Cos[d + e*x]*(9*e^2 + b^2*c^2*Log[F]^2) + 3*Cos[3*(d
 + e*x)]*(e^3 + b^2*c^2*e*Log[F]^2) - 2*b*c*Log[F]*(-13*e^2 - b^2*c^2*Log[F
]^2 + Cos[2*(d + e*x)]*(e^2 + b^2*c^2*Log[F]^2))*Sin[d + e*x]))/(4*(9*e^4 +
 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{3\left(\left(\ln(F)^2b^2c^2e+e^3\right)\cos(3ex+3d)-\frac{bc\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\sin(3ex+3d)}{3}+\left(9e^2+b^2c^2\ln(F)^2\right)\left(bc\ln(F)\sin(ex+d)-e\cos(ex+\right.\right.}{4b^4c^4\ln(F)^4+40b^2c^2e^2\ln(F)^2+36e^4}$
risch	$-\frac{3e\,F^{c(xb+a)}\cos(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{3bc\,F^{c(xb+a)}\ln(F)\sin(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{3e\,F^{c(xb+a)}\cos(3ex+3d)}{4\left(9e^2+b^2c^2\ln(F)^2\right)}-\frac{cb\ln(F)F^{c(xb+a)}\sin(3ex+3d)}{4\left(9e^2+b^2c^2\ln(F)^2\right)}$
norman	$\frac{-\frac{6e^3\mathrm{e}^{c(xb+a)\ln(F)}}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{6e^3\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{6e\left(2b^2c^2\ln(F)^2+3e^2\right)\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{6e\left(2b^2\right.}{}}{}$
default	$\frac{F^{ac}\left(\frac{-\frac{4e\,\mathrm{e}^{bcx\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{4e\,\mathrm{e}^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}+\frac{8bc\ln(F)\mathrm{e}^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}+\frac{4e\left(b^2c^2\ln(F)^2+3e^2\right)\mathrm{e}^{bcx\ln(F)}}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{4e\left(11b^2c\right.}{9e}\right.}{}$

```
[In] int(F^(c*(b*x+a))*sin(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*((ln(F)^2*b^2*c^2*e+e^3)*cos(3*e*x+3*d)-1/3*b*c*ln(F)*(e^2+b^2*c^2*ln(F)^
2)*sin(3*e*x+3*d)+(9*e^2+b^2*c^2*ln(F)^2)*(b*c*ln(F)*sin(e*x+d)-e*cos(e*x+d
)))*F^(c*(b*x+a))/(4*b^4*c^4*ln(F)^4+40*b^2*c^2*e^2*ln(F)^2+36*e^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

$$\int F^{c(a+bx)} \sin^3(d+ex)\,dx = \frac{\left(3\,e^3\cos\left(ex+d\right)^3 - 9\,e^3\cos\left(ex+d\right) + 3\left(b^2c^2e\cos\left(ex+d\right)^3 - b^2c^2e\cos\left(ex+d\right)\right)\log\left(F\right)^2 - \left(\left(b^3c^3\cos\left(ex\right.\right.\right.}{b^4c^4\log\left(F\right)^4 + 10\,b^2c^2e^2\log\left(F\right)}$$

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] (3*e^3*cos(e*x + d)^3 - 9*e^3*cos(e*x + d) + 3*(b^2*c^2*e*cos(e*x + d)^3 -
b^2*c^2*e*cos(e*x + d))*log(F)^2 - ((b^3*c^3*cos(e*x + d)^2 - b^3*c^3)*log(
F)^3 + (b*c*e^2*cos(e*x + d)^2 - 7*b*c*e^2)*log(F))*sin(e*x + d))*F^(b*c*x
+ a*c)/(b^4*c^4*log(F)^4 + 10*b^2*c^2*e^2*log(F)^2 + 9*e^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.30 (sec) , antiderivative size = 1681, normalized size of antiderivative = 8.45

$$\int F^{c(a+bx)} \sin^3(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F**(c*(b*x+a))*sin(e*x+d)**3,x)
```

```
[Out] Piecewise((x*sin(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sin(d)**3, Eq(b,
0) & Eq(e, 0)), (x*sin(d)**3, Eq(c, 0) & Eq(e, 0)), (-3*F**(a*c + b*c*x)*x*
sin(I*b*c*x*log(F) - d)**3/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F) -
d)**2*cos(I*b*c*x*log(F) - d)/8 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F) -
 d)*cos(I*b*c*x*log(F) - d)**2/8 + 3*I*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F
) - d)**3/8 + F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)**3/(8*b*c*log(F)) -
3*I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)**2*cos(I*b*c*x*log(F) - d)/(4*
b*c*log(F)) - 3*I*F**(a*c + b*c*x)*cos(I*b*c*x*log(F) - d)**3/(8*b*c*log(F)
), Eq(e, -I*b*c*log(F))), (-F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 - d)**3
/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 - d)**2*cos(I*b*c*x*log(F)
/3 - d)/8 + 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 - d)*cos(I*b*c*x*log(
F)/3 - d)**2/8 - I*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/3 - d)**3/8 + F**(
a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)) - 3*I*F**(a*c + b*
c*x)*sin(I*b*c*x*log(F)/3 - d)**2*cos(I*b*c*x*log(F)/3 - d)/(b*c*log(F)) -
15*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)*cos(I*b*c*x*log(F)/3 - d)**2/
(4*b*c*log(F)) + 11*I*F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/3 - d)**3/(8*b*c*
log(F)), Eq(e, -I*b*c*log(F)/3)), (F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3
```

```
+ d)**3/8 - 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 + d)**2*cos(I*b*c*x
*log(F)/3 + d)/8 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 + d)*cos(I*b*c
*x*log(F)/3 + d)**2/8 + I*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/3 + d)**3/8
 - F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 + d)**3/(8*b*c*log(F)) + 3*I*F**(a
*c + b*c*x)*sin(I*b*c*x*log(F)/3 + d)**2*cos(I*b*c*x*log(F)/3 + d)/(b*c*log
(F)) + 15*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 + d)*cos(I*b*c*x*log(F)/3 +
 d)**2/(4*b*c*log(F)) - 11*I*F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/3 + d)**3/
(8*b*c*log(F)), Eq(e, I*b*c*log(F)/3)), (3*F**(a*c + b*c*x)*x*sin(I*b*c*x*l
og(F) + d)**3/8 - 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F) + d)**2*cos(I*b
*c*x*log(F) + d)/8 + 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F) + d)*cos(I*b*c
*x*log(F) + d)**2/8 - 3*I*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F) + d)**3/8 -
 F**(a*c + b*c*x)*sin(I*b*c*x*log(F) + d)**3/(8*b*c*log(F)) + 3*I*F**(a*c +
 b*c*x)*sin(I*b*c*x*log(F) + d)**2*cos(I*b*c*x*log(F) + d)/(4*b*c*log(F)) +
 3*I*F**(a*c + b*c*x)*cos(I*b*c*x*log(F) + d)**3/(8*b*c*log(F)), Eq(e, I*b*
c*log(F))), (F**(a*c + b*c*x)*b**3*c**3*log(F)**3*sin(d + e*x)**3/(b**4*c**
4*log(F)**4 + 10*b**2*c**2*e**2*log(F)**2 + 9*e**4) - 3*F**(a*c + b*c*x)*b*
*2*c**2*e*log(F)**2*sin(d + e*x)**2*cos(d + e*x)/(b**4*c**4*log(F)**4 + 10*
b**2*c**2*e**2*log(F)**2 + 9*e**4) + 7*F**(a*c + b*c*x)*b*c*e**2*log(F)*sin
(d + e*x)**3/(b**4*c**4*log(F)**4 + 10*b**2*c**2*e**2*log(F)**2 + 9*e**4) +
 6*F**(a*c + b*c*x)*b*c*e**2*log(F)*sin(d + e*x)*cos(d + e*x)**2/(b**4*c**4
*log(F)**4 + 10*b**2*c**2*e**2*log(F)**2 + 9*e**4) - 9*F**(a*c + b*c*x)*e**
3*sin(d + e*x)**2*cos(d + e*x)/(b**4*c**4*log(F)**4 + 10*b**2*c**2*e**2*log
(F)**2 + 9*e**4) - 6*F**(a*c + b*c*x)*e**3*cos(d + e*x)**3/(b**4*c**4*log(F
)**4 + 10*b**2*c**2*e**2*log(F)**2 + 9*e**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(199) = 398$.

Time = 0.27 (sec) , antiderivative size = 813, normalized size of antiderivative = 4.09

$$\int F^{c(a+bx)}\sin^3(d+ex)\,dx =$$

$$-\frac{\left(F^{ac}b^3c^3\log\left(F\right)^3\sin\left(3\,d\right)-3\,F^{ac}b^2c^2e\cos\left(3\,d\right)\log\left(F\right)^2+F^{ac}bce^2\log\left(F\right)\sin\left(3\,d\right)-3\,F^{ac}e^3\cos\left(3\,d\right)\right)\ldots}{}$$

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) - 3*F^(a*c)*b^2*c^2*e*cos(3*d)*log
(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(3*d) - 3*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)
*cos(3*e*x) - (F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + 3*F^(a*c)*b^2*c^2*e*cos(
3*d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 3*F^(a*c)*e^3*cos(3*d))*F
^(b*c*x)*cos(3*e*x + 6*d) + 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + F^(a*c)*
b^2*c^2*e*cos(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 9*F^(a*c)
*e^3*cos(3*d))*F^(b*c*x)*cos(e*x + 4*d) - 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3
```

```
*d) - F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*
d) - 9*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*cos(e*x - 2*d) + (F^(a*c)*b^3*c^3*co
s(3*d)*log(F)^3 + 3*F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*d) + F^(a*c)*b*c*e^2*c
os(3*d)*log(F) + 3*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*sin(3*e*x) + (F^(a*c)*b^
3*c^3*cos(3*d)*log(F)^3 - 3*F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*d) + F^(a*c)*b
*c*e^2*cos(3*d)*log(F) - 3*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*sin(3*e*x + 6*d)
 - 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*
d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 9*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*
sin(e*x + 4*d) - 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 + F^(a*c)*b^2*c^2*e*l
og(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F) + 9*F^(a*c)*e^3*sin(3*
d))*F^(b*c*x)*sin(e*x - 2*d))/(b^4*c^4*cos(3*d)^2*log(F)^4 + b^4*c^4*log(F)
^4*sin(3*d)^2 + 9*(cos(3*d)^2 + sin(3*d)^2)*e^4 + 10*(b^2*c^2*cos(3*d)^2*lo
g(F)^2 + b^2*c^2*log(F)^2*sin(3*d)^2)*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1275, normalized size of antiderivative = 6.41

$$\int F^{c(a+bx)} \sin^3(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c
*sgn(F) - 1/2*pi*a*c + 3*e*x + 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(
F) - pi*b*c + 6*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 6*e)*cos(1/2*pi*b*c*x*sgn
(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 3*e*x + 3*d)/(4*b^2*c
^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 6*e)^2))*e^(b*c*x*log(abs(F))
+ a*c*log(abs(F))) + 3/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*p
i*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^
2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 2*e)*cos(
1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x +
 d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*
log(abs(F)) + a*c*log(abs(F))) - 3/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sg
n(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*
log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c
 - 2*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi
*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2
))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*(2*b*c*log(abs(F))*sin(1/2
*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 3*e*x -
3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 6*e)^2) - (pi*b*c
*sgn(F) - pi*b*c - 6*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c
*sgn(F) - 1/2*pi*a*c - 3*e*x - 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(
F) - pi*b*c - 6*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - (I*e^(1/2*
```

```
I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + 3
*I*e*x + 3*I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*I
*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) +
1/2*I*pi*a*c - 3*I*e*x - 3*I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*l
og(abs(F)) - 48*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 3*(-I*e^(1/
2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c +
 I*e*x + I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*I*e
) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/
2*I*pi*a*c - I*e*x - I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs
(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 3*(I*e^(1/2*I*pi*
b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - I*e*x
- I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 16*I*e) + I*e
^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*
a*c + I*e*x + I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) +
16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - (-I*e^(1/2*I*pi*b*c*x*sg
n(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - 3*I*e*x - 3*I*
d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 48*I*e) - I*e^(-1
/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c
+ 3*I*e*x + 3*I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) +
48*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 28.75 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.95

$$\begin{aligned}
&\int F^{c(a+bx)} \sin^3(d+ex)\,dx \\
&= -\frac{3\,F^{c\,(a+b\,x)}\,(\cos(e\,x)-\sin(e\,x)\,1\mathrm{i})\,(\cos(d)-\sin(d)\,1\mathrm{i})}{8\,(e+b\,c\,\ln(F)\,1\mathrm{i})} \\
&\quad+\frac{F^{c\,(a+b\,x)}\,(\cos(3\,e\,x)+\sin(3\,e\,x)\,1\mathrm{i})\,(\cos(3\,d)+\sin(3\,d)\,1\mathrm{i})\,1\mathrm{i}}{8\,(b\,c\,\ln(F)+e\,3\mathrm{i})} \\
&\quad+\frac{F^{c\,(a+b\,x)}\,(\cos(3\,e\,x)-\sin(3\,e\,x)\,1\mathrm{i})\,(\cos(3\,d)-\sin(3\,d)\,1\mathrm{i})}{8\,(3\,e+b\,c\,\ln(F)\,1\mathrm{i})} \\
&\quad-\frac{F^{c\,(a+b\,x)}\,(\cos(e\,x)+\sin(e\,x)\,1\mathrm{i})\,(\cos(d)+\sin(d)\,1\mathrm{i})\,3\mathrm{i}}{8\,(b\,c\,\ln(F)+e\,1\mathrm{i})}
\end{aligned}$$

```
[In] int(F^(c*(a + b*x))*sin(d + e*x)^3,x)
```

```
[Out] (F^(c*(a + b*x))*(cos(3*e*x) + sin(3*e*x)*1i)*(cos(3*d) + sin(3*d)*1i)*1i)/
(8*(e*3i + b*c*log(F))) - (3*F^(c*(a + b*x))*(cos(e*x) - sin(e*x)*1i)*(cos(
d) - sin(d)*1i))/(8*(e + b*c*log(F)*1i)) + (F^(c*(a + b*x))*(cos(3*e*x) - s
in(3*e*x)*1i)*(cos(3*d) - sin(3*d)*1i))/(8*(3*e + b*c*log(F)*1i)) - (F^(c*(
a + b*x))*(cos(e*x) + sin(e*x)*1i)*(cos(d) + sin(d)*1i)*3i)/(8*(e*1i + b*c*
log(F)))
```

3.3 $\int F^{c(a+bx)} \sin^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 128

$$\int F^{c(a+bx)} \sin^2(d+ex)\,dx = \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)} - \frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)} + \frac{bcF^{c(a+bx)}\log(F)\sin^2(d+ex)}{4e^2+b^2c^2\log^2(F)}$$

```
[Out] 2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)-2*e*F^(c*(b*x+a))*cos
(e*x+d)*sin(e*x+d)/(4*e^2+b^2*c^2*ln(F)^2)+b*c*F^(c*(b*x+a))*ln(F)*sin(e*x+
d)^2/(4*e^2+b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4519, 2225}

$$\int F^{c(a+bx)} \sin^2(d+ex)\,dx = \frac{bc\log(F)\sin^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2} - \frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2} + \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}$$

```
[In] Int[F^(c*(a + b*x))*Sin[d + e*x]^2,x]
```

```
[Out] (2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]^2)) - (2*e*F^(c
*(a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log[F]^2) + (b*c*F^
(c*(a + b*x))*Log[F]*Sin[d + e*x]^2)/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4519

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d +
 e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)} \\ &\quad+\frac{bcF^{c(a+bx)}\log(F)\sin^2(d+ex)}{4e^2+b^2c^2\log^2(F)}+\frac{(2e^2)\int F^{c(a+bx)}\,dx}{4e^2+b^2c^2\log^2(F)} \\ &= \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}-\frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)} \\ &\quad+\frac{bcF^{c(a+bx)}\log(F)\sin^2(d+ex)}{4e^2+b^2c^2\log^2(F)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx = \frac{F^{c(a+bx)}\left(4e^2+b^2c^2\log^2(F)-b^2c^2\cos(2(d+ex))\log^2(F)-2bce\log(F)\sin(2(d+ex))\right)}{8bce^2\log(F)+2b^3c^3\log^3(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Sin[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(4*e^2 + b^2*c^2*Log[F]^2 - b^2*c^2*Cos[2*(d + e*x)]*Log[F
]^2 - 2*b*c*e*Log[F]*Sin[2*(d + e*x)]))/(8*b*c*e^2*Log[F] + 2*b^3*c^3*Log[F
]^3)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

method	result
parallelrisch	$-\frac{F^{c(xb+a)}\left(\frac{b^{2}c^{2}\ln(F)^{2}\cos(2ex+2d)}{2}-\frac{b^{2}c^{2}\ln(F)^{2}}{2}+e\sin(2ex+2d)bc\ln(F)-2e^{2}\right)}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}$
risch	$\frac{F^{c(xb+a)}}{2bc\ln(F)}-\frac{F^{c(xb+a)}bc\ln(F)\cos(2ex+2d)}{2\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}-\frac{e\,F^{c(xb+a)}\sin(2ex+2d)}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}$
norman	$\frac{-\frac{4e\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}+\frac{4e\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{4e^{2}+b^{2}c^{2}\ln(F)^{2}}+\frac{2e^{2}\mathrm{e}^{c(xb+a)\ln(F)}}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}+\frac{2e^{2}\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{4}}{bc\ln(F)\left(4e^{2}+b^{2}c^{2}\ln(F)^{2}\right)}+\frac{4(e^{2}-}{}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}\right)^{2}}$

```
[In] int(F^(c*(b*x+a))*sin(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -F^(c*(b*x+a))*(1/2*b^2*c^2*ln(F)^2*cos(2*e*x+2*d)-1/2*b^2*c^2*ln(F)^2+e*si
n(2*e*x+2*d)*b*c*ln(F)-2*e^2)/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx = -\frac{\left(2\,bce\cos\left(ex+d\right)\log\left(F\right)\sin\left(ex+d\right)+\left(b^2c^2\cos\left(ex+d\right)^2-b^2c^2\right)\log\left(F\right)^2-2\,e^2\right)F^{bcx+ac}}{b^3c^3\log\left(F\right)^3+4\,bce^2\log\left(F\right)}$$

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*c*e*cos(e*x + d)*log(F)*sin(e*x + d) + (b^2*c^2*cos(e*x + d)^2 - b^2*
c^2)*log(F)^2 - 2*e^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + 4*b*c*e^2*log(F)
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 741, normalized size of antiderivative = 5.79

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx$$

$$= \begin{cases} x\sin^2(d) \\ \frac{x\sin^2(d+ex)}{2}+\frac{x\cos^2(d+ex)}{2}-\frac{\sin(d+ex)\cos(d+ex)}{2e} \\ F^{ac}\left(\frac{x\sin^2(d+ex)}{2}+\frac{x\cos^2(d+ex)}{2}-\frac{\sin(d+ex)\cos(d+ex)}{2e}\right) \\ \frac{x\sin^2(d+ex)}{2}+\frac{x\cos^2(d+ex)}{2}-\frac{\sin(d+ex)\cos(d+ex)}{2e} \\ \frac{F^{ac+bcx}x\sin^2\left(\frac{ibcx\log(F)}{2}-d\right)}{4}-\frac{iF^{ac+bcx}x\sin\left(\frac{ibcx\log(F)}{2}-d\right)\cos\left(\frac{ibcx\log(F)}{2}-d\right)}{2}-\frac{F^{ac+bcx}x\cos^2\left(\frac{ibcx\log(F)}{2}-d\right)}{4}+\frac{3iF^{ac+bcx}\sin}{} \\ \frac{F^{ac+bcx}x\sin^2\left(\frac{ibcx\log(F)}{2}+d\right)}{4}-\frac{iF^{ac+bcx}x\sin\left(\frac{ibcx\log(F)}{2}+d\right)\cos\left(\frac{ibcx\log(F)}{2}+d\right)}{2}-\frac{F^{ac+bcx}x\cos^2\left(\frac{ibcx\log(F)}{2}+d\right)}{4}+\frac{F^{ac+bcx}\sin^2}{bc} \\ \frac{F^{ac+bcx}b^2c^2\log(F)^2\sin^2(d+ex)}{b^3c^3\log(F)^3+4bce^2\log(F)}-\frac{2F^{ac+bcx}bce\log(F)\sin(d+ex)\cos(d+ex)}{b^3c^3\log(F)^3+4bce^2\log(F)}+\frac{2F^{ac+bcx}e^2\sin^2(d+ex)}{b^3c^3\log(F)^3+4bce^2\log(F)}+\frac{2F^{ac+bcx}e^2\cos^2(d+e}{b^3c^3\log(F)^3+4bce^2\log} \end{cases}$$

```
[In] integrate(F**(c*(b*x+a))*sin(e*x+d)**2,x)
```

```
[Out] Piecewise((x*sin(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (x*sin(
d + e*x)**2/2 + x*cos(d + e*x)**2/2 - sin(d + e*x)*cos(d + e*x)/(2*e), Eq(F
, 1)), (F**(a*c)*(x*sin(d + e*x)**2/2 + x*cos(d + e*x)**2/2 - sin(d + e*x)*
cos(d + e*x)/(2*e)), Eq(b, 0)), (x*sin(d + e*x)**2/2 + x*cos(d + e*x)**2/2
- sin(d + e*x)*cos(d + e*x)/(2*e), Eq(c, 0)), (F**(a*c + b*c*x)*x*sin(I*b*c
*x*log(F)/2 - d)**2/4 - I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 - d)*cos(
I*b*c*x*log(F)/2 - d)/2 - F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 - d)**2/4
 + 3*I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 - d)*cos(I*b*c*x*log(F)/2 - d)
/(2*b*c*log(F)) + F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/2 - d)**2/(b*c*log(F)
), Eq(e, -I*b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 + d)**
2/4 - I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 + d)*cos(I*b*c*x*log(F)/2 +
 d)/2 - F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 + d)**2/4 + F**(a*c + b*c*x
)*sin(I*b*c*x*log(F)/2 + d)**2/(b*c*log(F)) - I*F**(a*c + b*c*x)*sin(I*b*c*
x*log(F)/2 + d)*cos(I*b*c*x*log(F)/2 + d)/(2*b*c*log(F)), Eq(e, I*b*c*log(F
)/2)), (F**(a*c + b*c*x)*b**2*c**2*log(F)**2*sin(d + e*x)**2/(b**3*c**3*log
(F)**3 + 4*b*c*e**2*log(F)) - 2*F**(a*c + b*c*x)*b*c*e*log(F)*sin(d + e*x)*
cos(d + e*x)/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a*c + b*c*x)
*e**2*sin(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a*c
 + b*c*x)*e**2*cos(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)), T
rue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(128) = 256.

Time = 0.22 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.78

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx = -\frac{\left(F^{ac}b^2c^2\cos(2d)\log(F)^2 + 2F^{ac}bce\log(F)\sin(2d)\right)F^{bcx}\cos(2ex) + \left(F^{ac}b^2c^2\cos(2d)\log(F)^2 - 2F^{a}\right.}{}$$

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))
*F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c*
e*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2*s
in(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a*c)*
b^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(
2*e*x + 4*d) - 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c^2*log
(F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*F^(b*c*
x))/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 + 4*(b*c*cos
(2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 915, normalized size of antiderivative = 7.15

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/
2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(
F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*sin(1/2*pi*b*c*x*sgn
(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)/(4*b^2*c
^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(abs(F))
+ a*c*log(abs(F))) - 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/
2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(
F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 4*e)*
sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2
*e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2))*e
^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*cos(-1/2*pi*b*c*x*sgn(F) +
1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(a
```

```
bs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*
pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2
*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F))) + I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c
*sgn(F) - 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c +
 8*b*c*log(abs(F)) + 16*I*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x
 - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(-4*I*pi*b*c*sgn(F
) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F))) + I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c
*sgn(F) - 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c +
 8*b*c*log(abs(F)) - 16*I*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x
 - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(-4*I*pi*b*c*sgn(F
) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)))
 - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2
*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*
log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 29.59 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx = \frac{F^{a\,c+b\,c\,x}\,\left(2\,e^2+\frac{b^2\,c^2\,\ln(F)^2}{2}-\frac{b^2\,c^2\,\ln(F)^2\,\cos(2\,d+2\,e\,x)}{2}-b\,c\,e\,\ln\left(F\right)\,\sin\left(2\,d+2\,e\,x\right)\right)}{b\,c\,\ln\left(F\right)\,\left(b^2\,c^2\,\ln\left(F\right)^2+4\,e^2\right)}$$

```
[In] int(F^(c*(a + b*x))*sin(d + e*x)^2,x)
```

```
[Out] (F^(a*c + b*c*x)*(2*e^2 + (b^2*c^2*log(F)^2)/2 - (b^2*c^2*log(F)^2*cos(2*d
+ 2*e*x))/2 - b*c*e*log(F)*sin(2*d + 2*e*x)))/(b*c*log(F)*(4*e^2 + b^2*c^2*
log(F)^2))
```

3.4 $\int F^{c(a+bx)} \sin(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 73

$$\int F^{c(a+bx)} \sin(d+ex)\,dx = -\frac{eF^{c(a+bx)}\cos(d+ex)}{e^2+b^2c^2\log^2(F)} + \frac{bcF^{c(a+bx)}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

```
[Out] -e*F^(c*(b*x+a))*cos(e*x+d)/(e^2+b^2*c^2*ln(F)^2)+b*c*F^(c*(b*x+a))*ln(F)*s
in(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4517}

$$\int F^{c(a+bx)} \sin(d+ex)\,dx = \frac{bc\log(F)\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2} - \frac{e\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}$$

```
[In] Int[F^(c*(a + b*x))*Sin[d + e*x],x]
```

```
[Out] -((e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)) + (b*c*F^(c*(a
 + b*x))*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{eF^{c(a+bx)}\cos(d+ex)}{e^2+b^2c^2\log^2(F)} + \frac{bcF^{c(a+bx)}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int F^{c(a+bx)}\sin(d+ex)\,dx=\frac{F^{c(a+bx)}(-e\cos(d+ex)+bc\log(F)\sin(d+ex))}{e^2+b^2c^2\log^2(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Sin[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Cos[d + e*x]) + b*c*Log[F]*Sin[d + e*x]))/(e^2 + b^2*
c^2*Log[F]^2)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{F^{c(xb+a)}(bc\ln(F)\sin(ex+d)-e\cos(ex+d))}{e^2+b^2c^2\ln(F)^2}$	49
risch	$-\frac{e\,F^{c(xb+a)}\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{bc\,F^{c(xb+a)}\ln(F)\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}$	74
norman	$\frac{\frac{e\,e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}-\frac{e\,e^{c(xb+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2bc\ln(F)e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$	130

```
[In] int(F^(c*(b*x+a))*sin(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] F^(c*(b*x+a))*(b*c*ln(F)*sin(e*x+d)-e*cos(e*x+d))/(e^2+b^2*c^2*ln(F)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)}\sin(d+ex)\,dx=\frac{(bc\log(F)\sin(ex+d)-e\cos(ex+d))F^{bcx+ac}}{b^2c^2\log(F)^2+e^2}$$

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] (b*c*log(F)*sin(e*x + d) - e*cos(e*x + d))*F^(b*c*x + a*c)/(b^2*c^2*log(F)^
2 + e^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 347, normalized size of antiderivative = 4.75

$$\int F^{c(a+bx)}\sin(d+ex)\,dx$$

$$= \begin{cases} x\sin(d) & \text{for} \\ F^{ac}x\sin(d) & \text{for} \\ x\sin(d) & \text{for} \\ -\frac{F^{ac+bcx}x\sin(ibcx\log(F)-d)}{2} + \frac{iF^{ac+bcx}x\cos(ibcx\log(F)-d)}{2} + \frac{F^{ac+bcx}\sin(ibcx\log(F)-d)}{2bc\log(F)} - \frac{iF^{ac+bcx}\cos(ibcx\log(F)-d)}{bc\log(F)} & \text{for} \\ \frac{F^{ac+bcx}x\sin(ibcx\log(F)+d)}{2} - \frac{iF^{ac+bcx}x\cos(ibcx\log(F)+d)}{2} - \frac{F^{ac+bcx}\sin(ibcx\log(F)+d)}{2bc\log(F)} + \frac{iF^{ac+bcx}\cos(ibcx\log(F)+d)}{bc\log(F)} & \text{for} \\ \frac{F^{ac+bcx}bc\log(F)\sin(d+ex)}{b^2c^2\log(F)^2+e^2} - \frac{F^{ac+bcx}e\cos(d+ex)}{b^2c^2\log(F)^2+e^2} & \text{oth} \end{cases}$$

```
[In] integrate(F**(c*(b*x+a))*sin(e*x+d),x)
```

```
[Out] Piecewise((x*sin(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sin(d), Eq(b, 0) & E
q(e, 0)), (x*sin(d), Eq(c, 0) & Eq(e, 0)), (-F**(a*c + b*c*x)*x*sin(I*b*c*x
*log(F) - d)/2 + I*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F) - d)/2 + F**(a*c +
 b*c*x)*sin(I*b*c*x*log(F) - d)/(2*b*c*log(F)) - I*F**(a*c + b*c*x)*cos(I*b
*c*x*log(F) - d)/(b*c*log(F)), Eq(e, -I*b*c*log(F))), (F**(a*c + b*c*x)*x*s
in(I*b*c*x*log(F) + d)/2 - I*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F) + d)/2 -
 F**(a*c + b*c*x)*sin(I*b*c*x*log(F) + d)/(2*b*c*log(F)) + I*F**(a*c + b*c*
x)*cos(I*b*c*x*log(F) + d)/(b*c*log(F)), Eq(e, I*b*c*log(F))), (F**(a*c + b
*c*x)*b*c*log(F)*sin(d + e*x)/(b**2*c**2*log(F)**2 + e**2) - F**(a*c + b*c*
x)*e*cos(d + e*x)/(b**2*c**2*log(F)**2 + e**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(73) = 146.

Time = 0.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.66

$$\int F^{c(a+bx)}\sin(d+ex)\,dx =$$

$$-\frac{(F^{ac}bc\log(F)\sin(d) + F^{ac}e\cos(d))F^{bcx}\cos(ex+2d) - (F^{ac}bc\log(F)\sin(d) - F^{ac}e\cos(d))F^{bcx}\cos(e}{2\left(b^2c^2\cos(d)^2\log(F)^2 + b^2c^2\log}$$

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2*
d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) - (F
^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^
(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))/(b^2*c^2*co
s(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 634, normalized size of antiderivative = 8.68

$$\int F^{c(a+bx)}\sin(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="giac")
```

```
[Out] (2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(
F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b
*c + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 2*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2
*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F)
)^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs
(F))) - (2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*
a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c - 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c - 2*e)*cos(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*lo
g(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c
*log(abs(F))) - (-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*
c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*
b*c*log(abs(F)) + 4*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1
/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*
pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))
) - (I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F
)) - 4*I*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*s
gn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*
c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 28.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int F^{c(a+bx)}\sin(d+ex)\,dx = -\frac{F^{a\,c+b\,c\,x}\,\left(e\,\cos\left(d+e\,x\right)-b\,c\,\sin\left(d+e\,x\right)\,\ln\left(F\right)\right)}{b^2\,c^2\,{\ln\left(F\right)}^2+e^2}$$

```
[In] int(F^(c*(a + b*x))*sin(d + e*x),x)
```

```
[Out] -(F^(a*c + b*c*x)*(e*cos(d + e*x) - b*c*sin(d + e*x)*log(F)))/(e^2 + b^2*c^
2*log(F)^2)
```

3.5 $\int F^{c(a+bx)} \csc(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int F^{c(a+bx)}\csc(d+ex)\,dx = -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

```
[Out] -2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e],[3/2-1
/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))/(e-I*b*c*ln(F))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4538}

$$\int F^{c(a+bx)}\csc(d+ex)\,dx = -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x],x]
```

```
[Out] (-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])
/(2*e), (3 - (I*b*c*Log[F])/e)/2, E^((2*I)*(d + e*x))])/(e - I*b*c*Log[F])
```

Rule 4538

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
```

```
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\text{integral} = -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int F^{c(a+bx)}\csc(d+ex)\,dx$$
$$= \frac{iF^{c(a+bx)}\left(\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{e},1-\frac{ibc\log(F)}{e},-\cos(d+ex)-i\sin(d+ex)\right)-\text{Hypergeomet}\right.}{bc\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x],x]
[Out] (I*F^(c*(a + b*x))*(Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*Lo
g[F])/e, -Cos[d + e*x] - I*Sin[d + e*x]] - Hypergeometric2F1[1, ((-I)*b*c*L
og[F])/e, 1 - (I*b*c*Log[F])/e, Cos[d + e*x] + I*Sin[d + e*x]]))/(b*c*Log[F
])
```

Maple [F]

$$\int F^{c(xb+a)}\csc(ex+d)\,dx$$

```
[In] int(F^(c*(b*x+a))*csc(e*x+d),x)
[Out] int(F^(c*(b*x+a))*csc(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\csc(d+ex)\,dx = \int F^{(bx+a)c}\csc(ex+d)\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*csc(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\csc(d+ex)\,dx = \int F^{c(a+bx)}\csc\left(d+ex\right)dx$$

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))*csc(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\csc(d+ex)\,dx = \int F^{(bx+a)c}\csc\left(ex+d\right)\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*(F^(b*c*x)*F^(a*c)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*F^(a*c)*e*cos(e*x
+ d) - (F^(b*c*x)*F^(a*c)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*F^(a*c)*e*cos
(e*x + d))*cos(2*e*x + 2*d) - 2*(F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3 +
 (F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*cos(2*e*x + 2*d)^2 + (F^(a*c)*b
^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*sin(2*e*x + 2*d)^2 - 2*(F^(a*c)*b^2*c^2*e*
log(F)^2 + F^(a*c)*e^3)*cos(2*e*x + 2*d))*integrate((F^(b*c*x)*b*c*cos(e*x
+ d)*log(F) - F^(b*c*x)*e*sin(e*x + d) + (F^(b*c*x)*b*c*cos(e*x + d)*log(F)
 - F^(b*c*x)*e*sin(e*x + d))*cos(4*e*x + 4*d) - 2*(F^(b*c*x)*b*c*cos(e*x +
d)*log(F) - F^(b*c*x)*e*sin(e*x + d))*cos(2*e*x + 2*d) + (F^(b*c*x)*b*c*log
(F)*sin(e*x + d) + F^(b*c*x)*e*cos(e*x + d))*sin(4*e*x + 4*d) - 2*(F^(b*c*x
)*b*c*log(F)*sin(e*x + d) + F^(b*c*x)*e*cos(e*x + d))*sin(2*e*x + 2*d))/(b^
2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^2)*cos(4*e*x + 4*d)^2 + 4*(b^2*c^2*l
og(F)^2 + e^2)*cos(2*e*x + 2*d)^2 + (b^2*c^2*log(F)^2 + e^2)*sin(4*e*x + 4*
d)^2 - 4*(b^2*c^2*log(F)^2 + e^2)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(b^
2*c^2*log(F)^2 + e^2)*sin(2*e*x + 2*d)^2 + e^2 + 2*(b^2*c^2*log(F)^2 + e^2
- 2*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) - 4*(b^2*c^
2*log(F)^2 + e^2)*cos(2*e*x + 2*d)), x) + (F^(b*c*x)*F^(a*c)*b*c*cos(e*x +
d)*log(F) - F^(b*c*x)*F^(a*c)*e*sin(e*x + d))*sin(2*e*x + 2*d))/(b^2*c^2*lo
g(F)^2 + (b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)^2 + (b^2*c^2*log(F)^2 +
e^2)*sin(2*e*x + 2*d)^2 + e^2 - 2*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)
)
```

Giac [F]

$$\int F^{c(a+bx)}\csc(d+ex)\,dx = \int F^{(bx+a)c}\csc\left(ex+d\right)\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\csc(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)}\,dx$$

```
[In] int(F^(c*(a + b*x))/sin(d + e*x),x)
[Out] int(F^(c*(a + b*x))/sin(d + e*x), x)
```

3.6 $\int F^{c(a+bx)} \csc^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int F^{c(a+bx)} \csc^2(d+ex)\,dx = -\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

```
[Out] -4*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c*ln(F)/e],[2-1/2
*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))/(2*I*e+b*c*ln(F))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4538}

$$\int F^{c(a+bx)} \csc^2(d+ex)\,dx = -\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x]^2,x]
```

```
[Out] (-4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])/((2*I)*e + b*c*
Log[F])
```

Rule 4538

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
```

```
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\text{integral} = -\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.29

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx = -\frac{2iF^{c(a+bx)}\left(\left(-1+e^{2id}\right)\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)+\csc(d+ex)\sin(d)(\cdots}{e\left(-1+e^{2id}\right)}$$

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x]^2,x]
[Out] ((-2*I)*F^(c*(a + b*x))*((-1 + E^((2*I)*d))*Hypergeometric2F1[1, ((-1/2*I)*
b*c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))] + Csc[d + e*x
]*Sin[d]*(Cos[e*x] - I*Sin[e*x])))/(e*(-1 + E^((2*I)*d)))
```

Maple [F]

$$\int F^{c(xb+a)}\csc(ex+d)^2\,dx$$

```
[In] int(F^(c*(b*x+a))*csc(e*x+d)^2,x)
[Out] int(F^(c*(b*x+a))*csc(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx = \int F^{(bx+a)c}\csc(ex+d)^2\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*csc(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \csc^2(d+ex)\,dx = \int F^{c(a+bx)} \csc^2{(d+ex)}\,dx$$

[In] integrate(F**(c*(b*x+a))*csc(e*x+d)**2,x)

[Out] Integral(F**(c*(a + b*x))*csc(d + e*x)**2, x)

Maxima [F]

$$\int F^{c(a+bx)} \csc^2(d+ex)\,dx = \int F^{(bx+a)c} \csc{(ex+d)}^2\,dx$$

[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="maxima")

[Out] 4*(24*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^3*c^3*log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 - (F^(a*c)*b^3*c^3*log(F)^3 + 64*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 2*(5*F^(a*c)*b^2*c^2*e*log(F)^2 - 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x + 2*d) + (24*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) - (F^(a*c)*b^3*c^3*log(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 2*(F^(a*c)*b^2*c^2*e*log(F)^2 + 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x + 2*d))*cos(4*e*x + 4*d) + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(4*e*x + 4*d)^2 + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(2*e*x + 2*d)^2 + (F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(4*e*x + 4*d)^2 - 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F) - 2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) - 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(2*e*x + 2*d))*integrate((6*F^(b*c*x)*b*c*e*cos(6*e*x + 6*d)*log(F) - 18*F^(b*c*x)*b*c*e*cos(4*e*x + 4*d)*log(F) + 18*F^(b*c*x)*b*c*e*cos(2*e*x + 2*d)*log(F) - 6*F^(b*c*x)*b*c*e*log(F) - (b^2*c^2*log(F)^2 - 8*e^2)*F^(b*c*x)*sin(6*e*x + 6*d) + 3*(b^2*c^2*log(F)^2 - 8*e^2)*F^(b*c*x)*sin(4*e*x + 4*d) - 3*(b^2*c^2*log(F)^2 - 8*e^2)*F^(b*c*x)*sin(2*e*x + 2*d))/(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4 + (b^4*c^4*log(F)^4 +

```
 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(6*e*x + 6*d)^2 + 9*(b^4*c^4*log(F)^4
 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(4*e*x + 4*d)^2 + 9*(b^4*c^4*log(F)
^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*e*x + 2*d)^2 + (b^4*c^4*log(F)
^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(6*e*x + 6*d)^2 + 9*(b^4*c^4*log(
F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(4*e*x + 4*d)^2 - 18*(b^4*c^4*l
og(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(4*e*x + 4*d)*sin(2*e*x + 2*
d) + 9*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(2*e*x + 2*
d)^2 - 2*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4 + 3*(b^4*c^4*
log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(4*e*x + 4*d) - 3*(b^4*c^4*
log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*e*x + 2*d))*cos(6*e*x +
6*d) + 6*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4 - 3*(b^4*c^4*
log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*e*x + 2*d))*cos(4*e*x +
4*d) - 6*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*e*x +
2*d) - 6*((b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(4*e*x +
 4*d) - (b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(2*e*x + 2
*d))*sin(6*e*x + 6*d)), x) - (2*(F^(a*c)*b^2*c^2*e*log(F)^2 + 16*F^(a*c)*e^
3)*F^(b*c*x)*cos(2*e*x + 2*d) + (F^(a*c)*b^3*c^3*log(F)^3 + 16*F^(a*c)*b*c*
e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) + 4*(F^(a*c)*b^2*c^2*e*log(F)^2 - 8*
F^(a*c)*e^3)*F^(b*c*x))*sin(4*e*x + 4*d))/(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^
2*log(F)^2 + 64*e^4 + (b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)
*cos(4*e*x + 4*d)^2 + 4*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^
4)*cos(2*e*x + 2*d)^2 + (b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^
4)*sin(4*e*x + 4*d)^2 - 4*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*
e^4)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e
^2*log(F)^2 + 64*e^4)*sin(2*e*x + 2*d)^2 + 2*(b^4*c^4*log(F)^4 + 20*b^2*c^2
*e^2*log(F)^2 + 64*e^4 - 2*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64
*e^4)*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) - 4*(b^4*c^4*log(F)^4 + 20*b^2*c^2
*e^2*log(F)^2 + 64*e^4)*cos(2*e*x + 2*d))
```

Giac [F]

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx = \int F^{(bx+a)c}\csc\left(ex+d\right)^2\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)^2}\,dx$$

```
[In] int(F^(c*(a + b*x))/sin(d + e*x)^2,x)
[Out] int(F^(c*(a + b*x))/sin(d + e*x)^2, x)
```

3.7 $\int F^{c(a+bx)} \csc^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 137

$$\int F^{c(a+bx)} \csc^3(d+ex)\,dx = -\frac{F^{c(a+bx)} \cot(d+ex)\csc(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\csc(d+ex)\log(F)}{2e^2} - \frac{e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)(e+ibc\log(F))}{e^2}$$

```
[Out] -1/2*F^(c*(b*x+a))*cot(e*x+d)*csc(e*x+d)/e-1/2*b*c*F^(c*(b*x+a))*csc(e*x+d)
*ln(F)/e^2-exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e
],[3/2-1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))*(e+I*b*c*ln(F))/e^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4534, 4538}

$$\int F^{c(a+bx)} \csc^3(d+ex)\,dx = -\frac{e^{i(d+ex)}F^{c(a+bx)}(e+ibc\log(F))\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{e^2} - \frac{bc\log(F)\csc(d+ex)F^{c(a+bx)}}{2e^2} - \frac{\cot(d+ex)\csc(d+ex)F^{c(a+bx)}}{2e}$$

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x]^3,x]
```

```
[Out] -1/2*(F^(c*(a + b*x))*Cot[d + e*x]*Csc[d + e*x])/e - (b*c*F^(c*(a + b*x))*C
sc[d + e*x]*Log[F])/(2*e^2) - (E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometr
ic2F1[1, (e - I*b*c*Log[F])/(2*e), (3 - (I*b*c*Log[F])/e)/2, E^((2*I)*(d +
e*x))]*(e + I*b*c*Log[F]))/e^2
```

Rule 4534

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x] - Simp[F^(c*(a + b*x
))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4538

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{F^{c(a+bx)}\cot(d+ex)\csc(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\csc(d+ex)\log(F)}{2e^2} \\
&\quad + \frac{1}{2}\left(1+\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\csc(d+ex)\,dx \\
&= -\frac{F^{c(a+bx)}\cot(d+ex)\csc(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\csc(d+ex)\log(F)}{2e^2} \\
&\quad - \frac{e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)(e+ibc\log(F))}{e^2}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 334 vs. 2(137) = 274.

Time = 8.17 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.44

$$
\int F^{c(a+bx)}\csc^3(d+ex)\,dx
$$

$$
= \frac{F^{c(a+bx)}\left(-e\csc^2\left(\frac{1}{2}(d+ex)\right)-4bc\csc(d)\log(F)+\csc(d)\left(\frac{4e^2}{bc\log(F)}+4bc\log(F)\right)+e\sec^2\left(\frac{1}{2}(d+ex)\right)-\cdots\right.}{}
$$

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Csc[(d + e*x)/2]^2) - 4*b*c*Csc[d]*Log[F] + Csc[d]*((
4*e^2)/(b*c*Log[F]) + 4*b*c*Log[F]) + e*Sec[(d + e*x)/2]^2 - ((4*I)*(e^2 +
b^2*c^2*Log[F]^2)*(1 + Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c
*Log[F])/e, Cos[d + e*x] + I*Sin[d + e*x]]*(-1 + Cos[d] + I*Sin[d])))/(b*c*
Log[F]*(-1 + Cos[d] + I*Sin[d])) - ((4*I)*(e^2 + b^2*c^2*Log[F]^2)*(1 - Hyp
ergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*Log[F])/e, -Cos[d + e*x]
- I*Sin[d + e*x]]*(1 + Cos[d] + I*Sin[d])))/(b*c*Log[F]*(1 + Cos[d] + I*Sin
[d])) + 2*b*c*Csc[d/2]*Csc[(d + e*x)/2]*Log[F]*Sin[(e*x)/2] - 2*b*c*Log[F]*
Sec[d/2]*Sec[(d + e*x)/2]*Sin[(e*x)/2]))/(8*e^2)
```

Maple [F]

$$\int F^{c(xb+a)} \csc(ex+d)^3\, dx$$

```
[In] int(F^(c*(b*x+a))*csc(e*x+d)^3,x)
```

```
[Out] int(F^(c*(b*x+a))*csc(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \csc^3(d+ex)\, dx = \int F^{(bx+a)c} \csc(ex+d)^3\, dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csc(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \csc^3(d+ex)\, dx = \int F^{c(a+bx)} \csc^3(d+ex)\, dx$$

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d)**3,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csc(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx=\int F^{(bx+a)c}\csc{(ex+d)}^3\,dx$$

[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="maxima")

[Out] 8*(48*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F)*sin(e*x + d) - 6*(F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d) - (48*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F)*sin(e*x + d) - 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*cos(3*e*x + 3*d) - 6*(F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d) - (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(3*e*x + 3*d))*cos(6*e*x + 6*d) + 3*(48*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F)*sin(e*x + d) - 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*cos(3*e*x + 3*d) - 6*(F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d) - (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(3*e*x + 3*d))*cos(4*e*x + 4*d) + 3*(3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*cos(2*e*x + 2*d) - (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) - (F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x))*cos(3*e*x + 3*d) - 18*(8*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F)*sin(e*x + d) - (F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d))*cos(2*e*x + 2*d) - 6*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d) + (F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*cos(6*e*x + 6*d)^2 + 9*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*cos(4*e*x + 4*d)^2 + 9*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*cos(2*e*x + 2*d)^2 + (F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*sin(6*e*x + 6*d)^2 + 9*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*sin(4*e*x + 4*d)^2 - 18*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*sin(4*e*x + 4*d)*si

```
n(2*e*x + 2*d) + 9*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2
*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*
c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e
^6*cos(d))*sin(2*e*x + 2*d)^2 - 2*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a
*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) +
34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d)
+ 225*F^(a*c)*e^6*cos(d) + 3*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b
^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^
(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225
*F^(a*c)*e^6*cos(d))*cos(4*e*x + 4*d) - 3*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d
) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*s
in(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)
*sin(d) + 225*F^(a*c)*e^6*cos(d))*cos(2*e*x + 2*d))*cos(6*e*x + 6*d) + 6*(F
^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34
*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)
^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d) - 3*(F^(a*c
)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a
*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 +
225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*cos(2*e*x + 2*d
))*cos(4*e*x + 4*d) - 6*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^
4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)
*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a
*c)*e^6*cos(d))*cos(2*e*x + 2*d) - 6*((F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) +
F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d
) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin
(d) + 225*F^(a*c)*e^6*cos(d))*sin(4*e*x + 4*d) - (F^(a*c)*b^5*c^5*e*log(F)^
5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log
(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5
*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*sin(2*e*x + 2*d))*sin(6*e*x + 6*d)
)*integrate((8*F^(b*c*x)*b*c*e*cos(e*x)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2
)*F^(b*c*x)*sin(e*x) + (8*F^(b*c*x)*b*c*e*cos(e*x)*log(F) + (b^2*c^2*log(F)
^2 - 15*e^2)*F^(b*c*x)*sin(e*x))*cos(8*e*x + 8*d) - 4*(8*F^(b*c*x)*b*c*e*co
s(e*x)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(e*x))*cos(6*e*x +
 6*d) + 6*(8*F^(b*c*x)*b*c*e*cos(e*x)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2)*
F^(b*c*x)*sin(e*x))*cos(4*e*x + 4*d) - 4*(8*F^(b*c*x)*b*c*e*cos(e*x)*log(F)
 + (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(e*x))*cos(2*e*x + 2*d) + (8*F^
(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*cos(e
*x))*sin(8*e*x + 8*d) - 4*(8*F^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*log
(F)^2 - 15*e^2)*F^(b*c*x)*cos(e*x))*sin(6*e*x + 6*d) + 6*(8*F^(b*c*x)*b*c*e
*log(F)*sin(e*x) - (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*cos(e*x))*sin(4*e*
x + 4*d) - 4*(8*F^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*log(F)^2 - 15*e^
2)*F^(b*c*x)*cos(e*x))*sin(2*e*x + 2*d))/(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2
*log(F)^2 + 225*e^4 + (b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4
)*cos(8*e*x + 8*d)^2 + 16*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225
*e^4)*cos(6*e*x + 6*d)^2 + 36*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 +
```

225*e^4)*cos(4*e*x + 4*d)^2 + 16*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)
^2 + 225*e^4)*cos(2*e*x + 2*d)^2 + (b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F
)^2 + 225*e^4)*sin(8*e*x + 8*d)^2 + 16*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*l
og(F)^2 + 225*e^4)*sin(6*e*x + 6*d)^2 + 36*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e
^2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d)^2 - 48*(b^4*c^4*log(F)^4 + 34*b^2*c
^2*e^2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 16*(b^4*c^4*
log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2*e*x + 2*d)^2 + 2*(b^4*c
^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 - 4*(b^4*c^4*log(F)^4 + 34*
b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(6*e*x + 6*d) + 6*(b^4*c^4*log(F)^4 + 34
*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(4*e*x + 4*d) - 4*(b^4*c^4*log(F)^4 + 3
4*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(8*e*x + 8*d) - 8*(b
^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + 6*(b^4*c^4*log(F)^4 +
34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(4*e*x + 4*d) - 4*(b^4*c^4*log(F)^4
+ 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(6*e*x + 6*d) + 1
2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 - 4*(b^4*c^4*log(F)
^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(4*e*x + 4*d)
- 8*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d)
- 4*(2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(6*e*x +
6*d) - 3*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(4*e*x +
4*d) + 2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2*e*x
+ 2*d))*sin(8*e*x + 8*d) - 16*(3*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^
2 + 225*e^4)*sin(4*e*x + 4*d) - 2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)
^2 + 225*e^4)*sin(2*e*x + 2*d))*sin(6*e*x + 6*d)), x) - 6*(F^(a*c)*b^5*c^5*
e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^
3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c
)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d) + (F^(a*c)*b^5*c^5*e*cos(d
)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*c
os(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e
^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*cos(6*e*x + 6*d)^2 + 9*(F^(a*c)*
b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c
)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 22
5*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*cos(4*e*x + 4*d)^
2 + 9*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin
(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F
)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*co
s(2*e*x + 2*d)^2 + (F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2
*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*
c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e
^6*sin(d))*sin(6*e*x + 6*d)^2 + 9*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a
*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 -
34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F)
- 225*F^(a*c)*e^6*sin(d))*sin(4*e*x + 4*d)^2 - 18*(F^(a*c)*b^5*c^5*e*cos(d)
*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*co
s(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^
5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*sin(4*e*x + 4*d)*sin(2*e*x + 2*d)

```
 + 9*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(
d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)
^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*sin
(2*e*x + 2*d)^2 - 2*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^
2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2
*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*
e^6*sin(d) + 3*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log
(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*
e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*s
in(d))*cos(4*e*x + 4*d) - 3*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^
4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(
a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*
F^(a*c)*e^6*sin(d))*cos(2*e*x + 2*d))*cos(6*e*x + 6*d) + 6*(F^(a*c)*b^5*c^5
*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c
^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*
c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d) - 3*(F^(a*c)*b^5*c^5*e*co
s(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^
3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*
c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*cos(2*e*x + 2*d))*cos(4*e*x +
 4*d) - 6*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4
*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*l
og(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d)
)*cos(2*e*x + 2*d) - 6*((F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^
4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)
*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a
*c)*e^6*sin(d))*sin(4*e*x + 4*d) - (F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(
a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 -
 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F)
 - 225*F^(a*c)*e^6*sin(d))*sin(2*e*x + 2*d))*sin(6*e*x + 6*d))*integrate((8
*F^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*co
s(e*x) + (8*F^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*log(F)^2 - 15*e^2)*F
^(b*c*x)*cos(e*x))*cos(8*e*x + 8*d) - 4*(8*F^(b*c*x)*b*c*e*log(F)*sin(e*x)
- (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*cos(e*x))*cos(6*e*x + 6*d) + 6*(8*F
^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*cos(
e*x))*cos(4*e*x + 4*d) - 4*(8*F^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*lo
g(F)^2 - 15*e^2)*F^(b*c*x)*cos(e*x))*cos(2*e*x + 2*d) - (8*F^(b*c*x)*b*c*e*
cos(e*x)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(e*x))*sin(8*e*x
 + 8*d) + 4*(8*F^(b*c*x)*b*c*e*cos(e*x)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2
)*F^(b*c*x)*sin(e*x))*sin(6*e*x + 6*d) - 6*(8*F^(b*c*x)*b*c*e*cos(e*x)*log(
F) + (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(e*x))*sin(4*e*x + 4*d) + 4*(
8*F^(b*c*x)*b*c*e*cos(e*x)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*s
in(e*x))*sin(2*e*x + 2*d))/(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 22
5*e^4 + (b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(8*e*x +
8*d)^2 + 16*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(6*e*
x + 6*d)^2 + 36*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(
```

4*e*x + 4*d)^2 + 16*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*
cos(2*e*x + 2*d)^2 + (b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)
*sin(8*e*x + 8*d)^2 + 16*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*
e^4)*sin(6*e*x + 6*d)^2 + 36*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 +
225*e^4)*sin(4*e*x + 4*d)^2 - 48*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^
2 + 225*e^4)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 16*(b^4*c^4*log(F)^4 + 34*
b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2*e*x + 2*d)^2 + 2*(b^4*c^4*log(F)^4 +
34*b^2*c^2*e^2*log(F)^2 + 225*e^4 - 4*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*lo
g(F)^2 + 225*e^4)*cos(6*e*x + 6*d) + 6*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*l
og(F)^2 + 225*e^4)*cos(4*e*x + 4*d) - 4*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*
log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(8*e*x + 8*d) - 8*(b^4*c^4*log(F)^
4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + 6*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^
2*log(F)^2 + 225*e^4)*cos(4*e*x + 4*d) - 4*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e
^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(6*e*x + 6*d) + 12*(b^4*c^4*log
(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 - 4*(b^4*c^4*log(F)^4 + 34*b^2*c^
2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) - 8*(b^4*c^4*l
og(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d) - 4*(2*(b^4*c
^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(6*e*x + 6*d) - 3*(b^4*
c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d) + 2*(b^4
*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2*e*x + 2*d))*sin(8*
e*x + 8*d) - 16*(3*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*s
in(4*e*x + 4*d) - 2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*
sin(2*e*x + 2*d))*sin(6*e*x + 6*d)), x) + (48*F^(b*c*x)*F^(a*c)*b*c*e^2*cos
(e*x + d)*log(F) - (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F
^(b*c*x)*cos(3*e*x + 3*d) + 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)
*F^(b*c*x)*sin(3*e*x + 3*d) + 6*(F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^
3)*F^(b*c*x)*sin(e*x + d))*sin(6*e*x + 6*d) - 3*(48*F^(b*c*x)*F^(a*c)*b*c*e
^2*cos(e*x + d)*log(F) - (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log
(F))*F^(b*c*x)*cos(3*e*x + 3*d) + 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c
)*e^3)*F^(b*c*x)*sin(3*e*x + 3*d) + 6*(F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a
*c)*e^3)*F^(b*c*x)*sin(e*x + d))*sin(4*e*x + 4*d) + (3*(F^(a*c)*b^3*c^3*log
(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 9*(F^(a*c)*
b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x + 2*d) - (F^(a*c)*
b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x))*sin(3*e*x + 3*d) +
18*(8*F^(b*c*x)*F^(a*c)*b*c*e^2*cos(e*x + d)*log(F) + (F^(a*c)*b^2*c^2*e*l
og(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d))*sin(2*e*x + 2*d))/(b^4*c^
4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + (b^4*c^4*log(F)^4 + 34*b^2
*c^2*e^2*log(F)^2 + 225*e^4)*cos(6*e*x + 6*d)^2 + 9*(b^4*c^4*log(F)^4 + 34*
b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(4*e*x + 4*d)^2 + 9*(b^4*c^4*log(F)^4 +
34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d)^2 + (b^4*c^4*log(F)^4 +
34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(6*e*x + 6*d)^2 + 9*(b^4*c^4*log(F)^
4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d)^2 - 18*(b^4*c^4*log
(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d
) + 9*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2*e*x + 2*
d)^2 - 2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + 3*(b^4*c^4

```
*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(4*e*x + 4*d) - 3*(b^4*c^
4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(6*e*x
 + 6*d) + 6*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 - 3*(b^4*
c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(4*e
*x + 4*d) - 6*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*
e*x + 2*d) - 6*((b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(
4*e*x + 4*d) - (b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2
*e*x + 2*d))*sin(6*e*x + 6*d))
```

Giac [F]

$$\int F^{c(a+bx)} \csc^3(d+ex)\,dx = \int F^{(bx+a)c} \csc\left(ex+d\right)^3\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \csc^3(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)^3}\,dx$$

```
[In] int(F^(c*(a + b*x))/sin(d + e*x)^3,x)
[Out] int(F^(c*(a + b*x))/sin(d + e*x)^3, x)
```

3.8 $\int F^{c(a+bx)} \csc^4(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 141

$$\int F^{c(a+bx)} \csc^4(d+ex)\,dx = -\frac{F^{c(a+bx)}\cot(d+ex)\csc^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\csc^2(d+ex)\log(F)}{6e^2} + \frac{2e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)(2ie-bc\log(F))}{3e^2}$$

```
[Out] -1/3*F^(c*(b*x+a))*cot(e*x+d)*csc(e*x+d)^2/e-1/6*b*c*F^(c*(b*x+a))*csc(e*x+
d)^2*ln(F)/e^2+2/3*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c
*ln(F)/e],[2-1/2*I*b*c*ln(F)/e],exp(2*I*(e*x+d)))*(2*I*e-b*c*ln(F))/e^2
```

Rubi [A] (verified)

Time = 0.07 (sec), antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4534, 4538}

$$\int F^{c(a+bx)} \csc^4(d+ex)\,dx = \frac{2e^{2i(d+ex)}F^{c(a+bx)}(-bc\log(F)+2ie)\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{3e^2} - \frac{bc\log(F)\csc^2(d+ex)F^{c(a+bx)}}{6e^2} - \frac{\cot(d+ex)\csc^2(d+ex)F^{c(a+bx)}}{3e}$$

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x]^4,x]
```

```
[Out] -1/3*(F^(c*(a + b*x))*Cot[d + e*x]*Csc[d + e*x]^2)/e - (b*c*F^(c*(a + b*x))
*Csc[d + e*x]^2*Log[F])/(6*e^2) + (2*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hy
pergeometric2F1[2, 1 - ((I/2)*b*c*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, E^((
2*I)*(d + e*x))]*((2*I)*e - b*c*Log[F]))/(3*e^2)
```

Rule 4534

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csc[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x] - Simp[F^(c*(a + b*x
))*Csc[d + e*x]^(n - 1)*(Cos[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4538

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]
/(2*e)), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{F^{c(a+bx)}\cot(d+ex)\csc^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\csc^2(d+ex)\log(F)}{6e^2} \\
&\quad + \frac{1}{6}\left(4+\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\csc^2(d+ex)\,dx \\
&= -\frac{F^{c(a+bx)}\cot(d+ex)\csc^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\csc^2(d+ex)\log(F)}{6e^2} \\
&\quad + \frac{2e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)(2ie-bc\log(F))}{3e^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.23

$$\int F^{c(a+bx)}\csc^4(d+ex)\,dx = \frac{F^{c(a+bx)}\left(-e\csc^2(d+ex)(2e\cot(d)+bc\log(F)) - \frac{2i\left(1+\left(-1+e^{2id}\right)\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right.\right.}{-1+e^{2id}}\right.}{6e^3}$$

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Csc[d + e*x]^2*(2*e*Cot[d] + b*c*Log[F])) - ((2*I)*(1
 + (-1 + E^((2*I)*d))*Hypergeometric2F1[1, ((-1/2*I)*b*c*Log[F])/e, 1 - ((I
/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])*(4*e^2 + b^2*c^2*Log[F]^2))/(-1 +
E^((2*I)*d)) + 2*e^2*Csc[d]*Csc[d + e*x]^3*Sin[e*x] + Csc[d]*Csc[d + e*x]*(
4*e^2 + b^2*c^2*Log[F]^2)*Sin[e*x]))/(6*e^3)
```

Maple [F]

$$\int F^{c(xb+a)} \csc(ex+d)^4 \, dx$$

```
[In] int(F^(c*(b*x+a))*csc(e*x+d)^4,x)
[Out] int(F^(c*(b*x+a))*csc(e*x+d)^4,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \csc^4(d+ex) \, dx = \int F^{(bx+a)c} \csc(ex+d)^4 \, dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^4,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*csc(e*x + d)^4, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \csc^4(d+ex) \, dx = \int F^{c(a+bx)} \csc^4(d+ex) \, dx$$

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d)**4,x)
[Out] Integral(F**(c*(a + b*x))*csc(d + e*x)**4, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \csc^4(d+ex) \, dx = \int F^{(bx+a)c} \csc(ex+d)^4 \, dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^4,x, algorithm="maxima")
[Out] 16*(6*(F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F
^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*e*x + 4*d)^2 + 320*(F^(a*c)*b^3*c^3*
e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 + 6*
```

```
(F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)
*b*c*e^4*log(F))*F^(b*c*x)*sin(4*e*x + 4*d)^2 + 320*(F^(a*c)*b^3*c^3*e^2*lo
g(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 + 560*(F^(
a*c)*b^3*c^3*e^2*log(F)^3 - 32*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x
+ 2*d) - 40*(F^(a*c)*b^4*c^4*e*log(F)^4 - 104*F^(a*c)*b^2*c^2*e^3*log(F)^2)
*F^(b*c*x)*sin(2*e*x + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*F^(a*c
)*b*c*e^4*log(F))*F^(b*c*x) + ((F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*
c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*e*x + 4*d)
- 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*c
os(2*e*x + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 100*F^(a*c)*b^2*c^2*e^3*l
og(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x)*sin(4*e*x + 4*d) - 8*(F^(a*c)*b^4*c^4
*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 1536*F^(a*c)*e^5)*F^(b*c*x)
*sin(2*e*x + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*F^(a*c)*b*c*e^4*
log(F))*F^(b*c*x))*cos(8*e*x + 8*d) - 4*((F^(a*c)*b^5*c^5*log(F)^5 + 100*F^
(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*e
*x + 4*d) - 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F
^(b*c*x)*cos(2*e*x + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 100*F^(a*c)*b^2
*c^2*e^3*log(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x)*sin(4*e*x + 4*d) - 8*(F^(a*
c)*b^4*c^4*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 1536*F^(a*c)*e^5)
*F^(b*c*x)*sin(2*e*x + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*F^(a*c
)*b*c*e^4*log(F))*F^(b*c*x))*cos(6*e*x + 6*d) - (4*(F^(a*c)*b^5*c^5*log(F)^
5 + 220*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 9984*F^(a*c)*b*c*e^4*log(F))*F^(b*c*
x)*cos(2*e*x + 2*d) + 64*(F^(a*c)*b^4*c^4*e*log(F)^4 + 55*F^(a*c)*b^2*c^2*e
^3*log(F)^2 - 576*F^(a*c)*e^5)*F^(b*c*x)*sin(2*e*x + 2*d) - (F^(a*c)*b^5*c^
5*log(F)^5 - 860*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 21504*F^(a*c)*b*c*e^4*log(F
))*F^(b*c*x))*cos(4*e*x + 4*d) + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(
a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*
c)*b*c*e^8*log(F) + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4
*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F
))*cos(8*e*x + 8*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*
c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^
8*log(F))*cos(6*e*x + 6*d)^2 + 36*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*
c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)
*b*c*e^8*log(F))*cos(4*e*x + 4*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 11
6*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*
F^(a*c)*b*c*e^8*log(F))*cos(2*e*x + 2*d)^2 + (F^(a*c)*b^7*c^7*e^2*log(F)^7
+ 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36
864*F^(a*c)*b*c*e^8*log(F))*sin(8*e*x + 8*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*lo
g(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)
^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(6*e*x + 6*d)^2 + 36*(F^(a*c)*b^7*c^7
*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6
*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(4*e*x + 4*d)^2 - 48*(F^(a*c)*
b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*
c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(4*e*x + 4*d)*sin(2*e*x
 + 2*d) + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)
```

```
^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(
2*e*x + 2*d)^2 + 2*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*
log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F)
 - 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 390
4*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(6*e*x +
6*d) + 6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 +
 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(4*e*
x + 4*d) - 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)
^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(
2*e*x + 2*d))*cos(8*e*x + 8*d) - 8*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a
*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c
)*b*c*e^8*log(F) + 6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^
4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(
F))*cos(4*e*x + 4*d) - 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^
5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*
log(F))*cos(2*e*x + 2*d))*cos(6*e*x + 6*d) + 12*(F^(a*c)*b^7*c^7*e^2*log(F)
^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 +
 36864*F^(a*c)*b*c*e^8*log(F) - 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*
c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)
*b*c*e^8*log(F))*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) - 8*(F^(a*c)*b^7*c^7*e^
2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*lo
g(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(2*e*x + 2*d) - 4*(2*(F^(a*c)*b^7
*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3
*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(6*e*x + 6*d) - 3*(F^(a*c)
*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3
*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(4*e*x + 4*d) + 2*(F^(
a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)
*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(2*e*x + 2*d))*sin
(8*e*x + 8*d) - 16*(3*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e
^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log
(F))*sin(4*e*x + 4*d) - 2*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c
^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8
*log(F))*sin(2*e*x + 2*d))*sin(6*e*x + 6*d))*integrate(-((b^3*c^3*log(F)^3
 - 104*b*c*e^2*log(F))*F^(b*c*x)*cos(10*e*x + 10*d) - 5*(b^3*c^3*log(F)^3 -
104*b*c*e^2*log(F))*F^(b*c*x)*cos(8*e*x + 8*d) + 10*(b^3*c^3*log(F)^3 - 104
*b*c*e^2*log(F))*F^(b*c*x)*cos(6*e*x + 6*d) - 10*(b^3*c^3*log(F)^3 - 104*b*
c*e^2*log(F))*F^(b*c*x)*cos(4*e*x + 4*d) + 5*(b^3*c^3*log(F)^3 - 104*b*c*e^
2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 6*(3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^
(b*c*x)*sin(10*e*x + 10*d) - 30*(3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x)*s
in(8*e*x + 8*d) + 60*(3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x)*sin(6*e*x +
6*d) - 60*(3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x)*sin(4*e*x + 4*d) + 30*(
3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x)*sin(2*e*x + 2*d) - (b^3*c^3*log(F)
^3 - 104*b*c*e^2*log(F))*F^(b*c*x))/(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log
(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6 + (b^6*c^6*log(F)^6 + 116*b^4
*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(10*e*x + 10*
```

d)^2 + 25*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*l
og(F)^2 + 36864*e^6)*cos(8*e*x + 8*d)^2 + 100*(b^6*c^6*log(F)^6 + 116*b^4*c
^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6*d)^2
 + 100*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(
F)^2 + 36864*e^6)*cos(4*e*x + 4*d)^2 + 25*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e
^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d)^2 + (
b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 3
6864*e^6)*sin(10*e*x + 10*d)^2 + 25*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log
(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(8*e*x + 8*d)^2 + 100*(b^
6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 368
64*e^6)*sin(6*e*x + 6*d)^2 + 100*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)
^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d)^2 - 100*(b^6*c
^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*
e^6)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 25*(b^6*c^6*log(F)^6 + 116*b^4*c^4
*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d)^2 -
 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2
 + 36864*e^6 + 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^
2*e^4*log(F)^2 + 36864*e^6)*cos(8*e*x + 8*d) - 10*(b^6*c^6*log(F)^6 + 116*b
^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6*
d) + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log
(F)^2 + 36864*e^6)*cos(4*e*x + 4*d) - 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2
*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(10
*e*x + 10*d) + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c
^2*e^4*log(F)^2 + 36864*e^6 - 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)
^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6*d) + 10*(b^6*c^6*
log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6
)*cos(4*e*x + 4*d) - 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*
b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(8*e*x + 8*d) - 20*(
b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 3
6864*e^6 + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e
^4*log(F)^2 + 36864*e^6)*cos(4*e*x + 4*d) - 5*(b^6*c^6*log(F)^6 + 116*b^4*c
^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*
cos(6*e*x + 6*d) + 20*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b
^2*c^2*e^4*log(F)^2 + 36864*e^6 - 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log
(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(4*e*x
+ 4*d) - 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4
*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d) - 10*((b^6*c^6*log(F)^6 + 116*b^4*c
^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(8*e*x + 8*d) -
 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2
 + 36864*e^6)*sin(6*e*x + 6*d) + 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(
F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) - (b^6*c^6*l
og(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)
*sin(2*e*x + 2*d))*sin(10*e*x + 10*d) - 50*(2*(b^6*c^6*log(F)^6 + 116*b^4*c
^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(6*e*x + 6*d) -
 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2

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 + 36864*e^6)*sin(4*e*x + 4*d) + (b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)
^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d))*sin(8*e*x + 8
*d) - 100*(2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^
4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) - (b^6*c^6*log(F)^6 + 116*b^4*c^4*
e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d))*sin
(6*e*x + 6*d)), x) - 8*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^
3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*
log(F)^2 + (F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 +
 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*co
s(8*e*x + 8*d)^2 + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3
*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*l
og(F)^2)*cos(6*e*x + 6*d)^2 + 36*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*
b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^
2*c^2*e^7*log(F)^2)*cos(4*e*x + 4*d)^2 + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 1
16*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864
*F^(a*c)*b^2*c^2*e^7*log(F)^2)*cos(2*e*x + 2*d)^2 + (F^(a*c)*b^8*c^8*e*log(
F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4
 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin(8*e*x + 8*d)^2 + 16*(F^(a*c)*b^8
*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e
^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin(6*e*x + 6*d)^2 + 36*(
F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c
)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin(4*e*x + 4*
d)^2 - 48*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 +
3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin
(4*e*x + 4*d)*sin(2*e*x + 2*d) + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*
c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)
*b^2*c^2*e^7*log(F)^2)*sin(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^8*c^8*e*log(F)^8 +
 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 368
64*F^(a*c)*b^2*c^2*e^7*log(F)^2 - 4*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*
c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)
*b^2*c^2*e^7*log(F)^2)*cos(6*e*x + 6*d) + 6*(F^(a*c)*b^8*c^8*e*log(F)^8 + 1
16*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864
*F^(a*c)*b^2*c^2*e^7*log(F)^2)*cos(4*e*x + 4*d) - 4*(F^(a*c)*b^8*c^8*e*log(
F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4
 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*cos(2*e*x + 2*d))*cos(8*e*x + 8*d) -
 8*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^
(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2 + 6*(F^(a*c
)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*
c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*cos(4*e*x + 4*d) - 4
*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a
*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*cos(2*e*x +
2*d))*cos(6*e*x + 6*d) + 12*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c
^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2
*e^7*log(F)^2 - 4*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log
(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F
```

)^2)*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) - 8*(F^(a*c)*b^8*c^8*e*log(F)^8 + 1
16*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864
*F^(a*c)*b^2*c^2*e^7*log(F)^2)*cos(2*e*x + 2*d) - 4*(2*(F^(a*c)*b^8*c^8*e*l
og(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F
)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin(6*e*x + 6*d) - 3*(F^(a*c)*b^8
*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e
^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin(4*e*x + 4*d) + 2*(F^(
a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b
^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin(2*e*x + 2*d))
*sin(8*e*x + 8*d) - 16*(3*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6
*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e
^7*log(F)^2)*sin(4*e*x + 4*d) - 2*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)
*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b
^2*c^2*e^7*log(F)^2)*sin(2*e*x + 2*d))*sin(6*e*x + 6*d))*integrate((6*(3*b^
2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x)*cos(10*e*x + 10*d) - 30*(3*b^2*c^2*e*l
og(F)^2 - 32*e^3)*F^(b*c*x)*cos(8*e*x + 8*d) + 60*(3*b^2*c^2*e*log(F)^2 - 3
2*e^3)*F^(b*c*x)*cos(6*e*x + 6*d) - 60*(3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b
*c*x)*cos(4*e*x + 4*d) + 30*(3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x)*cos(2
*e*x + 2*d) - (b^3*c^3*log(F)^3 - 104*b*c*e^2*log(F))*F^(b*c*x)*sin(10*e*x
+ 10*d) + 5*(b^3*c^3*log(F)^3 - 104*b*c*e^2*log(F))*F^(b*c*x)*sin(8*e*x + 8
d) - 10(b^3*c^3*log(F)^3 - 104*b*c*e^2*log(F))*F^(b*c*x)*sin(6*e*x + 6*d)
+ 10*(b^3*c^3*log(F)^3 - 104*b*c*e^2*log(F))*F^(b*c*x)*sin(4*e*x + 4*d) -
5*(b^3*c^3*log(F)^3 - 104*b*c*e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) - 6*(3
*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x))/(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^
2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6 + (b^6*c^6*log(F)^6 + 11
6*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(10*e*x
+ 10*d)^2 + 25*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*
e^4*log(F)^2 + 36864*e^6)*cos(8*e*x + 8*d)^2 + 100*(b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6
d)^2 + 100(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4
*log(F)^2 + 36864*e^6)*cos(4*e*x + 4*d)^2 + 25*(b^6*c^6*log(F)^6 + 116*b^4*
c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d)^
2 + (b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^
2 + 36864*e^6)*sin(10*e*x + 10*d)^2 + 25*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^
2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(8*e*x + 8*d)^2 + 10
0*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2
+ 36864*e^6)*sin(6*e*x + 6*d)^2 + 100*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*l
og(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d)^2 - 100*(
b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 3
6864*e^6)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 25*(b^6*c^6*log(F)^6 + 116*b^
4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d
)^2 - 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log
(F)^2 + 36864*e^6 + 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b
^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(8*e*x + 8*d) - 10*(b^6*c^6*log(F)^6 +
116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x

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 + 6*d) + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^
4*log(F)^2 + 36864*e^6)*cos(4*e*x + 4*d) - 5*(b^6*c^6*log(F)^6 + 116*b^4*c^
4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*c
os(10*e*x + 10*d) + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*
b^2*c^2*e^4*log(F)^2 + 36864*e^6 - 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*l
og(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6*d) + 10*(b^6
*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 3686
4*e^6)*cos(4*e*x + 4*d) - 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 +
3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(8*e*x + 8*d) -
 20*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^
2 + 36864*e^6 + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*
c^2*e^4*log(F)^2 + 36864*e^6)*cos(4*e*x + 4*d) - 5*(b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2
*d))*cos(6*e*x + 6*d) + 20*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3
904*b^2*c^2*e^4*log(F)^2 + 36864*e^6 - 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^
2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(4
*e*x + 4*d) - 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^
2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d) - 10*((b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(8*e*x + 8
*d) - 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log
(F)^2 + 36864*e^6)*sin(6*e*x + 6*d) + 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2
*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) - (b^6*
c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864
*e^6)*sin(2*e*x + 2*d))*sin(10*e*x + 10*d) - 50*(2*(b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(6*e*x + 6
*d) - 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log
(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) + (b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*l
og(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d))*sin(8*e*
x + 8*d) - 100*(2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c
^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) - (b^6*c^6*log(F)^6 + 116*b^4
*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d)
)*sin(6*e*x + 6*d)), x) + (4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 100*F^(a*c)*b^2*
c^2*e^3*log(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x)*cos(4*e*x + 4*d) + 8*(F^(a*c
)*b^4*c^4*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 1536*F^(a*c)*e^5)*
F^(b*c*x)*cos(2*e*x + 2*d) + (F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^
3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(4*e*x + 4*d) -
80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin
(2*e*x + 2*d) + 8*(F^(a*c)*b^4*c^4*e*log(F)^4 - 140*F^(a*c)*b^2*c^2*e^3*log
(F)^2 + 384*F^(a*c)*e^5)*F^(b*c*x))*sin(8*e*x + 8*d) - 4*(4*(F^(a*c)*b^4*c^
4*e*log(F)^4 + 100*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*
x)*cos(4*e*x + 4*d) + 8*(F^(a*c)*b^4*c^4*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^
3*log(F)^2 - 1536*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) + (F^(a*c)*b^5*c^
5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F)
)*F^(b*c*x)*sin(4*e*x + 4*d) - 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c
)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) + 8*(F^(a*c)*b^4*c^4*e*log(F)^
```

```
4 - 140*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 384*F^(a*c)*e^5)*F^(b*c*x))*sin(6*e*
x + 6*d) + 4*(16*(F^(a*c)*b^4*c^4*e*log(F)^4 + 55*F^(a*c)*b^2*c^2*e^3*log(F
)^2 - 576*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) - (F^(a*c)*b^5*c^5*log(F)
^5 + 220*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 9984*F^(a*c)*b*c*e^4*log(F))*F^(b*c
*x)*sin(2*e*x + 2*d) + (11*F^(a*c)*b^4*c^4*e*log(F)^4 - 1780*F^(a*c)*b^2*c^
2*e^3*log(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x))*sin(4*e*x + 4*d))/(b^6*c^6*lo
g(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6 +
 (b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 +
 36864*e^6)*cos(8*e*x + 8*d)^2 + 16*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log
(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6*d)^2 + 36*(b^6
*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 3686
4*e^6)*cos(4*e*x + 4*d)^2 + 16*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4
 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d)^2 + (b^6*c^6*log
(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*s
in(8*e*x + 8*d)^2 + 16*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*
b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(6*e*x + 6*d)^2 + 36*(b^6*c^6*log(F)^6
 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(4*
e*x + 4*d)^2 - 48*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c
^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 16*(b^6*c^
6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e
^6)*sin(2*e*x + 2*d)^2 + 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3
904*b^2*c^2*e^4*log(F)^2 + 36864*e^6 - 4*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^
2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6*d) + 6*(b
^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36
864*e^6)*cos(4*e*x + 4*d) - 4*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4
+ 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(8*e*x + 8*d)
 - 8*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)
^2 + 36864*e^6 + 6*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*
c^2*e^4*log(F)^2 + 36864*e^6)*cos(4*e*x + 4*d) - 4*(b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2
*d))*cos(6*e*x + 6*d) + 12*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3
904*b^2*c^2*e^4*log(F)^2 + 36864*e^6 - 4*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^
2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(4
*e*x + 4*d) - 8*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2
*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d) - 4*(2*(b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(6*e*x + 6
*d) - 3*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log
(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) + 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2
*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d))*sin(8*
e*x + 8*d) - 16*(3*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*
c^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) - 2*(b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2
*d))*sin(6*e*x + 6*d))
```

Giac [F]

$$\int F^{c(a+bx)} \csc^4(d+ex)\,dx = \int F^{(bx+a)c} \csc\left(ex+d\right)^4\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^4,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \csc^4(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\sin\left(d+e\,x\right)^4}\,dx$$

```
[In] int(F^(c*(a + b*x))/sin(d + e*x)^4,x)
[Out] int(F^(c*(a + b*x))/sin(d + e*x)^4, x)
```

3.9 $\int e^x \sin^4(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 54

$$\int e^x \sin^4(x)\,dx = \frac{24e^x}{85} - \frac{24}{85}e^x\cos(x)\sin(x) + \frac{12}{85}e^x\sin^2(x) - \frac{4}{17}e^x\cos(x)\sin^3(x) + \frac{1}{17}e^x\sin^4(x)$$

```
[Out] 24/85*exp(x)-24/85*exp(x)*cos(x)*sin(x)+12/85*exp(x)*sin(x)^2-4/17*exp(x)*c
os(x)*sin(x)^3+1/17*exp(x)*sin(x)^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4519, 2225}

$$\int e^x \sin^4(x)\,dx = \frac{24e^x}{85} + \frac{1}{17}e^x\sin^4(x) + \frac{12}{85}e^x\sin^2(x) - \frac{4}{17}e^x\sin^3(x)\cos(x) - \frac{24}{85}e^x\sin(x)\cos(x)$$

```
[In] Int[E^x*Sin[x]^4,x]
```

```
[Out] (24*E^x)/85 - (24*E^x*Cos[x]*Sin[x])/85 + (12*E^x*Sin[x]^2)/85 - (4*E^x*Cos
[x]*Sin[x]^3)/17 + (E^x*Sin[x]^4)/17
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4519

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
```

```
g[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d +
 e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{4}{17}e^x\cos(x)\sin^3(x)+\frac{1}{17}e^x\sin^4(x)+\frac{12}{17}\int e^x\sin^2(x)\,dx\\ &= -\frac{24}{85}e^x\cos(x)\sin(x)+\frac{12}{85}e^x\sin^2(x)-\frac{4}{17}e^x\cos(x)\sin^3(x)+\frac{1}{17}e^x\sin^4(x)+\frac{24\int e^x\,dx}{85}\\ &= \frac{24e^x}{85}-\frac{24}{85}e^x\cos(x)\sin(x)+\frac{12}{85}e^x\sin^2(x)-\frac{4}{17}e^x\cos(x)\sin^3(x)+\frac{1}{17}e^x\sin^4(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int e^x\sin^4(x)\,dx=\frac{1}{680}e^x(255-68\cos(2x)+5\cos(4x)-136\sin(2x)+20\sin(4x))$$

```
[In] Integrate[E^x*Sin[x]^4,x]
[Out] (E^x*(255 - 68*Cos[2*x] + 5*Cos[4*x] - 136*Sin[2*x] + 20*Sin[4*x]))/680
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

method	result
parallelrisch	$-\frac{e^x(-255+136\sin(2x)-20\sin(4x)-5\cos(4x)+68\cos(2x))}{680}$
default	$\frac{(\sin(x)-4\cos(x))e^x\sin(x)^3}{17}+\frac{12(\sin(x)-2\cos(x))e^x\sin(x)}{85}+\frac{24\,e^x}{85}$
risch	$\frac{3\,e^x}{8}+\frac{e^{(1+4i)x}}{272}-\frac{ie^{(1+4i)x}}{68}-\frac{e^{(1+2i)x}}{20}+\frac{ie^{(1+2i)x}}{10}-\frac{e^{(1-2i)x}}{20}-\frac{ie^{(1-2i)x}}{10}+\frac{e^{(1-4i)x}}{272}+\frac{ie^{(1-4i)x}}{68}$
norman	$\frac{-\frac{48\,e^x\tan\left(\frac{x}{2}\right)}{85}+\frac{144\,e^x\tan\left(\frac{x}{2}\right)^2}{85}-\frac{208\,e^x\tan\left(\frac{x}{2}\right)^3}{85}+\frac{64\,e^x\tan\left(\frac{x}{2}\right)^4}{17}+\frac{208\,e^x\tan\left(\frac{x}{2}\right)^5}{85}+\frac{144\,e^x\tan\left(\frac{x}{2}\right)^6}{85}+\frac{48\,e^x\tan\left(\frac{x}{2}\right)^7}{85}+\frac{24\,e^x\tan\left(\frac{x}{2}\right)^8}{85}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^4}$

```
[In] int(exp(x)*sin(x)^4,x,method=_RETURNVERBOSE)
[Out] -1/680*exp(x)*(-255+136*sin(2*x)-20*sin(4*x)-5*cos(4*x)+68*cos(2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int e^x \sin^4(x)\,dx = \frac{4}{85}\left(5\cos(x)^3 - 11\cos(x)\right)e^x\sin(x) + \frac{1}{85}\left(5\cos(x)^4 - 22\cos(x)^2 + 41\right)e^x$$

```
[In] integrate(exp(x)*sin(x)^4,x, algorithm="fricas")
[Out] 4/85*(5*cos(x)^3 - 11*cos(x))*e^x*sin(x) + 1/85*(5*cos(x)^4 - 22*cos(x)^2 +
 41)*e^x
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int e^x \sin^4(x)\,dx = \frac{41e^x\sin^4(x)}{85} - \frac{44e^x\sin^3(x)\cos(x)}{85} + \frac{12e^x\sin^2(x)\cos^2(x)}{17} - \frac{24e^x\sin(x)\cos^3(x)}{85} + \frac{24e^x\cos^4(x)}{85}$$

```
[In] integrate(exp(x)*sin(x)**4,x)
[Out] 41*exp(x)*sin(x)**4/85 - 44*exp(x)*sin(x)**3*cos(x)/85 + 12*exp(x)*sin(x)**
2*cos(x)**2/17 - 24*exp(x)*sin(x)*cos(x)**3/85 + 24*exp(x)*cos(x)**4/85
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int e^x \sin^4(x)\,dx = \frac{1}{136}\cos(4x)e^x - \frac{1}{10}\cos(2x)e^x + \frac{1}{34}e^x\sin(4x) - \frac{1}{5}e^x\sin(2x) + \frac{3}{8}e^x$$

```
[In] integrate(exp(x)*sin(x)^4,x, algorithm="maxima")
[Out] 1/136*cos(4*x)*e^x - 1/10*cos(2*x)*e^x + 1/34*e^x*sin(4*x) - 1/5*e^x*sin(2*
x) + 3/8*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.65

$$\int e^x \sin^4(x)\,dx = \frac{1}{136}\left(\cos(4x) + 4\sin(4x)\right)e^x - \frac{1}{10}\left(\cos(2x) + 2\sin(2x)\right)e^x + \frac{3}{8}e^x$$

```
[In] integrate(exp(x)*sin(x)^4,x, algorithm="giac")
```

```
[Out] 1/136*(cos(4*x) + 4*sin(4*x))*e^x - 1/10*(cos(2*x) + 2*sin(2*x))*e^x + 3/8*
e^x
```

Mupad [B] (verification not implemented)

Time = 26.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int e^x \sin^4(x)\,dx = \frac{3\,e^x}{8} - \frac{e^x\left(\frac{4\cos(2x)}{5} + \frac{8\sin(2x)}{5} - \frac{2\cos(2x)^2}{17} - \frac{8\cos(2x)\sin(2x)}{17} + \frac{1}{17}\right)}{8}$$

```
[In] int(exp(x)*sin(x)^4,x)
```

```
[Out] (3*exp(x))/8 - (exp(x)*((4*cos(2*x))/5 + (8*sin(2*x))/5 - (2*cos(2*x)^2)/17
 - (8*cos(2*x)*sin(2*x))/17 + 1/17))/8
```

3.10 $\int F^{c(a+bx)} \cos^n(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int F^{c(a+bx)} \cos^n(d+ex)\,dx = -\frac{\left(1+e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)} \cos^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,-\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(2-n-\frac{ibc\log(F)}{e}\right),-\right.}{ien-bc\log(F)}$$

```
[Out] -F^(c*(b*x+a))*cos(e*x+d)^n*hypergeom([-n, 1/2*(-e*n-I*b*c*ln(F))/e],[1-1/2
*n-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/((1+exp(2*I*(e*x+d)))^n)/(I*e*n-b*
c*ln(F))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4526, 2291}

$$\int F^{c(a+bx)} \cos^n(d+ex)\,dx = -\frac{\left(1+e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)} \cos^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,-\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(-n-\frac{ibc\log(F)}{e}+2\right),\right.}{-bc\log(F)+ien}$$

```
[In] Int[F^(c*(a + b*x))*Cos[d + e*x]^n,x]
```

```
[Out] -((F^(c*(a + b*x))*Cos[d + e*x]^n*Hypergeometric2F1[-n, -1/2*(e*n + I*b*c*L
og[F])/e, (2 - n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))])/((1 + E^((2*
I)*(d + e*x)))^n*(I*e*n - b*c*Log[F])))
```

Rule 2291

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 4526

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Dist[E^(I*n*(d + e*x))*(Cos[d + e*x]^n/(1 + E^(2*I*(d + e*x)))^n), In
t[F^(c*(a + b*x))*((1 + E^(2*I*(d + e*x)))^n/E^(I*n*(d + e*x))), x], x] /;
FreeQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(e^{in(d+ex)}\left(1+e^{2i(d+ex)}\right)^{-n}\cos^n(d+ex)\right)\int e^{-in(d+ex)}\left(1+e^{2i(d+ex)}\right)^n F^{c(a+bx)}\,dx$$

$$= -\frac{\left(1+e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)}\cos^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,-\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(2-n-\frac{ibc\log(F}{e}\right.\right.}{ien-bc\log(F)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}\cos^n(d+ex)\,dx$$

$$= \frac{\left(1+e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)}\cos^n(d+ex)\,\text{Hypergeometric2F1}\left(-n,-\frac{i(-ien+bc\log(F))}{2e},1-\frac{i(-ien+bc\log(F))}{2e},-e^{2i(\right.}{-ien+bc\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Cos[d + e*x]^n,x]
```

```
[Out] (F^(c*(a + b*x))*Cos[d + e*x]^n*Hypergeometric2F1[-n, ((-1/2*I)*((-I)*e*n +
 b*c*Log[F]))/e, 1 - ((I/2)*((-I)*e*n + b*c*Log[F]))/e, -E^((2*I)*(d + e*x)
)])/((1 + E^((2*I)*(d + e*x)))^n*((-I)*e*n + b*c*Log[F]))
```

Maple [F]

$$\int F^{c(xb+a)} \cos(ex+d)^n \, dx$$

```
[In] int(F^(c*(b*x+a))*cos(e*x+d)^n,x)
[Out] int(F^(c*(b*x+a))*cos(e*x+d)^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \cos^n(d+ex) \, dx = \int F^{(bx+a)c} \cos(ex+d)^n \, dx$$

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^n,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*cos(e*x + d)^n, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \cos^n(d+ex) \, dx = \int F^{c(a+bx)} \cos^n(d+ex) \, dx$$

```
[In] integrate(F**(c*(b*x+a))*cos(e*x+d)**n,x)
[Out] Integral(F**(c*(a + b*x))*cos(d + e*x)**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \cos^n(d+ex) \, dx = \int F^{(bx+a)c} \cos(ex+d)^n \, dx$$

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^n,x, algorithm="maxima")
[Out] integrate(F^((b*x + a)*c)*cos(e*x + d)^n, x)
```

Giac [F]

$$\int F^{c(a+bx)} \cos^n(d+ex)\,dx = \int F^{(bx+a)c} \cos(ex+d)^n\,dx$$

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^n,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*cos(e*x + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \cos^n(d+ex)\,dx = \int F^{c\,(a+b\,x)} \cos(d+e\,x)^n\,dx$$

```
[In] int(F^(c*(a + b*x))*cos(d + e*x)^n,x)
[Out] int(F^(c*(a + b*x))*cos(d + e*x)^n, x)
```

3.11 $\int F^{c(a+bx)} \cos^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 199

$$\int F^{c(a+bx)} \cos^3(d+ex)\,dx = \frac{bcF^{c(a+bx)}\cos^3(d+ex)\log(F)}{9e^2+b^2c^2\log^2(F)} + \frac{6bce^2F^{c(a+bx)}\cos(d+ex)\log(F)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{3eF^{c(a+bx)}\cos^2(d+ex)\sin(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{6e^3F^{c(a+bx)}\sin(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}$$

```
[Out] b*c*F^(c*(b*x+a))*cos(e*x+d)^3*ln(F)/(9*e^2+b^2*c^2*ln(F)^2)+6*b*c*e^2*F^(c
*(b*x+a))*cos(e*x+d)*ln(F)/(9*e^4+10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^4)+3
*e*F^(c*(b*x+a))*cos(e*x+d)^2*sin(e*x+d)/(9*e^2+b^2*c^2*ln(F)^2)+6*e^3*F^(c
*(b*x+a))*sin(e*x+d)/(9*e^4+10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^4)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{4520, 4518\}$

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \frac{bc\log(F)\cos^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2} + \frac{3e\sin(d+ex)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2} + \frac{6bce^2\log(F)\cos(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)+10b^2c^2e^2\log^2(F)+9e^4} + \frac{6e^3\sin(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)+10b^2c^2e^2\log^2(F)+9e^4}$$

```
[In] Int[F^(c*(a + b*x))*Cos[d + e*x]^3,x]
```

```
[Out] (b*c*F^(c*(a + b*x))*Cos[d + e*x]^3*Log[F])/(9*e^2 + b^2*c^2*Log[F]^2) + (6
*b*c*e^2*F^(c*(a + b*x))*Cos[d + e*x]*Log[F])/(9*e^4 + 10*b^2*c^2*e^2*Log[F
]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cos[d + e*x]^2*Sin[d + e*x])
/(9*e^2 + b^2*c^2*Log[F]^2) + (6*e^3*F^(c*(a + b*x))*Sin[d + e*x])/(9*e^4 +
 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4)
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4520

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[e*m*F^(c*(a + b*x))*Sin[d +
 e*x]*(Cos[d + e*x]^(m - 1)/(e^2*m^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bcF^{c(a+bx)}\cos^3(d+ex)\log(F)}{9e^2+b^2c^2\log^2(F)} + \frac{3eF^{c(a+bx)}\cos^2(d+ex)\sin(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{(6e^2)\int F^{c(a+bx)}\cos(d+ex)\,dx}{9e^2+b^2c^2\log^2(F)}\\ &= \frac{bcF^{c(a+bx)}\cos^3(d+ex)\log(F)}{9e^2+b^2c^2\log^2(F)} + \frac{6bce^2F^{c(a+bx)}\cos(d+ex)\log(F)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{3eF^{c(a+bx)}\cos^2(d+ex)\sin(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{6e^3F^{c(a+bx)}\sin(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx$$
$$=\frac{F^{c(a+bx)}\left(bc\cos(3(d+ex))\log(F)\left(e^2+b^2c^2\log^2(F)\right)+3bc\cos(d+ex)\log(F)\left(9e^2+b^2c^2\log^2(F)\right)+6e(\right.}{4\left(9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F\right.}$$

```
[In] Integrate[F^(c*(a + b*x))*Cos[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(b*c*Cos[3*(d + e*x)]*Log[F]*(e^2 + b^2*c^2*Log[F]^2) + 3*
b*c*Cos[d + e*x]*Log[F]*(9*e^2 + b^2*c^2*Log[F]^2) + 6*e*(5*e^2 + b^2*c^2*L
og[F]^2 + Cos[2*(d + e*x)]*(e^2 + b^2*c^2*Log[F]^2))*Sin[d + e*x]))/(4*(9*e
^4 + 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{F^{c(xb+a)}\left(bc\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\cos(3ex+3d)+\left(3\ln(F)^2b^2c^2e+3e^3\right)\sin(3ex+3d)+3\left(9e^2+b^2c^2\ln(F)^2\right)(\cos(ex+d)\ln(F\right.}{4b^4c^4\ln(F)^4+40b^2c^2e^2\ln(F)^2+36e^4}$
risch	$\frac{3bc\,F^{c(xb+a)}\cos(ex+d)\ln(F)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{3e\,F^{c(xb+a)}\sin(ex+d)}{4\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{F^{c(xb+a)}bc\ln(F)\cos(3ex+3d)}{4b^2c^2\ln(F)^2+36e^2}+\frac{3e\,F^{c(xb+a)}\sin(3ex+3d)}{4\left(9e^2+b^2c^2\ln(F)^2\right)}$
norman	$\frac{\ln(F)bc\left(b^2c^2\ln(F)^2+7e^2\right)e^{c(xb+a)\ln(F)}}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}-\frac{12e\left(b^2c^2\ln(F)^2-e^2\right)e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4}+\frac{6e\left(b^2c^2\ln(F)^2+3e^2\right)e^{c(xb+a)\ln(F)}\tan}{9e^4+10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F}$

```
[In] int(F^(c*(b*x+a))*cos(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] F^(c*(b*x+a))*(b*c*ln(F)*(e^2+b^2*c^2*ln(F)^2)*cos(3*e*x+3*d)+(3*ln(F)^2*b^
2*c^2*e+3*e^3)*sin(3*e*x+3*d)+3*(9*e^2+b^2*c^2*ln(F)^2)*(cos(e*x+d)*ln(F)*b
*c+e*sin(e*x+d)))/(4*b^4*c^4*ln(F)^4+40*b^2*c^2*e^2*ln(F)^2+36*e^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \frac{\big(b^3c^3\cos(ex+d)^3\log(F)^3 + \big(bce^2\cos(ex+d)^3 + 6\,bce^2\cos(ex+d)\big)\log(F) + 3\,\big(b^2c^2e\cos(ex+d)^2\log(}{b^4c^4\log(F)^4 + 10\,b^2c^2e^2\log(F)^2 + 9\,e^4}$$

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] (b^3*c^3*cos(e*x + d)^3*log(F)^3 + (b*c*e^2*cos(e*x + d)^3 + 6*b*c*e^2*cos(
e*x + d))*log(F) + 3*(b^2*c^2*e*cos(e*x + d)^2*log(F)^2 + e^3*cos(e*x + d)^
2 + 2*e^3)*sin(e*x + d))*F^(b*c*x + a*c)/(b^4*c^4*log(F)^4 + 10*b^2*c^2*e^2
*log(F)^2 + 9*e^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.23 (sec) , antiderivative size = 1671, normalized size of antiderivative = 8.40

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F**(c*(b*x+a))*cos(e*x+d)**3,x)
```

```
[Out] Piecewise((x*cos(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cos(d)**3, Eq(b,
0) & Eq(e, 0)), (x*cos(d)**3, Eq(c, 0) & Eq(e, 0)), (3*I*F**(a*c + b*c*x)*x
*sin(I*b*c*x*log(F) - d)**3/8 + 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F) - d
)**2*cos(I*b*c*x*log(F) - d)/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)
- d)*cos(I*b*c*x*log(F) - d)**2/8 + 3*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)
 - d)**3/8 - 5*I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)**3/(8*b*c*log(F))
 - F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)**2*cos(I*b*c*x*log(F) - d)/(4*b
*c*log(F)) - I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) - d)*cos(I*b*c*x*log(F)
- d)**2/(b*c*log(F)) - 3*F**(a*c + b*c*x)*cos(I*b*c*x*log(F) - d)**3/(8*b*c
*log(F)), Eq(e, -I*b*c*log(F))), (-I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/
3 - d)**3/8 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 - d)**2*cos(I*b*c*x
*log(F)/3 - d)/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 - d)*cos(I*b
*c*x*log(F)/3 - d)**2/8 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/3 - d)**3/8
 + I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)) + 3*I*F**
(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 - d)*cos(I*b*c*x*log(F)/3 - d)**2/(4*b*c
*log(F)) + 9*F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)),
Eq(e, -I*b*c*log(F)/3)), (-I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 + d)**
```

```
3/8 - 3*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 + d)**2*cos(I*b*c*x*log(F)/
3 + d)/8 + 3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/3 + d)*cos(I*b*c*x*log
(F)/3 + d)**2/8 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/3 + d)**3/8 + I*F**
(a*c + b*c*x)*sin(I*b*c*x*log(F)/3 + d)**3/(8*b*c*log(F)) + 3*I*F**(a*c + b
*c*x)*sin(I*b*c*x*log(F)/3 + d)*cos(I*b*c*x*log(F)/3 + d)**2/(4*b*c*log(F))
 + 9*F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/3 + d)**3/(8*b*c*log(F)), Eq(e, I*
b*c*log(F)/3)), (3*I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F) + d)**3/8 + 3*F*
*(a*c + b*c*x)*x*sin(I*b*c*x*log(F) + d)**2*cos(I*b*c*x*log(F) + d)/8 + 3*I
*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F) + d)*cos(I*b*c*x*log(F) + d)**2/8 +
3*F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F) + d)**3/8 - 5*I*F**(a*c + b*c*x)*si
n(I*b*c*x*log(F) + d)**3/(8*b*c*log(F)) - F**(a*c + b*c*x)*sin(I*b*c*x*log(
F) + d)**2*cos(I*b*c*x*log(F) + d)/(4*b*c*log(F)) - I*F**(a*c + b*c*x)*sin(
I*b*c*x*log(F) + d)*cos(I*b*c*x*log(F) + d)**2/(b*c*log(F)) - 3*F**(a*c + b
*c*x)*cos(I*b*c*x*log(F) + d)**3/(8*b*c*log(F)), Eq(e, I*b*c*log(F))), (F**
(a*c + b*c*x)*b**3*c**3*log(F)**3*cos(d + e*x)**3/(b**4*c**4*log(F)**4 + 10
*b**2*c**2*e**2*log(F)**2 + 9*e**4) + 3*F**(a*c + b*c*x)*b**2*c**2*e*log(F)
**2*sin(d + e*x)*cos(d + e*x)**2/(b**4*c**4*log(F)**4 + 10*b**2*c**2*e**2*l
og(F)**2 + 9*e**4) + 6*F**(a*c + b*c*x)*b*c*e**2*log(F)*sin(d + e*x)**2*cos
(d + e*x)/(b**4*c**4*log(F)**4 + 10*b**2*c**2*e**2*log(F)**2 + 9*e**4) + 7*
F**(a*c + b*c*x)*b*c*e**2*log(F)*cos(d + e*x)**3/(b**4*c**4*log(F)**4 + 10*
b**2*c**2*e**2*log(F)**2 + 9*e**4) + 6*F**(a*c + b*c*x)*e**3*sin(d + e*x)**
3/(b**4*c**4*log(F)**4 + 10*b**2*c**2*e**2*log(F)**2 + 9*e**4) + 9*F**(a*c
+ b*c*x)*e**3*sin(d + e*x)*cos(d + e*x)**2/(b**4*c**4*log(F)**4 + 10*b**2*c
**2*e**2*log(F)**2 + 9*e**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(199) = 398$.

Time = 0.25 (sec) , antiderivative size = 813, normalized size of antiderivative = 4.09

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx$$
$$= \frac{\left(F^{ac}b^3c^3\cos\left(3\,d\right)\log\left(F\right)^3 + 3\,F^{ac}b^2c^2e\log\left(F\right)^2\sin\left(3\,d\right) + F^{ac}bce^2\cos\left(3\,d\right)\log\left(F\right) + 3\,F^{ac}e^3\sin\left(3\,d\right)\right)F^{bc}}{}$$

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 + 3*F^(a*c)*b^2*c^2*e*log(F)^2*sin(
3*d) + F^(a*c)*b*c*e^2*cos(3*d)*log(F) + 3*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*
cos(3*e*x) + (F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - 3*F^(a*c)*b^2*c^2*e*log(F
)^2*sin(3*d) + F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 3*F^(a*c)*e^3*sin(3*d))*F^
(b*c*x)*cos(3*e*x + 6*d) + 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - F^(a*c)*b
^2*c^2*e*log(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 9*F^(a*c)*
e^3*sin(3*d))*F^(b*c*x)*cos(e*x + 4*d) + 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)
```

```
^3 + F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F
) + 9*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*cos(e*x - 2*d) - (F^(a*c)*b^3*c^3*log
(F)^3*sin(3*d) - 3*F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + F^(a*c)*b*c*e^2*lo
g(F)*sin(3*d) - 3*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*sin(3*e*x) + (F^(a*c)*b^3
*c^3*log(F)^3*sin(3*d) + 3*F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + F^(a*c)*b*
c*e^2*log(F)*sin(3*d) + 3*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*sin(3*e*x + 6*d)
+ 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^
2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 9*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*s
in(e*x + 4*d) - 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) - F^(a*c)*b^2*c^2*e*co
s(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*d) - 9*F^(a*c)*e^3*cos(3*d
))*F^(b*c*x)*sin(e*x - 2*d))/(b^4*c^4*cos(3*d)^2*log(F)^4 + b^4*c^4*log(F)^
4*sin(3*d)^2 + 9*(cos(3*d)^2 + sin(3*d)^2)*e^4 + 10*(b^2*c^2*cos(3*d)^2*log
(F)^2 + b^2*c^2*log(F)^2*sin(3*d)^2)*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1271, normalized size of antiderivative = 6.39

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2
*pi*a*c + 3*e*x + 3*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c + 6*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 6*e)*sin(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 3*e*x + 3*d)/(4*b^2*c^
2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 6*e)^2))*e^(b*c*x*log(abs(F)) +
 a*c*log(abs(F))) + 3/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2
*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2
 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*sin(1
/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x +
d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*l
og(abs(F)) + a*c*log(abs(F))) + 3/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi
*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)*log(abs(F))/(4*b^2*c^2*l
og(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c
- 2*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*
a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2)
)*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*(2*b*c*cos(1/2*pi*b*c*x*sgn
(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 3*e*x - 3*d)*log(abs(
F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 6*e)^2) + (pi*b*c*
sgn(F) - pi*b*c - 6*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*
sgn(F) - 1/2*pi*a*c - 3*e*x - 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c - 6*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*e^(1/2
```

```
*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c +
3*I*e*x + 3*I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*
I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) +
 1/2*I*pi*a*c - 3*I*e*x - 3*I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*
log(abs(F)) - 48*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*I*(I*e^(
1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c
 + I*e*x + I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*I
*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) +
1/2*I*pi*a*c - I*e*x - I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(a
bs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*I*(I*e^(1/2*I
*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - I*
e*x - I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 16*I*e) -
 I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I
*pi*a*c + I*e*x + I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)
) + 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c
*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - 3*I*e*x -
 3*I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 48*I*e) - I*
e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi
*a*c + 3*I*e*x + 3*I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F
)) + 48*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 28.71 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.96

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx$$
$$= -\frac{F^{c\,(a+b\,x)}\,(\cos(e\,x)+\sin(e\,x)\ 1\mathrm{i})\ (\cos(d)+\sin(d)\ 1\mathrm{i})\ 3\mathrm{i}}{8\ (e-b\,c\,\ln(F)\ 1\mathrm{i})} - \frac{F^{c\,(a+b\,x)}\,(\cos(3\,e\,x)-\sin(3\,e\,x)\ 1\mathrm{i})\ (\cos(3\,d)-\sin(3\,d)\ 1\mathrm{i})}{8\ (-b\,c\,\ln(F)+e\,3\mathrm{i})} - \frac{F^{c\,(a+b\,x)}\,(\cos(3\,e\,x)+\sin(3\,e\,x)\ 1\mathrm{i})\ (\cos(3\,d)+\sin(3\,d)\ 1\mathrm{i})\ 1\mathrm{i}}{8\ (3\,e-b\,c\,\ln(F)\ 1\mathrm{i})} - \frac{3\,F^{c\,(a+b\,x)}\,(\cos(e\,x)-\sin(e\,x)\ 1\mathrm{i})\ (\cos(d)-\sin(d)\ 1\mathrm{i})}{8\ (-b\,c\,\ln(F)+e\,1\mathrm{i})}$$

```
[In] int(F^(c*(a + b*x))*cos(d + e*x)^3,x)
```

```
[Out] - (F^(c*(a + b*x))*(cos(e*x) + sin(e*x)*1i)*(cos(d) + sin(d)*1i)*3i)/(8*(e
- b*c*log(F)*1i)) - (F^(c*(a + b*x))*(cos(3*e*x) - sin(3*e*x)*1i)*(cos(3*d)
 - sin(3*d)*1i))/(8*(e*3i - b*c*log(F))) - (F^(c*(a + b*x))*(cos(3*e*x) + s
in(3*e*x)*1i)*(cos(3*d) + sin(3*d)*1i)*1i)/(8*(3*e - b*c*log(F)*1i)) - (3*F
^(c*(a + b*x))*(cos(e*x) - sin(e*x)*1i)*(cos(d) - sin(d)*1i))/(8*(e*1i - b*
c*log(F)))
```

3.12 $\int F^{c(a+bx)}\cos^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 128

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx=\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}+\frac{bcF^{c(a+bx)}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)}+\frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}$$

```
[Out] 2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+b*c*F^(c*(b*x+a))*cos
(e*x+d)^2*ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+2*e*F^(c*(b*x+a))*cos(e*x+d)*sin(e*
x+d)/(4*e^2+b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4520, 2225}

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx=\frac{bc\log(F)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}$$

```
[In] Int[F^(c*(a + b*x))*Cos[d + e*x]^2,x]
```

```
[Out] (2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]^2)) + (b*c*F^(c
*(a + b*x))*Cos[d + e*x]^2*Log[F])/(4*e^2 + b^2*c^2*Log[F]^2) + (2*e*F^(c*(
a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4520

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[e*m*F^(c*(a + b*x))*Sin[d +
 e*x]*(Cos[d + e*x]^(m - 1)/(e^2*m^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bcF^{c(a+bx)}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)} \\ &\quad+ \frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)} + \frac{(2e^2)\int F^{c(a+bx)}\,dx}{4e^2+b^2c^2\log^2(F)} \\ &= \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)} + \frac{bcF^{c(a+bx)}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)} \\ &\quad+ \frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.66

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx = \frac{F^{c(a+bx)}\left(4e^2+b^2c^2\log^2(F)+b^2c^2\cos(2(d+ex))\log^2(F)+2bce\log(F)\sin(2(d+ex))\right)}{8bce^2\log(F)+2b^3c^3\log^3(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Cos[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(4*e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cos[2*(d + e*x)]*Log[F
]^2 + 2*b*c*e*Log[F]*Sin[2*(d + e*x)]))/(8*b*c*e^2*Log[F] + 2*b^3*c^3*Log[F
]^3)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{F^{c(xb+a)}\left(b^{2} c^{2} \ln (F)^{2} \cos (2 e x+2 d)+b^{2} c^{2} \ln (F)^{2}+2 e \sin (2 e x+2 d) b c \ln (F)+4 e^{2}\right)}{2 b c \ln (F)\left(4 e^{2}+b^{2} c^{2} \ln (F)^{2}\right)}$
risch	$\frac{F^{c(xb+a)}}{2 b c \ln (F)}+\frac{F^{c(xb+a)} b c \ln (F) \cos (2 e x+2 d)}{2 b^{2} c^{2} \ln (F)^{2}+8 e^{2}}+\frac{e F^{c(xb+a)} \sin (2 e x+2 d)}{4 e^{2}+b^{2} c^{2} \ln (F)^{2}}$
norman	$\frac{\frac{\left(b^{2} c^{2} \ln (F)^{2}+2 e^{2}\right) \mathrm{e}^{c(xb+a) \ln (F)}}{b c \ln (F)\left(4 e^{2}+b^{2} c^{2} \ln (F)^{2}\right)}+\frac{\left(b^{2} c^{2} \ln (F)^{2}+2 e^{2}\right) \mathrm{e}^{c(xb+a) \ln (F)} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{4}}{b c \ln (F)\left(4 e^{2}+b^{2} c^{2} \ln (F)^{2}\right)}+\frac{4 e\, \mathrm{e}^{c(xb+a) \ln (F)} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{4 e^{2}+b^{2} c^{2} \ln (F)^{2}}-\frac{4 e\, \mathrm{e}^{c(xb+a) \ln (F)} \mathrm{t}}{4 e^{2}+b^{2} c^{2} \ln}}{\left(1+\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}\right)^{2}}$

```
[In] int(F^(c*(b*x+a))*cos(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*F^(c*(b*x+a))*(b^2*c^2*ln(F)^2*cos(2*e*x+2*d)+b^2*c^2*ln(F)^2+2*e*sin(2
*e*x+2*d)*b*c*ln(F)+4*e^2)/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int F^{c(a+bx)} \cos^2(d+ex)\, dx = \frac{\left(b^2c^2\cos\left(ex+d\right)^2\log\left(F\right)^2+2\,bce\cos\left(ex+d\right)\log\left(F\right)\sin\left(ex+d\right)+2\,e^2\right)F^{bcx+ac}}{b^3c^3\log\left(F\right)^3+4\,bce^2\log\left(F\right)}$$

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c^2*cos(e*x + d)^2*log(F)^2 + 2*b*c*e*cos(e*x + d)*log(F)*sin(e*x + d)
 + 2*e^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + 4*b*c*e^2*log(F))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 741, normalized size of antiderivative = 5.79

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx$$

$$= \begin{cases} x\cos^2(d) \\ \frac{x\sin^2(d+ex)}{2} + \frac{x\cos^2(d+ex)}{2} + \frac{\sin(d+ex)\cos(d+ex)}{2e} \\ F^{ac}\left(\frac{x\sin^2(d+ex)}{2} + \frac{x\cos^2(d+ex)}{2} + \frac{\sin(d+ex)\cos(d+ex)}{2e}\right) \\ \frac{x\sin^2(d+ex)}{2} + \frac{x\cos^2(d+ex)}{2} + \frac{\sin(d+ex)\cos(d+ex)}{2e} \\ -\frac{F^{ac+bcx}x\sin^2\left(\frac{ibcx\log(F)}{2}-d\right)}{4} + \frac{iF^{ac+bcx}x\sin\left(\frac{ibcx\log(F)}{2}-d\right)\cos\left(\frac{ibcx\log(F)}{2}-d\right)}{2} + \frac{F^{ac+bcx}x\cos^2\left(\frac{ibcx\log(F)}{2}-d\right)}{4} + \frac{iF^{ac+bcx}\mathrm{s}}{} \\ -\frac{F^{ac+bcx}x\sin^2\left(\frac{ibcx\log(F)}{2}+d\right)}{4} + \frac{iF^{ac+bcx}x\sin\left(\frac{ibcx\log(F)}{2}+d\right)\cos\left(\frac{ibcx\log(F)}{2}+d\right)}{2} + \frac{F^{ac+bcx}x\cos^2\left(\frac{ibcx\log(F)}{2}+d\right)}{4} + \frac{F^{ac+bcx}\mathrm{si}}{} \\ \frac{F^{ac+bcx}b^2c^2\log(F)^2\cos^2(d+ex)}{b^3c^3\log(F)^3+4bce^2\log(F)} + \frac{2F^{ac+bcx}bce\log(F)\sin(d+ex)\cos(d+ex)}{b^3c^3\log(F)^3+4bce^2\log(F)} + \frac{2F^{ac+bcx}e^2\sin^2(d+ex)}{b^3c^3\log(F)^3+4bce^2\log(F)} + \frac{2F^{ac+bcx}e^2\cos^2(d+e}{b^3c^3\log(F)^3+4bce^2\log} \end{cases}$$

```
[In] integrate(F**(c*(b*x+a))*cos(e*x+d)**2,x)
```

```
[Out] Piecewise((x*cos(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (x*sin(
d + e*x)**2/2 + x*cos(d + e*x)**2/2 + sin(d + e*x)*cos(d + e*x)/(2*e), Eq(F
, 1)), (F**(a*c)*(x*sin(d + e*x)**2/2 + x*cos(d + e*x)**2/2 + sin(d + e*x)*
cos(d + e*x)/(2*e)), Eq(b, 0)), (x*sin(d + e*x)**2/2 + x*cos(d + e*x)**2/2
+ sin(d + e*x)*cos(d + e*x)/(2*e), Eq(c, 0)), (-F**(a*c + b*c*x)*x*sin(I*b*
c*x*log(F)/2 - d)**2/4 + I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 - d)*cos
(I*b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 - d)**2/
4 + I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F)/2 - d)*cos(I*b*c*x*log(F)/2 - d)/
(2*b*c*log(F)) + F**(a*c + b*c*x)*cos(I*b*c*x*log(F)/2 - d)**2/(b*c*log(F))
, Eq(e, -I*b*c*log(F)/2)), (-F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 + d)**
2/4 + I*F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F)/2 + d)*cos(I*b*c*x*log(F)/2 +
 d)/2 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F)/2 + d)**2/4 + F**(a*c + b*c*x
)*sin(I*b*c*x*log(F)/2 + d)**2/(b*c*log(F)) - 3*I*F**(a*c + b*c*x)*sin(I*b*
c*x*log(F)/2 + d)*cos(I*b*c*x*log(F)/2 + d)/(2*b*c*log(F)), Eq(e, I*b*c*log
(F)/2)), (F**(a*c + b*c*x)*b**2*c**2*log(F)**2*cos(d + e*x)**2/(b**3*c**3*l
og(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a*c + b*c*x)*b*c*e*log(F)*sin(d + e*x
)*cos(d + e*x)/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a*c + b*c*
x)*e**2*sin(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a
*c + b*c*x)*e**2*cos(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)),
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(128) = 256.

Time = 0.23 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.78

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx$$
$$= \frac{\left(F^{ac}b^2c^2\cos(2d)\log(F)^2 + 2F^{ac}bce\log(F)\sin(2d)\right)F^{bcx}\cos(2ex) + \left(F^{ac}b^2c^2\cos(2d)\log(F)^2 - 2F^{ac}bc\right.}{}$$

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))*
F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c*e
*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2*si
n(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a*c)*b
^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2
*e*x + 4*d) + 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c^2*log(
F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*F^(b*c*x
))/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 + 4*(b*c*cos(
2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 915, normalized size of antiderivative = 7.15

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2
*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*sin(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)/(4*b^2*c^
2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(abs(F)) +
 a*c*log(abs(F))) + 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2
*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F
))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 4*e)*s
in(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*
e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2))*e^
(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1
/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(ab
```

```
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*p
i*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*
log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log
(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*s
gn(F) - 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8
*b*c*log(abs(F)) + 16*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(-4*I*pi*b*c*sgn(F)
+ 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(
abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sg
n(F) - 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*
b*c*log(abs(F)) - 16*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(-4*I*pi*b*c*sgn(F) +
 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(a
bs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn
(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) -
I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*
pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log
(abs(F)) + a*c*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 28.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.77

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx = \frac{2\,F^{a\,c+b\,c\,x}\,e^2+F^{a\,c+b\,c\,x}\,b^2\,c^2\cos\left(d+e\,x\right)^2\ln\left(F\right)^2+2\,F^{a\,c+b\,c\,x}\,b\,c\,e\,\cos\left(d+e\,x\right)\,\sin\left(d+e\,x\right)\,\ln\left(F\right)}{b^3\,c^3\ln\left(F\right)^3+4\,b\,c\,e^2\,\ln\left(F\right)}$$

```
[In] int(F^(c*(a + b*x))*cos(d + e*x)^2,x)
```

```
[Out] (2*F^(a*c + b*c*x)*e^2 + F^(a*c + b*c*x)*b^2*c^2*cos(d + e*x)^2*log(F)^2 +
2*F^(a*c + b*c*x)*b*c*e*cos(d + e*x)*sin(d + e*x)*log(F))/(b^3*c^3*log(F)^3
 + 4*b*c*e^2*log(F))
```

3.13 $\int F^{c(a+bx)}\cos(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int F^{c(a+bx)}\cos(d+ex)\,dx=\frac{bcF^{c(a+bx)}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{eF^{c(a+bx)}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

```
[Out] b*c*F^(c*(b*x+a))*cos(e*x+d)*ln(F)/(e^2+b^2*c^2*ln(F)^2)+e*F^(c*(b*x+a))*si
n(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4518}

$$\int F^{c(a+bx)}\cos(d+ex)\,dx=\frac{e\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}$$

```
[In] Int[F^(c*(a + b*x))*Cos[d + e*x],x]
```

```
[Out] (b*c*F^(c*(a + b*x))*Cos[d + e*x]*Log[F])/(e^2 + b^2*c^2*Log[F]^2) + (e*F^(
c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral}=\frac{bcF^{c(a+bx)}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{eF^{c(a+bx)}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

$$\int F^{c(a+bx)}\cos(d+ex)\,dx=\frac{F^{c(a+bx)}(bc\cos(d+ex)\log(F)+e\sin(d+ex))}{e^2+b^2c^2\log^2(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Cos[d + e*x],x]
[Out] (F^(c*(a + b*x))*(b*c*Cos[d + e*x]*Log[F] + e*Sin[d + e*x]))/(e^2 + b^2*c^2
*Log[F]^2)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{F^{c(xb+a)}(\cos(ex+d)\ln(F)bc+e\sin(ex+d))}{e^2+b^2c^2\ln(F)^2}$	48
risch	$\frac{bc\,F^{c(xb+a)}\cos(ex+d)\ln(F)}{e^2+b^2c^2\ln(F)^2}+\frac{e\,F^{c(xb+a)}\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}$	73
norman	$\frac{\frac{bc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2e\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}-\frac{bc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$	133

```
[In] int(F^(c*(b*x+a))*cos(e*x+d),x,method=_RETURNVERBOSE)
[Out] F^(c*(b*x+a))*(cos(e*x+d)*ln(F)*b*c+e*sin(e*x+d))/(e^2+b^2*c^2*ln(F)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)}\cos(d+ex)\,dx=\frac{(bc\cos(ex+d)\log(F)+e\sin(ex+d))F^{bcx+ac}}{b^2c^2\log(F)^2+e^2}$$

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="fricas")
[Out] (b*c*cos(e*x + d)*log(F) + e*sin(e*x + d))*F^(b*c*x + a*c)/(b^2*c^2*log(F)^
2 + e^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.97

$$\int F^{c(a+bx)}\cos(d+ex)\,dx = \begin{cases} x\cos(d) & \text{for } F=1 \wedge e=0 \\ F^{ac}x\cos(d) & \text{for } b=0 \wedge e=0 \\ x\cos(d) & \text{for } c=0 \wedge e=0 \\ \frac{iF^{ac+bcx}x\sin(ibcx\log(F)-d)}{2}+\frac{F^{ac+bcx}x\cos(ibcx\log(F)-d)}{2}-\frac{iF^{ac+bcx}\sin(ibcx\log(F)-d)}{2bc\log(F)} & \text{for } e=-ibc\log(F) \\ \frac{iF^{ac+bcx}x\sin(ibcx\log(F)+d)}{2}+\frac{F^{ac+bcx}x\cos(ibcx\log(F)+d)}{2}-\frac{iF^{ac+bcx}\sin(ibcx\log(F)+d)}{2bc\log(F)} & \text{for } e=ibc\log(F) \\ \frac{F^{ac+bcx}bc\log(F)\cos(d+ex)}{b^2c^2\log(F)^2+e^2}+\frac{F^{ac+bcx}e\sin(d+ex)}{b^2c^2\log(F)^2+e^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(F**(c*(b*x+a))*cos(e*x+d),x)
```

```
[Out] Piecewise((x*cos(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cos(d), Eq(b, 0) & E
q(e, 0)), (x*cos(d), Eq(c, 0) & Eq(e, 0)), (I*F**(a*c + b*c*x)*x*sin(I*b*c*
x*log(F) - d)/2 + F**(a*c + b*c*x)*x*cos(I*b*c*x*log(F) - d)/2 - I*F**(a*c
+ b*c*x)*sin(I*b*c*x*log(F) - d)/(2*b*c*log(F)), Eq(e, -I*b*c*log(F))), (I*
F**(a*c + b*c*x)*x*sin(I*b*c*x*log(F) + d)/2 + F**(a*c + b*c*x)*x*cos(I*b*c
*x*log(F) + d)/2 - I*F**(a*c + b*c*x)*sin(I*b*c*x*log(F) + d)/(2*b*c*log(F)
), Eq(e, I*b*c*log(F))), (F**(a*c + b*c*x)*b*c*log(F)*cos(d + e*x)/(b**2*c*
*2*log(F)**2 + e**2) + F**(a*c + b*c*x)*e*sin(d + e*x)/(b**2*c**2*log(F)**2
 + e**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(72) = 144.

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.67

$$\int F^{c(a+bx)}\cos(d+ex)\,dx = \frac{(F^{ac}bc\cos(d)\log(F)-F^{ac}e\sin(d))F^{bcx}\cos(ex+2\,d)+(F^{ac}bc\cos(d)\log(F)+F^{ac}e\sin(d))F^{bcx}\cos(ex)}{2\,(b^2c^2\cos(d)^2\log(F)^2+b^2c^2\log(}$$

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x + 2*d
) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x) + (F^
(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^(
a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x))/(b^2*c^2*cos
(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 631, normalized size of antiderivative = 8.76

$$\int F^{c(a+bx)} \cos(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="giac")
```

```
[Out] (2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*
a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b
*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2
*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F)
)^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs
(F))) + (2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) -
 1/2*pi*a*c - e*x - d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 2*e)*sin(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*lo
g(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c
*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a
*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4
*b*c*log(abs(F)) + 4*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I
*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)
)) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) -
 1/2*I*pi*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(ab
s(F)) - 4*I*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*
c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4
*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 27.93 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)} \cos(d+ex)\,dx = \frac{F^{a\,c+b\,c\,x}\,\left(e\,\sin\left(d+e\,x\right)+b\,c\,\cos\left(d+e\,x\right)\,\ln\left(F\right)\right)}{b^2\,c^2\,{\ln\left(F\right)}^2+e^2}$$

```
[In] int(F^(c*(a + b*x))*cos(d + e*x),x)
```

```
[Out] (F^(a*c + b*c*x)*(e*sin(d + e*x) + b*c*cos(d + e*x)*log(F)))/(e^2 + b^2*c^2
*log(F)^2)
```

3.14 $\int F^{c(a+bx)} \sec(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int F^{c(a+bx)} \sec(d+ex)\,dx = \frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{ie+bc\log(F)}$$

```
[Out] 2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e],[3/2-1/
2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/(b*c*ln(F)+I*e)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4536}

$$\int F^{c(a+bx)} \sec(d+ex)\,dx = \frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{bc\log(F)+ie}$$

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x],x]
```

```
[Out] (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])/
(2*e), (3 - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))])/(I*e + b*c*Log[F])
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
```

```
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{ie+bc\log(F)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\sec(d+ex)\,dx = \frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{ibc\log(F)}{2e},\frac{3}{2}-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{ie+bc\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x],x]
[Out] (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, 1/2 - ((I/2)*b*c*Lo
g[F])/e, 3/2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/(I*e + b*c*Log[
F])
```

Maple [F]

$$\int F^{c(xb+a)}\sec(ex+d)\,dx$$

```
[In] int(F^(c*(b*x+a))*sec(e*x+d),x)
[Out] int(F^(c*(b*x+a))*sec(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sec(d+ex)\,dx = \int F^{(bx+a)c}\sec(ex+d)\,dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*sec(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)} \sec(d+ex)\,dx = \int F^{c(a+bx)} \sec\left(d+ex\right)dx$$

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))*sec(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)} \sec(d+ex)\,dx = \int F^{(bx+a)c} \sec\left(ex+d\right)\,dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*(F^(b*c*x)*F^(a*c)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*F^(a*c)*e*sin(e*x
+ d) + (F^(b*c*x)*F^(a*c)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*F^(a*c)*e*sin
(e*x + d))*cos(2*e*x + 2*d) + 2*(F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3 +
 (F^(a*c)*b^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*cos(2*e*x + 2*d)^2 + (F^(a*c)*b
^2*c^2*e*log(F)^2 + F^(a*c)*e^3)*sin(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^2*c^2*e*
log(F)^2 + F^(a*c)*e^3)*cos(2*e*x + 2*d))*integrate((F^(b*c*x)*b*c*log(F)*s
in(e*x + d) + F^(b*c*x)*e*cos(e*x + d) + (F^(b*c*x)*b*c*log(F)*sin(e*x + d)
 + F^(b*c*x)*e*cos(e*x + d))*cos(4*e*x + 4*d) + 2*(F^(b*c*x)*b*c*log(F)*sin
(e*x + d) + F^(b*c*x)*e*cos(e*x + d))*cos(2*e*x + 2*d) - (F^(b*c*x)*b*c*cos
(e*x + d)*log(F) - F^(b*c*x)*e*sin(e*x + d))*sin(4*e*x + 4*d) - 2*(F^(b*c*x
)*b*c*cos(e*x + d)*log(F) - F^(b*c*x)*e*sin(e*x + d))*sin(2*e*x + 2*d))/(b^
2*c^2*log(F)^2 + (b^2*c^2*log(F)^2 + e^2)*cos(4*e*x + 4*d)^2 + 4*(b^2*c^2*l
og(F)^2 + e^2)*cos(2*e*x + 2*d)^2 + (b^2*c^2*log(F)^2 + e^2)*sin(4*e*x + 4*
d)^2 + 4*(b^2*c^2*log(F)^2 + e^2)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(b^
2*c^2*log(F)^2 + e^2)*sin(2*e*x + 2*d)^2 + e^2 + 2*(b^2*c^2*log(F)^2 + e^2
+ 2*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) + 4*(b^2*c^
2*log(F)^2 + e^2)*cos(2*e*x + 2*d)), x) + (F^(b*c*x)*F^(a*c)*b*c*log(F)*sin
(e*x + d) + F^(b*c*x)*F^(a*c)*e*cos(e*x + d))*sin(2*e*x + 2*d))/(b^2*c^2*lo
g(F)^2 + (b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)^2 + (b^2*c^2*log(F)^2 +
e^2)*sin(2*e*x + 2*d)^2 + e^2 + 2*(b^2*c^2*log(F)^2 + e^2)*cos(2*e*x + 2*d)
)
```

Giac [F]

$$\int F^{c(a+bx)}\sec(d+ex)\,dx = \int F^{(bx+a)c}\sec\left(ex+d\right)\,dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sec(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cos\left(d+e\,x\right)}\,dx$$

```
[In] int(F^(c*(a + b*x))/cos(d + e*x),x)
[Out] int(F^(c*(a + b*x))/cos(d + e*x), x)
```

3.15 $\int F^{c(a+bx)} \sec^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int F^{c(a+bx)} \sec^2(d+ex)\,dx = \frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

```
[Out] 4*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c*ln(F)/e],[2-1/2*
I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))/(2*I*e+b*c*ln(F))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4536}

$$\int F^{c(a+bx)} \sec^2(d+ex)\,dx = \frac{4e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x]^2,x]
```

```
[Out] (4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c*
Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/((2*I)*e + b*c*
Log[F])
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
```

```
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{4e^{2i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx = \frac{4e^{2i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x]^2,x]
[Out] (4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c*
Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/((2*I)*e + b*c*
Log[F])
```

Maple [F]

$$\int F^{c(xb+a)}\sec\left(ex+d\right)^{2}dx$$

```
[In] int(F^(c*(b*x+a))*sec(e*x+d)^2,x)
[Out] int(F^(c*(b*x+a))*sec(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx = \int F^{(bx+a)c}\sec\left(ex+d\right)^{2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*sec(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \sec^2(d+ex)\,dx = \int F^{c(a+bx)} \sec^2(d+ex)\,dx$$

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d)**2,x)
```

```
[Out] Integral(F**(c*(a + b*x))*sec(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \sec^2(d+ex)\,dx = \int F^{(bx+a)c} \sec(ex+d)^2\,dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 4*(24*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3 + 16*F
^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^3*c^3*lo
g(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 + (F^(a*c)
*b^3*c^3*log(F)^3 + 64*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) -
 2*(5*F^(a*c)*b^2*c^2*e*log(F)^2 - 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x + 2*
d) + (24*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + (F^(a*c)*b^3*c^3*log(F)^3 + 16*
F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) - 2*(F^(a*c)*b^2*c^2*e*l
og(F)^2 + 16*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x + 2*d))*cos(4*e*x + 4*d) - 4*
(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*
b*c*e^5*log(F) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F
)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(4*e*x + 4*d)^2 + 4*(F^(a*c)*b^5*c^5*e*
log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos
(2*e*x + 2*d)^2 + (F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(
F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(4*e*x + 4*d)^2 + 4*(F^(a*c)*b^5*c^5*e
*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*si
n(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c
)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*sin(2*e*x + 2*d)^2 + 2*
(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*
b*c*e^5*log(F) + 2*(F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log
(F)^3 + 64*F^(a*c)*b*c*e^5*log(F))*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) + 4*(
F^(a*c)*b^5*c^5*e*log(F)^5 + 20*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 64*F^(a*c)*b
*c*e^5*log(F))*cos(2*e*x + 2*d))*integrate((6*F^(b*c*x)*b*c*e*cos(6*e*x + 6
*d)*log(F) + 18*F^(b*c*x)*b*c*e*cos(4*e*x + 4*d)*log(F) + 18*F^(b*c*x)*b*c*
e*cos(2*e*x + 2*d)*log(F) + 6*F^(b*c*x)*b*c*e*log(F) - (b^2*c^2*log(F)^2 -
8*e^2)*F^(b*c*x)*sin(6*e*x + 6*d) - 3*(b^2*c^2*log(F)^2 - 8*e^2)*F^(b*c*x)*
sin(4*e*x + 4*d) - 3*(b^2*c^2*log(F)^2 - 8*e^2)*F^(b*c*x)*sin(2*e*x + 2*d))
/(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4 + (b^4*c^4*log(F)^4 +
```

```
20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(6*e*x + 6*d)^2 + 9*(b^4*c^4*log(F)^4
 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(4*e*x + 4*d)^2 + 9*(b^4*c^4*log(F)
^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*e*x + 2*d)^2 + (b^4*c^4*log(F)
^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(6*e*x + 6*d)^2 + 9*(b^4*c^4*log(
F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(4*e*x + 4*d)^2 + 18*(b^4*c^4*l
og(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(4*e*x + 4*d)*sin(2*e*x + 2*
d) + 9*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(2*e*x + 2*
d)^2 + 2*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4 + 3*(b^4*c^4*
log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(4*e*x + 4*d) + 3*(b^4*c^4*
log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*e*x + 2*d))*cos(6*e*x +
6*d) + 6*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4 + 3*(b^4*c^4*
log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*e*x + 2*d))*cos(4*e*x +
4*d) + 6*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*cos(2*e*x +
2*d) + 6*((b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(4*e*x +
 4*d) + (b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)*sin(2*e*x + 2
*d))*sin(6*e*x + 6*d)), x) + (2*(F^(a*c)*b^2*c^2*e*log(F)^2 + 16*F^(a*c)*e^
3)*F^(b*c*x)*cos(2*e*x + 2*d) + (F^(a*c)*b^3*c^3*log(F)^3 + 16*F^(a*c)*b*c*
e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) - 4*(F^(a*c)*b^2*c^2*e*log(F)^2 - 8*
F^(a*c)*e^3)*F^(b*c*x))*sin(4*e*x + 4*d))/(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^
2*log(F)^2 + 64*e^4 + (b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^4)
*cos(4*e*x + 4*d)^2 + 4*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^
4)*cos(2*e*x + 2*d)^2 + (b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*e^
4)*sin(4*e*x + 4*d)^2 + 4*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64*
e^4)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e
^2*log(F)^2 + 64*e^4)*sin(2*e*x + 2*d)^2 + 2*(b^4*c^4*log(F)^4 + 20*b^2*c^2
*e^2*log(F)^2 + 64*e^4 + 2*(b^4*c^4*log(F)^4 + 20*b^2*c^2*e^2*log(F)^2 + 64
*e^4)*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) + 4*(b^4*c^4*log(F)^4 + 20*b^2*c^2
*e^2*log(F)^2 + 64*e^4)*cos(2*e*x + 2*d))
```

Giac [F]

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx=\int F^{(bx+a)c}\sec\left(ex+d\right)^2\,dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cos\left(d+e\,x\right)^2}\,dx$$

```
[In] int(F^(c*(a + b*x))/cos(d + e*x)^2,x)
[Out] int(F^(c*(a + b*x))/cos(d + e*x)^2, x)
```

3.16 $\int F^{c(a+bx)} \sec^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 141

$$\int F^{c(a+bx)} \sec^3(d+ex)\,dx = -\frac{e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)(ie-bc\log(F))}{e^2} - \frac{bcF^{c(a+bx)}\log(F)\sec(d+ex)}{2e^2} + \frac{F^{c(a+bx)}\sec(d+ex)\tan(d+ex)}{2e}$$

```
[Out] -exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([1, 1/2*(e-I*b*c*ln(F))/e],[3/2-1/2
*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))*(I*e-b*c*ln(F))/e^2-1/2*b*c*F^(c*(b*x+a)
)*ln(F)*sec(e*x+d)/e^2+1/2*F^(c*(b*x+a))*sec(e*x+d)*tan(e*x+d)/e
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4533, 4536}

$$\int F^{c(a+bx)} \sec^3(d+ex)\,dx = -\frac{e^{i(d+ex)}F^{c(a+bx)}(-bc\log(F)+ie)\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{e^2} - \frac{bc\log(F)\sec(d+ex)F^{c(a+bx)}}{2e^2} + \frac{\tan(d+ex)\sec(d+ex)F^{c(a+bx)}}{2e}$$

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x]^3,x]
```

```
[Out] -((E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])/
(2*e), (3 - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]*(I*e - b*c*Log[F]))/
e^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d + e*x])/(2*e^2) + (F^(c*(a + b*x))
*Sec[d + e*x]*Tan[d + e*x])/(2*e)
```

Rule 4533

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{bcF^{c(a+bx)}\log(F)\sec(d+ex)}{2e^2}+\frac{F^{c(a+bx)}\sec(d+ex)\tan(d+ex)}{2e}\\&\quad+\frac{1}{2}\left(1+\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\sec(d+ex)\,dx\\&= -\frac{e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)(ie-bc\log(F))}{e^2}\\&\quad-\frac{bcF^{c(a+bx)}\log(F)\sec(d+ex)}{2e^2}+\frac{F^{c(a+bx)}\sec(d+ex)\tan(d+ex)}{2e}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(2e^{i(d+ex)}\,\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{3}{2}-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)(-ie+bc\log(F))+\sec(d+\right.}{2e^2}$$

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(2*E^(I*(d + e*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])
/(2*e), 3/2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((-I)*e + b*c*Log
[F]) + Sec[d + e*x]*(-(b*c*Log[F]) + e*Tan[d + e*x])))/(2*e^2)
```

Maple [F]

$$\int F^{c(xb+a)} \sec(ex+d)^3 \, dx$$

```
[In] int(F^(c*(b*x+a))*sec(e*x+d)^3,x)
[Out] int(F^(c*(b*x+a))*sec(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \sec^3(d+ex) \, dx = \int F^{(bx+a)c} \sec(ex+d)^3 \, dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*sec(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \sec^3(d+ex) \, dx = \int F^{c(a+bx)} \sec^3(d+ex) \, dx$$

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d)**3,x)
[Out] Integral(F**(c*(a + b*x))*sec(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \sec^3(d+ex) \, dx = \int F^{(bx+a)c} \sec(ex+d)^3 \, dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="maxima")
[Out] 8*(48*F^(b*c*x)*F^(a*c)*b*c*e^2*cos(e*x + d)*log(F) + 6*(F^(a*c)*b^2*c^2*e*
log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d) + (48*F^(b*c*x)*F^(a*c)*b
*c*e^2*cos(e*x + d)*log(F) + (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2
*log(F))*F^(b*c*x)*cos(3*e*x + 3*d) - 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^
(a*c)*e^3)*F^(b*c*x)*sin(3*e*x + 3*d) + 6*(F^(a*c)*b^2*c^2*e*log(F)^2 - 15*
```

```
F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d))*cos(6*e*x + 6*d) + 3*(48*F^(b*c*x)*F^(
a*c)*b*c*e^2*cos(e*x + d)*log(F) + (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b
*c*e^2*log(F))*F^(b*c*x)*cos(3*e*x + 3*d) - 3*(F^(a*c)*b^2*c^2*e*log(F)^2 +
 25*F^(a*c)*e^3)*F^(b*c*x)*sin(3*e*x + 3*d) + 6*(F^(a*c)*b^2*c^2*e*log(F)^2
 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d))*cos(4*e*x + 4*d) + (3*(F^(a*c)*b
^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 9
*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*sin(2*e*x + 2*d) +
 (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x))*cos(3*e*
x + 3*d) + 18*(8*F^(b*c*x)*F^(a*c)*b*c*e^2*cos(e*x + d)*log(F) + (F^(a*c)*b
^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d))*cos(2*e*x + 2*d
) - 6*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin
(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F
)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d) + (
F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 3
4*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin
(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*cos(6*e*x
 + 6*d)^2 + 9*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(
F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e
^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*si
n(d))*cos(4*e*x + 4*d)^2 + 9*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b
^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^
(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225
*F^(a*c)*e^6*sin(d))*cos(2*e*x + 2*d)^2 + (F^(a*c)*b^5*c^5*e*cos(d)*log(F)^
5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log
(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)
*log(F) - 225*F^(a*c)*e^6*sin(d))*sin(6*e*x + 6*d)^2 + 9*(F^(a*c)*b^5*c^5*e
*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3
*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)
*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*sin(4*e*x + 4*d)^2 + 18*(F
^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34
*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(
d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*sin(4*e*x
+ 4*d)*sin(2*e*x + 2*d) + 9*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^
4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(
a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*
F^(a*c)*e^6*sin(d))*sin(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)
^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*lo
g(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d
)*log(F) - 225*F^(a*c)*e^6*sin(d) + 3*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 -
F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^
3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log
(F) - 225*F^(a*c)*e^6*sin(d))*cos(4*e*x + 4*d) + 3*(F^(a*c)*b^5*c^5*e*cos(d
)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*c
os(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e
^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*cos(2*e*x + 2*d))*cos(6*e*x + 6*
```

```
d) + 6*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*si
n(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(
F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d) +
3*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d)
+ 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*
sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*cos(2*
e*x + 2*d))*cos(4*e*x + 4*d) + 6*(F^(a*c)*b^5*c^5*e*cos(d)*log(F)^5 - F^(a*
c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d)*log(F)^3 - 3
4*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*cos(d)*log(F) -
 225*F^(a*c)*e^6*sin(d))*cos(2*e*x + 2*d) + 6*((F^(a*c)*b^5*c^5*e*cos(d)*lo
g(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^3*e^3*cos(d
)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c)*b*c*e^5*c
os(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*sin(4*e*x + 4*d) + (F^(a*c)*b^5*c^5*
e*cos(d)*log(F)^5 - F^(a*c)*b^4*c^4*e^2*log(F)^4*sin(d) + 34*F^(a*c)*b^3*c^
3*e^3*cos(d)*log(F)^3 - 34*F^(a*c)*b^2*c^2*e^4*log(F)^2*sin(d) + 225*F^(a*c
)*b*c*e^5*cos(d)*log(F) - 225*F^(a*c)*e^6*sin(d))*sin(2*e*x + 2*d))*sin(6*e
*x + 6*d))*integrate((8*F^(b*c*x)*b*c*e*cos(e*x)*log(F) + (b^2*c^2*log(F)^2
 - 15*e^2)*F^(b*c*x)*sin(e*x) + (8*F^(b*c*x)*b*c*e*cos(e*x)*log(F) + (b^2*c
^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(e*x))*cos(8*e*x + 8*d) + 4*(8*F^(b*c*x)
*b*c*e*cos(e*x)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(e*x))*co
s(6*e*x + 6*d) + 6*(8*F^(b*c*x)*b*c*e*cos(e*x)*log(F) + (b^2*c^2*log(F)^2 -
 15*e^2)*F^(b*c*x)*sin(e*x))*cos(4*e*x + 4*d) + 4*(8*F^(b*c*x)*b*c*e*cos(e*
x)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(e*x))*cos(2*e*x + 2*d
) + (8*F^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c
*x)*cos(e*x))*sin(8*e*x + 8*d) + 4*(8*F^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^
2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*cos(e*x))*sin(6*e*x + 6*d) + 6*(8*F^(b*c
*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*cos(e*x))
*sin(4*e*x + 4*d) + 4*(8*F^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*log(F)^
2 - 15*e^2)*F^(b*c*x)*cos(e*x))*sin(2*e*x + 2*d))/(b^4*c^4*log(F)^4 + 34*b^
2*c^2*e^2*log(F)^2 + 225*e^4 + (b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2
+ 225*e^4)*cos(8*e*x + 8*d)^2 + 16*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F
)^2 + 225*e^4)*cos(6*e*x + 6*d)^2 + 36*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*l
og(F)^2 + 225*e^4)*cos(4*e*x + 4*d)^2 + 16*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e
^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d)^2 + (b^4*c^4*log(F)^4 + 34*b^2*c^2*
e^2*log(F)^2 + 225*e^4)*sin(8*e*x + 8*d)^2 + 16*(b^4*c^4*log(F)^4 + 34*b^2*
c^2*e^2*log(F)^2 + 225*e^4)*sin(6*e*x + 6*d)^2 + 36*(b^4*c^4*log(F)^4 + 34*
b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d)^2 + 48*(b^4*c^4*log(F)^4 +
 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 16*
(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2*e*x + 2*d)^2 +
 2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + 4*(b^4*c^4*log(F
)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(6*e*x + 6*d) + 6*(b^4*c^4*log(
F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(4*e*x + 4*d) + 4*(b^4*c^4*log
(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(8*e*x + 8*
d) + 8*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + 6*(b^4*c^4*l
og(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(4*e*x + 4*d) + 4*(b^4*c^4*
```

log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(6*e*x + 6*d) + 12*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + 4*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) + 8*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d) + 4*(2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(6*e*x + 6*d) + 3*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d) + 2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2*e*x + 2*d))*sin(8*e*x + 8*d) + 16*(3*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d) + 2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2*e*x + 2*d))*sin(6*e*x + 6*d)), x) + 6*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d) + (F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*cos(6*e*x + 6*d)^2 + 9*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*cos(4*e*x + 4*d)^2 + 9*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*cos(2*e*x + 2*d)^2 + (F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*sin(6*e*x + 6*d)^2 + 9*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*sin(4*e*x + 4*d)^2 + 18*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 9*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*sin(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d) + 3*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*cos(4*e*x + 4*d) + 3*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*cos(2*e*x + 2*d))*cos(6*e*x + 6*d) + 6*(F^(a*c

```
)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a
*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 +
225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d) + 3*(F^(a*c)*b^5
*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b
^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F
^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*cos(2*e*x + 2*d))*co
s(4*e*x + 4*d) + 6*(F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*c)*b^4*c^4*e^2
*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 34*F^(a*c)*b^2*
c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) + 225*F^(a*c)*e
^6*cos(d))*cos(2*e*x + 2*d) + 6*((F^(a*c)*b^5*c^5*e*log(F)^5*sin(d) + F^(a*
c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3*sin(d) + 3
4*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(F)*sin(d) +
 225*F^(a*c)*e^6*cos(d))*sin(4*e*x + 4*d) + (F^(a*c)*b^5*c^5*e*log(F)^5*sin
(d) + F^(a*c)*b^4*c^4*e^2*cos(d)*log(F)^4 + 34*F^(a*c)*b^3*c^3*e^3*log(F)^3
*sin(d) + 34*F^(a*c)*b^2*c^2*e^4*cos(d)*log(F)^2 + 225*F^(a*c)*b*c*e^5*log(
F)*sin(d) + 225*F^(a*c)*e^6*cos(d))*sin(2*e*x + 2*d))*sin(6*e*x + 6*d))*int
egrate((8*F^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*log(F)^2 - 15*e^2)*F^(
b*c*x)*cos(e*x) + (8*F^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*log(F)^2 -
15*e^2)*F^(b*c*x)*cos(e*x))*cos(8*e*x + 8*d) + 4*(8*F^(b*c*x)*b*c*e*log(F)*
sin(e*x) - (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*cos(e*x))*cos(6*e*x + 6*d)
 + 6*(8*F^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*
c*x)*cos(e*x))*cos(4*e*x + 4*d) + 4*(8*F^(b*c*x)*b*c*e*log(F)*sin(e*x) - (b
^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*cos(e*x))*cos(2*e*x + 2*d) - (8*F^(b*c*
x)*b*c*e*cos(e*x)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(e*x))*
sin(8*e*x + 8*d) - 4*(8*F^(b*c*x)*b*c*e*cos(e*x)*log(F) + (b^2*c^2*log(F)^2
 - 15*e^2)*F^(b*c*x)*sin(e*x))*sin(6*e*x + 6*d) - 6*(8*F^(b*c*x)*b*c*e*cos(
e*x)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2)*F^(b*c*x)*sin(e*x))*sin(4*e*x + 4
*d) - 4*(8*F^(b*c*x)*b*c*e*cos(e*x)*log(F) + (b^2*c^2*log(F)^2 - 15*e^2)*F^
(b*c*x)*sin(e*x))*sin(2*e*x + 2*d))/(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(
F)^2 + 225*e^4 + (b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos
(8*e*x + 8*d)^2 + 16*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)
*cos(6*e*x + 6*d)^2 + 36*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*
e^4)*cos(4*e*x + 4*d)^2 + 16*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 +
225*e^4)*cos(2*e*x + 2*d)^2 + (b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 +
 225*e^4)*sin(8*e*x + 8*d)^2 + 16*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)
^2 + 225*e^4)*sin(6*e*x + 6*d)^2 + 36*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*lo
g(F)^2 + 225*e^4)*sin(4*e*x + 4*d)^2 + 48*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^
2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 16*(b^4*c^4*log(F
)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2*e*x + 2*d)^2 + 2*(b^4*c^4*lo
g(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + 4*(b^4*c^4*log(F)^4 + 34*b^2*c
^2*e^2*log(F)^2 + 225*e^4)*cos(6*e*x + 6*d) + 6*(b^4*c^4*log(F)^4 + 34*b^2*
c^2*e^2*log(F)^2 + 225*e^4)*cos(4*e*x + 4*d) + 4*(b^4*c^4*log(F)^4 + 34*b^2
*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(8*e*x + 8*d) + 8*(b^4*c^
4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + 6*(b^4*c^4*log(F)^4 + 34*b
^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(4*e*x + 4*d) + 4*(b^4*c^4*log(F)^4 + 34*
```

```
b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(6*e*x + 6*d) + 12*(b^
4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + 4*(b^4*c^4*log(F)^4 +
34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) + 8*(
b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d) + 4*
(2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(6*e*x + 6*d)
+ 3*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d)
 + 2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2*e*x + 2*d
))*sin(8*e*x + 8*d) + 16*(3*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 2
25*e^4)*sin(4*e*x + 4*d) + 2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 +
225*e^4)*sin(2*e*x + 2*d))*sin(6*e*x + 6*d)), x) + (48*F^(b*c*x)*F^(a*c)*b*
c*e^2*log(F)*sin(e*x + d) + 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)
*F^(b*c*x)*cos(3*e*x + 3*d) - 6*(F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a*c)*e^
3)*F^(b*c*x)*cos(e*x + d) + (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*
log(F))*F^(b*c*x)*sin(3*e*x + 3*d))*sin(6*e*x + 6*d) + 3*(48*F^(b*c*x)*F^(a
*c)*b*c*e^2*log(F)*sin(e*x + d) + 3*(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c
)*e^3)*F^(b*c*x)*cos(3*e*x + 3*d) - 6*(F^(a*c)*b^2*c^2*e*log(F)^2 - 15*F^(a
*c)*e^3)*F^(b*c*x)*cos(e*x + d) + (F^(a*c)*b^3*c^3*log(F)^3 + 25*F^(a*c)*b*
c*e^2*log(F))*F^(b*c*x)*sin(3*e*x + 3*d))*sin(4*e*x + 4*d) - 3*(3*(F^(a*c)*
b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x)*cos(2*e*x + 2*d) - (F^(a*c)*
b^3*c^3*log(F)^3 + 25*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) +
(F^(a*c)*b^2*c^2*e*log(F)^2 + 25*F^(a*c)*e^3)*F^(b*c*x))*sin(3*e*x + 3*d) +
 18*(8*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F)*sin(e*x + d) - (F^(a*c)*b^2*c^2*e*l
og(F)^2 - 15*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d))*sin(2*e*x + 2*d))/(b^4*c^
4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + (b^4*c^4*log(F)^4 + 34*b^2
*c^2*e^2*log(F)^2 + 225*e^4)*cos(6*e*x + 6*d)^2 + 9*(b^4*c^4*log(F)^4 + 34*
b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(4*e*x + 4*d)^2 + 9*(b^4*c^4*log(F)^4 +
34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d)^2 + (b^4*c^4*log(F)^4 +
 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(6*e*x + 6*d)^2 + 9*(b^4*c^4*log(F)^
4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d)^2 + 18*(b^4*c^4*log
(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d
) + 9*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2*e*x + 2*
d)^2 + 2*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + 3*(b^4*c^4
*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(4*e*x + 4*d) + 3*(b^4*c^
4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(6*e*x
 + 6*d) + 6*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4 + 3*(b^4*
c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*e*x + 2*d))*cos(4*e
*x + 4*d) + 6*(b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*cos(2*
e*x + 2*d) + 6*((b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(
4*e*x + 4*d) + (b^4*c^4*log(F)^4 + 34*b^2*c^2*e^2*log(F)^2 + 225*e^4)*sin(2
*e*x + 2*d))*sin(6*e*x + 6*d))
```

Giac [F]

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx=\int F^{(bx+a)c}\sec\left(ex+d\right)^3\,dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cos\left(d+e\,x\right)^3}\,dx$$

```
[In] int(F^(c*(a + b*x))/cos(d + e*x)^3,x)
[Out] int(F^(c*(a + b*x))/cos(d + e*x)^3, x)
```

3.17 $\int F^{c(a+bx)}\sec^4(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 143

$$\int F^{c(a+bx)}\sec^4(d+ex)\,dx = -\frac{2e^{2i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)(2ie-bc\log(F))}{3e^2} - \frac{bcF^{c(a+bx)}\log(F)\sec^2(d+ex)}{6e^2} + \frac{F^{c(a+bx)}\sec^2(d+ex)\tan(d+ex)}{3e}$$

```
[Out] -2/3*exp(2*I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-1/2*I*b*c*ln(F)/e],[2-1
/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))*(2*I*e-b*c*ln(F))/e^2-1/6*b*c*F^(c*(b*
x+a))*ln(F)*sec(e*x+d)^2/e^2+1/3*F^(c*(b*x+a))*sec(e*x+d)^2*tan(e*x+d)/e
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4533, 4536}

$$\int F^{c(a+bx)}\sec^4(d+ex)\,dx = -\frac{2e^{2i(d+ex)}F^{c(a+bx)}(-bc\log(F)+2ie)\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{3e^2} - \frac{bc\log(F)\sec^2(d+ex)F^{c(a+bx)}}{6e^2} + \frac{\tan(d+ex)\sec^2(d+ex)F^{c(a+bx)}}{3e}$$

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x]^4,x]
```

```
[Out] (-2*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((2*I)*e - b*c*
Log[F]))/(3*e^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d + e*x]^2)/(6*e^2) + (F
^(c*(a + b*x))*Sec[d + e*x]^2*Tan[d + e*x])/(3*e)
```

Rule 4533

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{bcF^{c(a+bx)}\log(F)\sec^2(d+ex)}{6e^2}+\frac{F^{c(a+bx)}\sec^2(d+ex)\tan(d+ex)}{3e} \\ &\quad+\frac{1}{6}\left(4+\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\sec^2(d+ex)\,dx \\ &= -\frac{2e^{2i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)(2ie-bc\log(F)}{3e^2} \\ &\quad-\frac{bcF^{c(a+bx)}\log(F)\sec^2(d+ex)}{6e^2}+\frac{F^{c(a+bx)}\sec^2(d+ex)\tan(d+ex)}{3e}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)}\sec^4(d+ex)\,dx$$

$$=\frac{F^{c(a+bx)}\left(4e^{2i(d+ex)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)(-2ie+bc\log(F))+\text{sec}\right.}{6e^2}$$

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(4*E^((2*I)*(d + e*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((-2*I)*e + b*c
*Log[F]) + Sec[d + e*x]^2*(-(b*c*Log[F]) + 2*e*Tan[d + e*x])))/(6*e^2)
```

Maple [F]

$$\int F^{c(xb+a)} \sec(ex+d)^4 \, dx$$

```
[In] int(F^(c*(b*x+a))*sec(e*x+d)^4,x)
```

```
[Out] int(F^(c*(b*x+a))*sec(e*x+d)^4,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \sec^4(d+ex) \, dx = \int F^{(bx+a)c} \sec(ex+d)^4 \, dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sec(e*x + d)^4, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \sec^4(d+ex) \, dx = \int F^{c(a+bx)} \sec^4(d+ex) \, dx$$

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d)**4,x)
```

```
[Out] Integral(F**(c*(a + b*x))*sec(d + e*x)**4, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \sec^4(d+ex) \, dx = \int F^{(bx+a)c} \sec(ex+d)^4 \, dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 16*(6*(F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F
^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*e*x + 4*d)^2 + 320*(F^(a*c)*b^3*c^3*
e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 + 6*
(F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)
*b*c*e^4*log(F))*F^(b*c*x)*sin(4*e*x + 4*d)^2 + 320*(F^(a*c)*b^3*c^3*e^2*lo
```

g(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 - 560*(F^(
a*c)*b^3*c^3*e^2*log(F)^3 - 32*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x
+ 2*d) + 40*(F^(a*c)*b^4*c^4*e*log(F)^4 - 104*F^(a*c)*b^2*c^2*e^3*log(F)^2)
*F^(b*c*x)*sin(2*e*x + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*F^(a*c
)*b*c*e^4*log(F))*F^(b*c*x) + ((F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*
c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*e*x + 4*d)
+ 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*c
os(2*e*x + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 100*F^(a*c)*b^2*c^2*e^3*l
og(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x)*sin(4*e*x + 4*d) + 8*(F^(a*c)*b^4*c^4
*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 1536*F^(a*c)*e^5)*F^(b*c*x)
*sin(2*e*x + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*F^(a*c)*b*c*e^4*
log(F))*F^(b*c*x))*cos(8*e*x + 8*d) + 4*((F^(a*c)*b^5*c^5*log(F)^5 + 100*F^
(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(4*e
*x + 4*d) + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F
^(b*c*x)*cos(2*e*x + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 100*F^(a*c)*b^2
*c^2*e^3*log(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x)*sin(4*e*x + 4*d) + 8*(F^(a*
c)*b^4*c^4*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 1536*F^(a*c)*e^5)
*F^(b*c*x)*sin(2*e*x + 2*d) - 160*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 20*F^(a*c
)*b*c*e^4*log(F))*F^(b*c*x))*cos(6*e*x + 6*d) + (4*(F^(a*c)*b^5*c^5*log(F)^
5 + 220*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 9984*F^(a*c)*b*c*e^4*log(F))*F^(b*c*
x)*cos(2*e*x + 2*d) + 64*(F^(a*c)*b^4*c^4*e*log(F)^4 + 55*F^(a*c)*b^2*c^2*e
^3*log(F)^2 - 576*F^(a*c)*e^5)*F^(b*c*x)*sin(2*e*x + 2*d) + (F^(a*c)*b^5*c^
5*log(F)^5 - 860*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 21504*F^(a*c)*b*c*e^4*log(F
))*F^(b*c*x))*cos(4*e*x + 4*d) - 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(
a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*
c)*b*c*e^8*log(F) + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4
*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F
))*cos(8*e*x + 8*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*
c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^
8*log(F))*cos(6*e*x + 6*d)^2 + 36*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*
c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)
*b*c*e^8*log(F))*cos(4*e*x + 4*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 11
6*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*
F^(a*c)*b*c*e^8*log(F))*cos(2*e*x + 2*d)^2 + (F^(a*c)*b^7*c^7*e^2*log(F)^7
+ 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36
864*F^(a*c)*b*c*e^8*log(F))*sin(8*e*x + 8*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*lo
g(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)
^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(6*e*x + 6*d)^2 + 36*(F^(a*c)*b^7*c^7
*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6
*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(4*e*x + 4*d)^2 + 48*(F^(a*c)*
b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*
c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(4*e*x + 4*d)*sin(2*e*x
 + 2*d) + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)
^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(
2*e*x + 2*d)^2 + 2*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*

log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F)
 + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 390
4*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(6*e*x +
6*d) + 6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 +
 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(4*e*
x + 4*d) + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)
^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(
2*e*x + 2*d))*cos(8*e*x + 8*d) + 8*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a
*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c
)*b*c*e^8*log(F) + 6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^
4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(
F))*cos(4*e*x + 4*d) + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^
5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*
log(F))*cos(2*e*x + 2*d))*cos(6*e*x + 6*d) + 12*(F^(a*c)*b^7*c^7*e^2*log(F)
^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 +
 36864*F^(a*c)*b*c*e^8*log(F) + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*
c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)
*b*c*e^8*log(F))*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) + 8*(F^(a*c)*b^7*c^7*e^
2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*lo
g(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*cos(2*e*x + 2*d) + 4*(2*(F^(a*c)*b^7
*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3
*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(6*e*x + 6*d) + 3*(F^(a*c)
*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3
*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(4*e*x + 4*d) + 2*(F^(
a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 3904*F^(a*c)
*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log(F))*sin(2*e*x + 2*d))*sin
(8*e*x + 8*d) + 16*(3*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c^5*e
^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8*log
(F))*sin(4*e*x + 4*d) + 2*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 116*F^(a*c)*b^5*c
^5*e^4*log(F)^5 + 3904*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 36864*F^(a*c)*b*c*e^8
*log(F))*sin(2*e*x + 2*d))*sin(6*e*x + 6*d))*integrate(-((b^3*c^3*log(F)^3
 - 104*b*c*e^2*log(F))*F^(b*c*x)*cos(10*e*x + 10*d) + 5*(b^3*c^3*log(F)^3 -
104*b*c*e^2*log(F))*F^(b*c*x)*cos(8*e*x + 8*d) + 10*(b^3*c^3*log(F)^3 - 104
*b*c*e^2*log(F))*F^(b*c*x)*cos(6*e*x + 6*d) + 10*(b^3*c^3*log(F)^3 - 104*b*
c*e^2*log(F))*F^(b*c*x)*cos(4*e*x + 4*d) + 5*(b^3*c^3*log(F)^3 - 104*b*c*e^
2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 6*(3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^
(b*c*x)*sin(10*e*x + 10*d) + 30*(3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x)*s
in(8*e*x + 8*d) + 60*(3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x)*sin(6*e*x +
6*d) + 60*(3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x)*sin(4*e*x + 4*d) + 30*(
3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x)*sin(2*e*x + 2*d) + (b^3*c^3*log(F)
^3 - 104*b*c*e^2*log(F))*F^(b*c*x))/(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log
(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6 + (b^6*c^6*log(F)^6 + 116*b^4
*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(10*e*x + 10*
d)^2 + 25*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*l
og(F)^2 + 36864*e^6)*cos(8*e*x + 8*d)^2 + 100*(b^6*c^6*log(F)^6 + 116*b^4*c

```
^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6*d)^2
 + 100*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(
F)^2 + 36864*e^6)*cos(4*e*x + 4*d)^2 + 25*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e
^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d)^2 + (
b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 3
6864*e^6)*sin(10*e*x + 10*d)^2 + 25*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log
(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(8*e*x + 8*d)^2 + 100*(b^
6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 368
64*e^6)*sin(6*e*x + 6*d)^2 + 100*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)
^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d)^2 + 100*(b^6*c
^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*
e^6)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 25*(b^6*c^6*log(F)^6 + 116*b^4*c^4
*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d)^2 +
 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2
 + 36864*e^6 + 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^
2*e^4*log(F)^2 + 36864*e^6)*cos(8*e*x + 8*d) + 10*(b^6*c^6*log(F)^6 + 116*b
^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6*
d) + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log
(F)^2 + 36864*e^6)*cos(4*e*x + 4*d) + 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2
*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(10
*e*x + 10*d) + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c
^2*e^4*log(F)^2 + 36864*e^6 + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)
^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6*d) + 10*(b^6*c^6*
log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6
)*cos(4*e*x + 4*d) + 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*
b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(8*e*x + 8*d) + 20*(
b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 3
6864*e^6 + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e
^4*log(F)^2 + 36864*e^6)*cos(4*e*x + 4*d) + 5*(b^6*c^6*log(F)^6 + 116*b^4*c
^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*
cos(6*e*x + 6*d) + 20*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b
^2*c^2*e^4*log(F)^2 + 36864*e^6 + 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log
(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(4*e*x
+ 4*d) + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4
*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d) + 10*((b^6*c^6*log(F)^6 + 116*b^4*c
^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(8*e*x + 8*d) +
 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2
 + 36864*e^6)*sin(6*e*x + 6*d) + 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(
F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) + (b^6*c^6*l
og(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)
*sin(2*e*x + 2*d))*sin(10*e*x + 10*d) + 50*(2*(b^6*c^6*log(F)^6 + 116*b^4*c
^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(6*e*x + 6*d) +
 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2
 + 36864*e^6)*sin(4*e*x + 4*d) + (b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)
^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d))*sin(8*e*x + 8
```

d) + 100(2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^
4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) + (b^6*c^6*log(F)^6 + 116*b^4*c^4*
e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d))*sin
(6*e*x + 6*d)), x) + 8*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^
3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*
log(F)^2 + (F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 +
 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*co
s(8*e*x + 8*d)^2 + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3
*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*l
og(F)^2)*cos(6*e*x + 6*d)^2 + 36*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*
b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^
2*c^2*e^7*log(F)^2)*cos(4*e*x + 4*d)^2 + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 1
16*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864
*F^(a*c)*b^2*c^2*e^7*log(F)^2)*cos(2*e*x + 2*d)^2 + (F^(a*c)*b^8*c^8*e*log(
F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4
 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin(8*e*x + 8*d)^2 + 16*(F^(a*c)*b^8
*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e
^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin(6*e*x + 6*d)^2 + 36*(
F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c
)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin(4*e*x + 4*
d)^2 + 48*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 +
3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin
(4*e*x + 4*d)*sin(2*e*x + 2*d) + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*
c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)
*b^2*c^2*e^7*log(F)^2)*sin(2*e*x + 2*d)^2 + 2*(F^(a*c)*b^8*c^8*e*log(F)^8 +
 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 368
64*F^(a*c)*b^2*c^2*e^7*log(F)^2 + 4*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*
c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)
*b^2*c^2*e^7*log(F)^2)*cos(6*e*x + 6*d) + 6*(F^(a*c)*b^8*c^8*e*log(F)^8 + 1
16*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864
*F^(a*c)*b^2*c^2*e^7*log(F)^2)*cos(4*e*x + 4*d) + 4*(F^(a*c)*b^8*c^8*e*log(
F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4
 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*cos(2*e*x + 2*d))*cos(8*e*x + 8*d) +
 8*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^
(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2 + 6*(F^(a*c
)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*
c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*cos(4*e*x + 4*d) + 4
*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a
*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*cos(2*e*x +
2*d))*cos(6*e*x + 6*d) + 12*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c
^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2
*e^7*log(F)^2 + 4*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log
(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F
)^2)*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) + 8*(F^(a*c)*b^8*c^8*e*log(F)^8 + 1
16*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864

*F^(a*c)*b^2*c^2*e^7*log(F)^2)*cos(2*e*x + 2*d) + 4*(2*(F^(a*c)*b^8*c^8*e*l
og(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F
)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin(6*e*x + 6*d) + 3*(F^(a*c)*b^8
*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e
^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin(4*e*x + 4*d) + 2*(F^(
a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b
^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e^7*log(F)^2)*sin(2*e*x + 2*d))
*sin(8*e*x + 8*d) + 16*(3*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)*b^6*c^6
*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b^2*c^2*e
^7*log(F)^2)*sin(4*e*x + 4*d) + 2*(F^(a*c)*b^8*c^8*e*log(F)^8 + 116*F^(a*c)
*b^6*c^6*e^3*log(F)^6 + 3904*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 36864*F^(a*c)*b
^2*c^2*e^7*log(F)^2)*sin(2*e*x + 2*d))*sin(6*e*x + 6*d))*integrate((6*(3*b^
2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x)*cos(10*e*x + 10*d) + 30*(3*b^2*c^2*e*l
og(F)^2 - 32*e^3)*F^(b*c*x)*cos(8*e*x + 8*d) + 60*(3*b^2*c^2*e*log(F)^2 - 3
2*e^3)*F^(b*c*x)*cos(6*e*x + 6*d) + 60*(3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b
*c*x)*cos(4*e*x + 4*d) + 30*(3*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x)*cos(2
*e*x + 2*d) - (b^3*c^3*log(F)^3 - 104*b*c*e^2*log(F))*F^(b*c*x)*sin(10*e*x
+ 10*d) - 5*(b^3*c^3*log(F)^3 - 104*b*c*e^2*log(F))*F^(b*c*x)*sin(8*e*x + 8
d) - 10(b^3*c^3*log(F)^3 - 104*b*c*e^2*log(F))*F^(b*c*x)*sin(6*e*x + 6*d)
 - 10*(b^3*c^3*log(F)^3 - 104*b*c*e^2*log(F))*F^(b*c*x)*sin(4*e*x + 4*d) -
5*(b^3*c^3*log(F)^3 - 104*b*c*e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) + 6*(3
*b^2*c^2*e*log(F)^2 - 32*e^3)*F^(b*c*x))/(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^
2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6 + (b^6*c^6*log(F)^6 + 11
6*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(10*e*x
+ 10*d)^2 + 25*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*
e^4*log(F)^2 + 36864*e^6)*cos(8*e*x + 8*d)^2 + 100*(b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6
d)^2 + 100(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4
*log(F)^2 + 36864*e^6)*cos(4*e*x + 4*d)^2 + 25*(b^6*c^6*log(F)^6 + 116*b^4*
c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d)^
2 + (b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^
2 + 36864*e^6)*sin(10*e*x + 10*d)^2 + 25*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^
2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(8*e*x + 8*d)^2 + 10
0*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2
+ 36864*e^6)*sin(6*e*x + 6*d)^2 + 100*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*l
og(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d)^2 + 100*(
b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 3
6864*e^6)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 25*(b^6*c^6*log(F)^6 + 116*b^
4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d
)^2 + 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log
(F)^2 + 36864*e^6 + 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b
^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(8*e*x + 8*d) + 10*(b^6*c^6*log(F)^6 +
116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x
 + 6*d) + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^
4*log(F)^2 + 36864*e^6)*cos(4*e*x + 4*d) + 5*(b^6*c^6*log(F)^6 + 116*b^4*c^

4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*c
os(10*e*x + 10*d) + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*
b^2*c^2*e^4*log(F)^2 + 36864*e^6 + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*l
og(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6*d) + 10*(b^6
*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 3686
4*e^6)*cos(4*e*x + 4*d) + 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 +
3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(8*e*x + 8*d) +
20*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^
2 + 36864*e^6 + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*
c^2*e^4*log(F)^2 + 36864*e^6)*cos(4*e*x + 4*d) + 5*(b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2
*d))*cos(6*e*x + 6*d) + 20*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3
904*b^2*c^2*e^4*log(F)^2 + 36864*e^6 + 5*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^
2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(4
*e*x + 4*d) + 10*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^
2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d) + 10*((b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(8*e*x + 8
d) + 2(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log
(F)^2 + 36864*e^6)*sin(6*e*x + 6*d) + 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2
*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) + (b^6*
c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864
*e^6)*sin(2*e*x + 2*d))*sin(10*e*x + 10*d) + 50*(2*(b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(6*e*x + 6
d) + 2(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log
(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) + (b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*l
og(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d))*sin(8*e*
x + 8*d) + 100*(2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c
^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) + (b^6*c^6*log(F)^6 + 116*b^4
*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d)
)*sin(6*e*x + 6*d)), x) + (4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 100*F^(a*c)*b^2*
c^2*e^3*log(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x)*cos(4*e*x + 4*d) - 8*(F^(a*c
)*b^4*c^4*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 1536*F^(a*c)*e^5)*
F^(b*c*x)*cos(2*e*x + 2*d) + (F^(a*c)*b^5*c^5*log(F)^5 + 100*F^(a*c)*b^3*c^
3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(4*e*x + 4*d) +
80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin
(2*e*x + 2*d) + 8*(F^(a*c)*b^4*c^4*e*log(F)^4 - 140*F^(a*c)*b^2*c^2*e^3*log
(F)^2 + 384*F^(a*c)*e^5)*F^(b*c*x))*sin(8*e*x + 8*d) + 4*(4*(F^(a*c)*b^4*c^
4*e*log(F)^4 + 100*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*
x)*cos(4*e*x + 4*d) - 8*(F^(a*c)*b^4*c^4*e*log(F)^4 + 40*F^(a*c)*b^2*c^2*e^
3*log(F)^2 - 1536*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) + (F^(a*c)*b^5*c^
5*log(F)^5 + 100*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 2304*F^(a*c)*b*c*e^4*log(F)
)*F^(b*c*x)*sin(4*e*x + 4*d) + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 64*F^(a*c
)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) + 8*(F^(a*c)*b^4*c^4*e*log(F)^
4 - 140*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 384*F^(a*c)*e^5)*F^(b*c*x))*sin(6*e*
x + 6*d) - 4*(16*(F^(a*c)*b^4*c^4*e*log(F)^4 + 55*F^(a*c)*b^2*c^2*e^3*log(F

```
)^2 - 576*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) - (F^(a*c)*b^5*c^5*log(F)
^5 + 220*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 9984*F^(a*c)*b*c*e^4*log(F))*F^(b*c
*x)*sin(2*e*x + 2*d) - (11*F^(a*c)*b^4*c^4*e*log(F)^4 - 1780*F^(a*c)*b^2*c^
2*e^3*log(F)^2 + 2304*F^(a*c)*e^5)*F^(b*c*x))*sin(4*e*x + 4*d))/(b^6*c^6*lo
g(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6 +
 (b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 +
 36864*e^6)*cos(8*e*x + 8*d)^2 + 16*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log
(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6*d)^2 + 36*(b^6
*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 3686
4*e^6)*cos(4*e*x + 4*d)^2 + 16*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4
 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d)^2 + (b^6*c^6*log
(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*s
in(8*e*x + 8*d)^2 + 16*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*
b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(6*e*x + 6*d)^2 + 36*(b^6*c^6*log(F)^6
 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(4*
e*x + 4*d)^2 + 48*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c
^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 16*(b^6*c^
6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e
^6)*sin(2*e*x + 2*d)^2 + 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3
904*b^2*c^2*e^4*log(F)^2 + 36864*e^6 + 4*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^
2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(6*e*x + 6*d) + 6*(b
^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36
864*e^6)*cos(4*e*x + 4*d) + 4*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4
+ 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(8*e*x + 8*d)
 + 8*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)
^2 + 36864*e^6 + 6*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*
c^2*e^4*log(F)^2 + 36864*e^6)*cos(4*e*x + 4*d) + 4*(b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2
*d))*cos(6*e*x + 6*d) + 12*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3
904*b^2*c^2*e^4*log(F)^2 + 36864*e^6 + 4*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^
2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d))*cos(4
*e*x + 4*d) + 8*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2
*e^4*log(F)^2 + 36864*e^6)*cos(2*e*x + 2*d) + 4*(2*(b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(6*e*x + 6
*d) + 3*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log
(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) + 2*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2
*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2*d))*sin(8*
e*x + 8*d) + 16*(3*(b^6*c^6*log(F)^6 + 116*b^4*c^4*e^2*log(F)^4 + 3904*b^2*
c^2*e^4*log(F)^2 + 36864*e^6)*sin(4*e*x + 4*d) + 2*(b^6*c^6*log(F)^6 + 116*
b^4*c^4*e^2*log(F)^4 + 3904*b^2*c^2*e^4*log(F)^2 + 36864*e^6)*sin(2*e*x + 2
*d))*sin(6*e*x + 6*d))
```

Giac [F]

$$\int F^{c(a+bx)}\sec^4(d+ex)\,dx = \int F^{(bx+a)c}\sec(ex+d)^4\,dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^4,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\sec^4(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cos\left(d+e\,x\right)^4}\,dx$$

```
[In] int(F^(c*(a + b*x))/cos(d + e*x)^4,x)
[Out] int(F^(c*(a + b*x))/cos(d + e*x)^4, x)
```

3.18 $\int e^x \cos^4(x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 54

$$\int e^x \cos^4(x)\,dx = \frac{24e^x}{85} + \frac{12}{85}e^x\cos^2(x) + \frac{1}{17}e^x\cos^4(x) + \frac{24}{85}e^x\cos(x)\sin(x) + \frac{4}{17}e^x\cos^3(x)\sin(x)$$

```
[Out] 24/85*exp(x)+12/85*exp(x)*cos(x)^2+1/17*exp(x)*cos(x)^4+24/85*exp(x)*cos(x)
*sin(x)+4/17*exp(x)*cos(x)^3*sin(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4520, 2225}

$$\int e^x \cos^4(x)\,dx = \frac{24e^x}{85} + \frac{1}{17}e^x\cos^4(x) + \frac{12}{85}e^x\cos^2(x) + \frac{4}{17}e^x\sin(x)\cos^3(x) + \frac{24}{85}e^x\sin(x)\cos(x)$$

```
[In] Int[E^x*Cos[x]^4,x]
```

```
[Out] (24*E^x)/85 + (12*E^x*Cos[x]^2)/85 + (E^x*Cos[x]^4)/17 + (24*E^x*Cos[x]*Sin
[x])/85 + (4*E^x*Cos[x]^3*Sin[x])/17
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4520

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[e*m*F^(c*(a + b*x))*Sin[d +
 e*x]*(Cos[d + e*x]^(m - 1)/(e^2*m^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{17}e^x\cos^4(x)+\frac{4}{17}e^x\cos^3(x)\sin(x)+\frac{12}{17}\int e^x\cos^2(x)\,dx\\
&= \frac{12}{85}e^x\cos^2(x)+\frac{1}{17}e^x\cos^4(x)+\frac{24}{85}e^x\cos(x)\sin(x)+\frac{4}{17}e^x\cos^3(x)\sin(x)+\frac{24\int e^x\,dx}{85}\\
&= \frac{24e^x}{85}+\frac{12}{85}e^x\cos^2(x)+\frac{1}{17}e^x\cos^4(x)+\frac{24}{85}e^x\cos(x)\sin(x)+\frac{4}{17}e^x\cos^3(x)\sin(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int e^x\cos^4(x)\,dx=\frac{1}{680}e^x(255+68\cos(2x)+5\cos(4x)+136\sin(2x)+20\sin(4x))$$

```
[In] Integrate[E^x*Cos[x]^4,x]
[Out] (E^x*(255 + 68*Cos[2*x] + 5*Cos[4*x] + 136*Sin[2*x] + 20*Sin[4*x]))/680
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{e^x(255+5\cos(4x)+68\cos(2x)+136\sin(2x)+20\sin(4x))}{680}$
default	$\frac{(\cos(x)+4\sin(x))e^x\cos(x)^3}{17}+\frac{12(\cos(x)+2\sin(x))e^x\cos(x)}{85}+\frac{24\,e^x}{85}$
risch	$\frac{3\,e^x}{8}+\frac{e^{(1+4i)x}}{272}-\frac{ie^{(1+4i)x}}{68}+\frac{e^{(1+2i)x}}{20}-\frac{ie^{(1+2i)x}}{10}+\frac{e^{(1-2i)x}}{20}+\frac{ie^{(1-2i)x}}{10}+\frac{e^{(1-4i)x}}{272}+\frac{ie^{(1-4i)x}}{68}$
norman	$\frac{\frac{88\,e^x\tan(\frac{x}{2})}{85}+\frac{76\,e^x\tan(\frac{x}{2})^2}{85}-\frac{72\,e^x\tan(\frac{x}{2})^3}{85}+\frac{30\,e^x\tan(\frac{x}{2})^4}{17}+\frac{72\,e^x\tan(\frac{x}{2})^5}{85}+\frac{76\,e^x\tan(\frac{x}{2})^6}{85}-\frac{88\,e^x\tan(\frac{x}{2})^7}{85}+\frac{41\,e^x\tan(\frac{x}{2})^8}{85}+\frac{41\,e}{85}}{(1+\tan(\frac{x}{2})^2)^4}$

```
[In] int(exp(x)*cos(x)^4,x,method=_RETURNVERBOSE)
[Out] 1/680*exp(x)*(255+5*cos(4*x)+68*cos(2*x)+136*sin(2*x)+20*sin(4*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int e^x \cos^4(x)\,dx = \frac{4}{85}\left(5\cos\left(x\right)^3 + 6\cos\left(x\right)\right)e^x \sin\left(x\right) + \frac{1}{85}\left(5\cos\left(x\right)^4 + 12\cos\left(x\right)^2 + 24\right)e^x$$

```
[In] integrate(exp(x)*cos(x)^4,x, algorithm="fricas")
```

```
[Out] 4/85*(5*cos(x)^3 + 6*cos(x))*e^x*sin(x) + 1/85*(5*cos(x)^4 + 12*cos(x)^2 +
24)*e^x
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int e^x \cos^4(x)\,dx = \frac{24e^x \sin^4\left(x\right)}{85} + \frac{24e^x \sin^3\left(x\right)\cos\left(x\right)}{85} + \frac{12e^x \sin^2\left(x\right)\cos^2\left(x\right)}{17} + \frac{44e^x \sin\left(x\right)\cos^3\left(x\right)}{85} + \frac{41e^x \cos^4\left(x\right)}{85}$$

```
[In] integrate(exp(x)*cos(x)**4,x)
```

```
[Out] 24*exp(x)*sin(x)**4/85 + 24*exp(x)*sin(x)**3*cos(x)/85 + 12*exp(x)*sin(x)**
2*cos(x)**2/17 + 44*exp(x)*sin(x)*cos(x)**3/85 + 41*exp(x)*cos(x)**4/85
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int e^x \cos^4(x)\,dx = \frac{1}{136}\cos\left(4x\right)e^x + \frac{1}{10}\cos\left(2x\right)e^x + \frac{1}{34}e^x \sin\left(4x\right) + \frac{1}{5}e^x \sin\left(2x\right) + \frac{3}{8}e^x$$

```
[In] integrate(exp(x)*cos(x)^4,x, algorithm="maxima")
```

```
[Out] 1/136*cos(4*x)*e^x + 1/10*cos(2*x)*e^x + 1/34*e^x*sin(4*x) + 1/5*e^x*sin(2*
x) + 3/8*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.65

$$\int e^x \cos^4(x)\,dx = \frac{1}{136}\left(\cos\left(4\,x\right) + 4\,\sin\left(4\,x\right)\right)e^x + \frac{1}{10}\left(\cos\left(2\,x\right) + 2\,\sin\left(2\,x\right)\right)e^x + \frac{3}{8}\,e^x$$

```
[In] integrate(exp(x)*cos(x)^4,x, algorithm="giac")
```

```
[Out] 1/136*(cos(4*x) + 4*sin(4*x))*e^x + 1/10*(cos(2*x) + 2*sin(2*x))*e^x + 3/8*
e^x
```

Mupad [B] (verification not implemented)

Time = 27.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int e^x \cos^4(x)\,dx = \frac{3\,\mathrm{e}^x}{8} + \frac{\mathrm{e}^x\,\left(\frac{4\,\cos(2\,x)}{5} + \frac{8\,\sin(2\,x)}{5} + \frac{2\,\cos(2\,x)^2}{17} + \frac{8\,\cos(2\,x)\,\sin(2\,x)}{17} - \frac{1}{17}\right)}{8}$$

```
[In] int(exp(x)*cos(x)^4,x)
```

```
[Out] (3*exp(x))/8 + (exp(x)*((4*cos(2*x))/5 + (8*sin(2*x))/5 + (2*cos(2*x)^2)/17
 + (8*cos(2*x)*sin(2*x))/17 - 1/17))/8
```

3.19 $\int e^{c(a+bx)} \tan^3(d+ex) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 194

$$\int e^{c(a+bx)} \tan^3(d+ex) \, dx = \frac{ie^{c(a+bx)}}{bc} - \frac{6ie^{c(a+bx)} \text{Hypergeometric2F1}\left(1, -\frac{ibc}{2e}, 1-\frac{ibc}{2e}, -e^{2i(d+ex)}\right)}{bc} + \frac{12ie^{c(a+bx)} \text{Hypergeometric2F1}\left(2, -\frac{ibc}{2e}, 1-\frac{ibc}{2e}, -e^{2i(d+ex)}\right)}{bc} - \frac{8ie^{c(a+bx)} \text{Hypergeometric2F1}\left(3, -\frac{ibc}{2e}, 1-\frac{ibc}{2e}, -e^{2i(d+ex)}\right)}{bc}$$

```
[Out] I*exp(c*(b*x+a))/b/c-6*I*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2*
I*b*c/e],-exp(2*I*(e*x+d)))/b/c+12*I*exp(c*(b*x+a))*hypergeom([2, -1/2*I*b*
c/e],[1-1/2*I*b*c/e],-exp(2*I*(e*x+d)))/b/c-8*I*exp(c*(b*x+a))*hypergeom([3
, -1/2*I*b*c/e],[1-1/2*I*b*c/e],-exp(2*I*(e*x+d)))/b/c
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4527, 2225, 2283}

$$\int e^{c(a+bx)} \tan^3(d+ex) \, dx = -\frac{6ie^{c(a+bx)} \text{Hypergeometric2F1}\left(1, -\frac{ibc}{2e}, 1-\frac{ibc}{2e}, -e^{2i(d+ex)}\right)}{bc} + \frac{12ie^{c(a+bx)} \text{Hypergeometric2F1}\left(2, -\frac{ibc}{2e}, 1-\frac{ibc}{2e}, -e^{2i(d+ex)}\right)}{bc} - \frac{8ie^{c(a+bx)} \text{Hypergeometric2F1}\left(3, -\frac{ibc}{2e}, 1-\frac{ibc}{2e}, -e^{2i(d+ex)}\right)}{bc} + \frac{ie^{c(a+bx)}}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Tan[d + e*x]^3,x]
```

```
[Out] (I*E^(c*(a + b*x)))/(b*c) - ((6*I)*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-
1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c) + ((12*I)*E^
(c*(a + b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^
((2*I)*(d + e*x))])/(b*c) - ((8*I)*E^(c*(a + b*x))*Hypergeometric2F1[3, ((-
1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \left(-e^{c(a+bx)}+\frac{8e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^3}-\frac{12e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}+\frac{6e^{c(a+bx)}}{1+e^{2i(d+ex)}}\right)dx\right)\\
&= i\int e^{c(a+bx)}\,dx-6i\int \frac{e^{c(a+bx)}}{1+e^{2i(d+ex)}}\,dx-8i\int \frac{e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^3}\,dx+12i\int \frac{e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}\,dx\\
&= \frac{ie^{c(a+bx)}}{bc}-\frac{6ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}\\
&\quad+\frac{12ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}\\
&\quad-\frac{8ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.09

$$\int e^{c(a+bx)}\tan^3(d+ex)\,dx = \frac{1}{2}e^{c(a+bx)}\left(\frac{2(b^2c^2-2e^2)\,e^{2id}\left(bce^{2iex}\,\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},-e^{2i(d+ex)}\right)-(bc+2ie)\,\text{Hyper}\ldots\right.}{bc(ibc-2e)e^2\left(1+e^{2id}\right)}+\frac{\sec^2(d+ex)}{e}-\frac{bc\sec(d)\sec(d+ex)\sin(ex)}{e^2}-\frac{2\tan(d)}{bc}\right)$$

```
[In] Integrate[E^(c*(a + b*x))*Tan[d + e*x]^3,x]
```

```
[Out] (E^(c*(a + b*x))*((2*(b^2*c^2 - 2*e^2)*E^((2*I)*d)*(b*c*E^((2*I)*e*x)*Hyper
geometric2F1[1, 1 - ((I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))]
 - (b*c + (2*I)*e)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e
, -E^((2*I)*(d + e*x))]))/(b*c*(I*b*c - 2*e)*e^2*(1 + E^((2*I)*d))) + Sec[d
 + e*x]^2/e - (b*c*Sec[d]*Sec[d + e*x]*Sin[e*x])/e^2 - (2*Tan[d])/(b*c)))/2
```

Maple [F]

$$\int e^{c(xb+a)}\tan\left(ex+d\right)^3\,dx$$

```
[In] int(exp(c*(b*x+a))*tan(e*x+d)^3,x)
```

```
[Out] int(exp(c*(b*x+a))*tan(e*x+d)^3,x)
```

Fricas [F]

$$\int e^{c(a+bx)}\tan^3(d+ex)\,dx = \int e^{((bx+a)c)}\tan\left(ex+d\right)^3\,dx$$

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(e^(b*c*x + a*c)*tan(e*x + d)^3, x)
```

Sympy [F]

$$\int e^{c(a+bx)} \tan^3(d+ex)\,dx = e^{ac} \int e^{bcx} \tan^3{(d+ex)}\,dx$$

[In] integrate(exp(c*(b*x+a))*tan(e*x+d)**3,x)

[Out] exp(a*c)*Integral(exp(b*c*x)*tan(d + e*x)**3, x)

Maxima [F]

$$\int e^{c(a+bx)} \tan^3(d+ex)\,dx = \int e^{((bx+a)c)} \tan{(ex+d)}^3\,dx$$

[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="maxima")

[Out] (4*e*cos(2*e*x + 2*d)^2*e^(b*c*x + a*c) - b*c*e^(b*c*x + a*c)*sin(2*e*x + 2*d) + 4*e*e^(b*c*x + a*c)*sin(2*e*x + 2*d)^2 + 2*e*cos(2*e*x + 2*d)*e^(b*c*x + a*c) + (b*c*e^(b*c*x + a*c)*sin(2*e*x + 2*d) + 2*e*cos(2*e*x + 2*d)*e^(b*c*x + a*c))*cos(4*e*x + 4*d) + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(4*e*x + 4*d)^2 + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d)^2 + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(4*e*x + 4*d)^2 + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(2*e*x + 2*d)^2 + 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) + 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d))*integrate(e^(b*c*x)*sin(2*e*x + 2*d)/(e^4*cos(2*e*x + 2*d)^2 + e^4*sin(2*e*x + 2*d)^2 + 2*e^4*cos(2*e*x + 2*d) + e^4), x) - (b*c*cos(2*e*x + 2*d)*e^(b*c*x + a*c) + b*c*e^(b*c*x + a*c) - 2*e*e^(b*c*x + a*c)*sin(2*e*x + 2*d))*sin(4*e*x + 4*d))/(e^2*cos(4*e*x + 4*d)^2 + 4*e^2*cos(2*e*x + 2*d)^2 + e^2*sin(4*e*x + 4*d)^2 + 4*e^2*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*e^2*sin(2*e*x + 2*d)^2 + 4*e^2*cos(2*e*x + 2*d) + e^2 + 2*(2*e^2*cos(2*e*x + 2*d) + e^2)*cos(4*e*x + 4*d))

Giac [F]

$$\int e^{c(a+bx)} \tan^3(d+ex)\,dx = \int e^{((bx+a)c)} \tan{(ex+d)}^3\,dx$$

[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="giac")

[Out] integrate(e^((b*x + a)*c)*tan(e*x + d)^3, x)

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \tan^3(d+ex)\,dx = \int e^{c\,(a+b\,x)} \tan(d+e\,x)^3\,dx$$

```
[In] int(exp(c*(a + b*x))*tan(d + e*x)^3,x)
[Out] int(exp(c*(a + b*x))*tan(d + e*x)^3, x)
```

3.20 $\int e^{c(a+bx)}\tan^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 130

$$\int e^{c(a+bx)}\tan^2(d+ex)\,dx=-\frac{e^{c(a+bx)}}{bc}+\frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}$$

```
[Out] -exp(c*(b*x+a))/b/c+4*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2*I*b
*c/e],-exp(2*I*(e*x+d)))/b/c-4*exp(c*(b*x+a))*hypergeom([2, -1/2*I*b*c/e],[
1-1/2*I*b*c/e],-exp(2*I*(e*x+d)))/b/c
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4527, 2225, 2283}

$$\int e^{c(a+bx)}\tan^2(d+ex)\,dx=\frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}-\frac{e^{c(a+bx)}}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Tan[d + e*x]^2,x]
```

```
[Out] -(E^(c*(a + b*x))/(b*c)) + (4*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-1/2*I
)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c) - (4*E^(c*(a + b*
```

```
x))*Hypergeometric2F1[2, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d
+ e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \left(e^{c(a+bx)} + \frac{4e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2} - \frac{4e^{c(a+bx)}}{1+e^{2i(d+ex)}}\right) dx \\ &= -\left(4\int \frac{e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}\,dx\right) + 4\int \frac{e^{c(a+bx)}}{1+e^{2i(d+ex)}}\,dx - \int e^{c(a+bx)}\,dx \\ &= -\frac{e^{c(a+bx)}}{bc} + \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc} \\ &\quad - \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.35

$$\int e^{c(a+bx)}\tan^2(d+ex)\,dx = e^{c(a+bx)}\left(-\frac{1}{bc} + \frac{2e^{2id}\left(ibce^{2iex}\,\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},-e^{2i(d+ex)}\right) + (-ibc+2e)\,\text{Hypergeometric2F1}\left(1,\right.\right.}{(bc+2ie)e\left(1+e^{2id}\right)} + \frac{\sec(d)\sec(d+ex)\sin(ex)}{e}\right)$$

```
[In] Integrate[E^(c*(a + b*x))*Tan[d + e*x]^2,x]
```

```
[Out] E^(c*(a + b*x))*(-(1/(b*c)) + (2*E^((2*I)*d)*(I*b*c*E^((2*I)*e*x)*Hypergeom
etric2F1[1, 1 - ((I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))] + (
(-I)*b*c + 2*e)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -
E^((2*I)*(d + e*x))]))/((b*c + (2*I)*e)*e*(1 + E^((2*I)*d))) + (Sec[d]*Sec[
d + e*x]*Sin[e*x])/e)
```

Maple [F]

$$\int \mathrm{e}^{c(xb+a)} \tan (ex+d)^2 \, dx$$

```
[In] int(exp(c*(b*x+a))*tan(e*x+d)^2,x)
```

```
[Out] int(exp(c*(b*x+a))*tan(e*x+d)^2,x)
```

Fricas [F]

$$\int e^{c(a+bx)} \tan^2(d+ex) \, dx = \int e^{((bx+a)c)} \tan (ex+d)^2 \, dx$$

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(e^(b*c*x + a*c)*tan(e*x + d)^2, x)
```

Sympy [F]

$$\int e^{c(a+bx)} \tan^2(d+ex) \, dx = e^{ac} \int e^{bcx} \tan^2 (d+ex) \, dx$$

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)**2,x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*tan(d + e*x)**2, x)
```

Maxima [F]

$$\int e^{c(a+bx)} \tan^2(d+ex) \, dx = \int e^{((bx+a)c)} \tan (ex+d)^2 \, dx$$

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(e*cos(2*e*x + 2*d)^2*e^(b*c*x + a*c) - 2*b*c*e^(b*c*x + a*c)*sin(2*e*x +
2*d) + e*e^(b*c*x + a*c)*sin(2*e*x + 2*d)^2 + 2*e*cos(2*e*x + 2*d)*e^(b*c*x
```

```
 + a*c) + e*e^(b*c*x + a*c) + 2*(b^2*c^2*e^2*cos(2*e*x + 2*d)^2 + b^2*c^2*e
^2*sin(2*e*x + 2*d)^2 + 2*b^2*c^2*e^2*cos(2*e*x + 2*d) + b^2*c^2*e^2)*integ
rate(e^(b*c*x + a*c)*sin(2*e*x + 2*d)/(e^2*cos(2*e*x + 2*d)^2 + e^2*sin(2*e
*x + 2*d)^2 + 2*e^2*cos(2*e*x + 2*d) + e^2), x))/(b*c*e*cos(2*e*x + 2*d)^2
+ b*c*e*sin(2*e*x + 2*d)^2 + 2*b*c*e*cos(2*e*x + 2*d) + b*c*e)
```

Giac [F]

$$\int e^{c(a+bx)}\tan^2(d+ex)\,dx=\int e^{((bx+a)c)}\tan{(ex+d)}^2\,dx$$

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="giac")
[Out] integrate(e^((b*x + a)*c)*tan(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\tan^2(d+ex)\,dx=\int \mathrm{e}^{c\,(a+b\,x)}\,\tan(d+e\,x)^2\,dx$$

```
[In] int(exp(c*(a + b*x))*tan(d + e*x)^2,x)
[Out] int(exp(c*(a + b*x))*tan(d + e*x)^2, x)
```

3.21 $\int e^{c(a+bx)} \tan(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int e^{c(a+bx)} \tan(d+ex)\,dx = -\frac{ie^{c(a+bx)}}{bc} + \frac{2ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}$$

```
[Out] -I*exp(c*(b*x+a))/b/c+2*I*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2
*I*b*c/e],-exp(2*I*(e*x+d)))/b/c
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4527, 2225, 2283}

$$\int e^{c(a+bx)} \tan(d+ex)\,dx = \frac{2ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc} - \frac{ie^{c(a+bx)}}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Tan[d + e*x],x]
```

```
[Out] ((-I)*E^(c*(a + b*x)))/(b*c) + ((2*I)*E^(c*(a + b*x))*Hypergeometric2F1[1,
((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i\int\left(-e^{c(a+bx)}+\frac{2e^{c(a+bx)}}{1+e^{2i(d+ex)}}\right)dx\\ &= -\left(i\int e^{c(a+bx)}\,dx\right)+2i\int\frac{e^{c(a+bx)}}{1+e^{2i(d+ex)}}\,dx\\ &= -\frac{ie^{c(a+bx)}}{bc}+\frac{2ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 166 vs. $2(78) = 156$.

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.13

$$\int e^{c(a+bx)}\tan(d+ex)\,dx$$
$$= \frac{e^{c(a+bx)}\left(2bce^{2i(d+ex)}\,\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},-e^{2i(d+ex)}\right)-(bc+2ie)\left(1-e^{2id}+2e^{2id}\,\text{Hype}\right.\right.}{bc(ibc-2e)\left(1+e^{2id}\right)}$$

```
[In] Integrate[E^(c*(a + b*x))*Tan[d + e*x],x]
```

```
[Out] (E^(c*(a + b*x))*(2*b*c*E^((2*I)*(d + e*x))*Hypergeometric2F1[1, 1 - ((I/2)
*b*c)/e, 2 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))] - (b*c + (2*I)*e)*(1 - E^
((2*I)*d) + 2*E^((2*I)*d)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)
*b*c)/e, -E^((2*I)*(d + e*x))])))/(b*c*(I*b*c - 2*e)*(1 + E^((2*I)*d)))
```

Maple [F]

$$\int \mathrm{e}^{c(xb+a)} \tan (ex+d)\, dx$$

```
[In] int(exp(c*(b*x+a))*tan(e*x+d),x)
[Out] int(exp(c*(b*x+a))*tan(e*x+d),x)
```

Fricas [F]

$$\int e^{c(a+bx)} \tan(d+ex)\, dx = \int e^{((bx+a)c)} \tan (ex+d)\; dx$$

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d),x, algorithm="fricas")
[Out] integral(e^(b*c*x + a*c)*tan(e*x + d), x)
```

Sympy [F]

$$\int e^{c(a+bx)} \tan(d+ex)\, dx = e^{ac} \int e^{bcx} \tan (d+ex)\, dx$$

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d),x)
[Out] exp(a*c)*Integral(exp(b*c*x)*tan(d + e*x), x)
```

Maxima [F]

$$\int e^{c(a+bx)} \tan(d+ex)\, dx = \int e^{((bx+a)c)} \tan (ex+d)\; dx$$

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d),x, algorithm="maxima")
[Out] integrate(e^((b*x + a)*c)*tan(e*x + d), x)
```

Giac [F]

$$\int e^{c(a+bx)} \tan(d+ex)\,dx = \int e^{((bx+a)c)} \tan(ex+d)\,dx$$

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d),x, algorithm="giac")
[Out] integrate(e^((b*x + a)*c)*tan(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \tan(d+ex)\,dx = \int e^{c\,(a+b\,x)} \tan(d+e\,x)\,dx$$

```
[In] int(exp(c*(a + b*x))*tan(d + e*x),x)
[Out] int(exp(c*(a + b*x))*tan(d + e*x), x)
```

3.22 $\int e^{c(a+bx)} \cot(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int e^{c(a+bx)}\cot(d+ex)\,dx=\frac{ie^{c(a+bx)}}{bc}-\frac{2ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}$$

```
[Out] I*exp(c*(b*x+a))/b/c-2*I*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2*
I*b*c/e],exp(2*I*(e*x+d)))/b/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4528, 2225, 2283}

$$\int e^{c(a+bx)}\cot(d+ex)\,dx=\frac{ie^{c(a+bx)}}{bc}-\frac{2ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Cot[d + e*x],x]
```

```
[Out] (I*E^(c*(a + b*x)))/(b*c) - ((2*I)*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-
1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
```

```
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4528

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d + e
*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(i\int\left(-e^{c(a+bx)}-\frac{2e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\right)dx\right)\\ &= i\int e^{c(a+bx)}\,dx+2i\int\frac{e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\,dx\\ &= \frac{ie^{c(a+bx)}}{bc}-\frac{2ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 163 vs. 2(76) = 152.

Time = 1.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.14

$$\int e^{c(a+bx)}\cot(d+ex)\,dx$$
$$=\frac{e^{c(a+bx)}\left(2ibce^{2i(d+ex)}\,\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},e^{2i(d+ex)}\right)+i(bc+2ie)\left(1+e^{2id}-2e^{2id}\,\text{Hype}\right.\right.}{bc(bc+2ie)\left(-1+e^{2id}\right)}$$

```
[In] Integrate[E^(c*(a + b*x))*Cot[d + e*x],x]
```

```
[Out] (E^(c*(a + b*x))*((2*I)*b*c*E^((2*I)*(d + e*x))*Hypergeometric2F1[1, 1 - ((
I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))] + I*(b*c + (2*I)*e)*(1
 + E^((2*I)*d) - 2*E^((2*I)*d)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - (
(I/2)*b*c)/e, E^((2*I)*(d + e*x))])))/(b*c*(b*c + (2*I)*e)*(-1 + E^((2*I)*d
)))
```

Maple [F]

$$\int \mathrm{e}^{c(xb+a)}\cot (ex+d)\, dx$$

```
[In] int(exp(c*(b*x+a))*cot(e*x+d),x)
[Out] int(exp(c*(b*x+a))*cot(e*x+d),x)
```

Fricas [F]

$$\int e^{c(a+bx)}\cot(d+ex)\, dx = \int \cot (ex+d)\, e^{((bx+a)c)}\, dx$$

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d),x, algorithm="fricas")
[Out] integral(cot(e*x + d)*e^(b*c*x + a*c), x)
```

Sympy [F]

$$\int e^{c(a+bx)}\cot(d+ex)\, dx = e^{ac}\int e^{bcx}\cot (d+ex)\, dx$$

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d),x)
[Out] exp(a*c)*Integral(exp(b*c*x)*cot(d + e*x), x)
```

Maxima [F]

$$\int e^{c(a+bx)}\cot(d+ex)\, dx = \int \cot (ex+d)\, e^{((bx+a)c)}\, dx$$

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d),x, algorithm="maxima")
[Out] integrate(cot(e*x + d)*e^((b*x + a)*c), x)
```

Giac [F]

$$\int e^{c(a+bx)} \cot(d+ex)\,dx = \int \cot(ex+d)\,e^{((bx+a)c)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d),x, algorithm="giac")
[Out] integrate(cot(e*x + d)*e^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \cot(d+ex)\,dx = \int \cot(d+e\,x)\,\mathrm{e}^{c\,(a+b\,x)}\,dx$$

```
[In] int(cot(d + e*x)*exp(c*(a + b*x)),x)
[Out] int(cot(d + e*x)*exp(c*(a + b*x)), x)
```

3.23 $\int e^{c(a+bx)} \cot^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 126

$$\int e^{c(a+bx)} \cot^2(d+ex)\,dx = -\frac{e^{c(a+bx)}}{bc} + \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} - \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}$$

```
[Out] -exp(c*(b*x+a))/b/c+4*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2*I*b
*c/e],exp(2*I*(e*x+d)))/b/c-4*exp(c*(b*x+a))*hypergeom([2, -1/2*I*b*c/e],[1
-1/2*I*b*c/e],exp(2*I*(e*x+d)))/b/c
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4528, 2225, 2283}

$$\int e^{c(a+bx)} \cot^2(d+ex)\,dx = \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} - \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} - \frac{e^{c(a+bx)}}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Cot[d + e*x]^2,x]
```

```
[Out] -(E^(c*(a + b*x))/(b*c)) + (4*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-1/2*I
)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c) - (4*E^(c*(a + b*x
```

```
))*Hypergeometric2F1[2, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d +
e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4528

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d + e
*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \left(e^{c(a+bx)}+\frac{4e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^2}+\frac{4e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\right)dx\\ &= -\left(4\int \frac{e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^2}\,dx\right)-4\int \frac{e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\,dx-\int e^{c(a+bx)}\,dx\\ &= -\frac{e^{c(a+bx)}}{bc}+\frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}\\ &\quad-\frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.36

$$\begin{aligned}\int e^{c(a+bx)}\cot^2(d+ex)\,dx &= e^{c(a+bx)}\left(-\frac{1}{bc}\right.\\ &\quad+\frac{2e^{2id}\left(ibce^{2iex}\,\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},e^{2i(d+ex)}\right)+(-ibc+2e)\,\text{Hypergeometric2F1}\left(1,-\right.\right.}{(bc+2ie)e\left(-1+e^{2id}\right)}\\ &\quad\left.+\frac{\csc(d)\csc(d+ex)\sin(ex)}{e}\right)\end{aligned}$$

```
[In] Integrate[E^(c*(a + b*x))*Cot[d + e*x]^2,x]
```

```
[Out] E^(c*(a + b*x))*(-(1/(b*c)) + (2*E^((2*I)*d)*(I*b*c*E^((2*I)*e*x)*Hypergeom
etric2F1[1, 1 - ((I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))] + ((
-I)*b*c + 2*e)*Hypergeometric2F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^
((2*I)*(d + e*x))]))/((b*c + (2*I)*e)*e*(-1 + E^((2*I)*d))) + (Csc[d]*Csc[d
 + e*x]*Sin[e*x])/e)
```

Maple [F]

$$\int \mathrm{e}^{c(xb+a)} \cot (ex+d)^2 \, dx$$

```
[In] int(exp(c*(b*x+a))*cot(e*x+d)^2,x)
```

```
[Out] int(exp(c*(b*x+a))*cot(e*x+d)^2,x)
```

Fricas [F]

$$\int e^{c(a+bx)} \cot^2(d+ex) \, dx = \int \cot (ex+d)^2 e^{((bx+a)c)} \, dx$$

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(cot(e*x + d)^2*e^(b*c*x + a*c), x)
```

Sympy [F]

$$\int e^{c(a+bx)} \cot^2(d+ex) \, dx = e^{ac} \int e^{bcx} \cot^2 (d+ex) \, dx$$

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)**2,x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*cot(d + e*x)**2, x)
```

Maxima [F]

$$\int e^{c(a+bx)} \cot^2(d+ex) \, dx = \int \cot (ex+d)^2 e^{((bx+a)c)} \, dx$$

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(e*cos(2*e*x + 2*d)^2*e^(b*c*x + a*c) + 2*b*c*e^(b*c*x + a*c)*sin(2*e*x +
2*d) + e*e^(b*c*x + a*c)*sin(2*e*x + 2*d)^2 - 2*e*cos(2*e*x + 2*d)*e^(b*c*x
```

```
 + a*c) + e*e^(b*c*x + a*c) + (b^2*c^2*e^2*cos(2*e*x + 2*d)^2 + b^2*c^2*e^2
*sin(2*e*x + 2*d)^2 - 2*b^2*c^2*e^2*cos(2*e*x + 2*d) + b^2*c^2*e^2)*integra
te(e^(b*c*x + a*c)*sin(e*x + d)/(e^2*cos(e*x + d)^2 + e^2*sin(e*x + d)^2 +
2*e^2*cos(e*x + d) + e^2), x) - (b^2*c^2*e^2*cos(2*e*x + 2*d)^2 + b^2*c^2*e
^2*sin(2*e*x + 2*d)^2 - 2*b^2*c^2*e^2*cos(2*e*x + 2*d) + b^2*c^2*e^2)*integ
rate(e^(b*c*x + a*c)*sin(e*x + d)/(e^2*cos(e*x + d)^2 + e^2*sin(e*x + d)^2
- 2*e^2*cos(e*x + d) + e^2), x))/(b*c*e*cos(2*e*x + 2*d)^2 + b*c*e*sin(2*e*
x + 2*d)^2 - 2*b*c*e*cos(2*e*x + 2*d) + b*c*e)
```

Giac [F]

$$\int e^{c(a+bx)} \cot^2(d+ex)\,dx = \int \cot(ex+d)^2 e^{((bx+a)c)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="giac")
[Out] integrate(cot(e*x + d)^2*e^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \cot^2(d+ex)\,dx = \int \cot(d+e\,x)^2\,\mathrm{e}^{c\,(a+b\,x)}\,dx$$

```
[In] int(cot(d + e*x)^2*exp(c*(a + b*x)),x)
[Out] int(cot(d + e*x)^2*exp(c*(a + b*x)), x)
```

3.24 $\int e^{c(a+bx)} \cot^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 188

$$\int e^{c(a+bx)} \cot^3(d+ex)\,dx = -\frac{ie^{c(a+bx)}}{bc} + \frac{6ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} - \frac{12ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} + \frac{8ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}$$

```
[Out] -I*exp(c*(b*x+a))/b/c+6*I*exp(c*(b*x+a))*hypergeom([1, -1/2*I*b*c/e],[1-1/2
*I*b*c/e],exp(2*I*(e*x+d)))/b/c-12*I*exp(c*(b*x+a))*hypergeom([2, -1/2*I*b*
c/e],[1-1/2*I*b*c/e],exp(2*I*(e*x+d)))/b/c+8*I*exp(c*(b*x+a))*hypergeom([3,
 -1/2*I*b*c/e],[1-1/2*I*b*c/e],exp(2*I*(e*x+d)))/b/c
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4528, 2225, 2283}

$$\int e^{c(a+bx)} \cot^3(d+ex)\,dx = \frac{6ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} - \frac{12ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} + \frac{8ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc} - \frac{ie^{c(a+bx)}}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Cot[d + e*x]^3,x]
```

```
[Out] ((-I)*E^(c*(a + b*x)))/(b*c) + ((6*I)*E^(c*(a + b*x))*Hypergeometric2F1[1,
((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c) - ((12*I)*
E^(c*(a + b*x))*Hypergeometric2F1[2, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E
^((2*I)*(d + e*x))])/(b*c) + ((8*I)*E^(c*(a + b*x))*Hypergeometric2F1[3, ((
-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4528

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d + e
*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i\int \left(-e^{c(a+bx)}-\frac{8e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^3}-\frac{12e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^2}-\frac{6e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\right)dx\\
&= -\left(i\int e^{c(a+bx)}\,dx\right)-6i\int \frac{e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\,dx\\
&\quad -8i\int \frac{e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^3}\,dx-12i\int \frac{e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^2}\,dx\\
&= -\frac{ie^{c(a+bx)}}{bc}+\frac{6ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}\\
&\quad -\frac{12ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}\\
&\quad +\frac{8ie^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.12

$$\int e^{c(a+bx)}\cot^3(d+ex)\,dx = \frac{1}{2}e^{c(a+bx)}\left(-\frac{2\cot(d)}{bc}-\frac{\csc^2(d+ex)}{e}+\frac{2(b^2c^2-2e^2)\,e^{2id}\left(ibce^{2iex}\,\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},e^{2i(d+ex)}\right)+(-ibc+2e)\,\text{Hypergeometr}\ldots\right)}{bc(bc+2ie)e^2\left(-1+e^{2id}\right)}+\frac{bc\csc(d)\csc(d+ex)\sin(ex)}{e^2}\right)$$

```
[In] Integrate[E^(c*(a + b*x))*Cot[d + e*x]^3,x]
```

```
[Out] (E^(c*(a + b*x))*((-2*Cot[d])/(b*c) - Csc[d + e*x]^2/e + (2*(b^2*c^2 - 2*e^
2)*E^((2*I)*d)*(I*b*c*E^((2*I)*e*x)*Hypergeometric2F1[1, 1 - ((I/2)*b*c)/e,
 2 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))] + ((-I)*b*c + 2*e)*Hypergeometric2
F1[1, ((-1/2*I)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))]))/(b*c*(b*c
 + (2*I)*e)*e^2*(-1 + E^((2*I)*d))) + (b*c*Csc[d]*Csc[d + e*x]*Sin[e*x])/e^
2))/2
```

Maple [F]

$$\int \mathrm{e}^{c(xb+a)}\cot\left(ex+d\right)^3 dx$$

```
[In] int(exp(c*(b*x+a))*cot(e*x+d)^3,x)
```

```
[Out] int(exp(c*(b*x+a))*cot(e*x+d)^3,x)
```

Fricas [F]

$$\int e^{c(a+bx)}\cot^3(d+ex)\,dx = \int \cot\left(ex+d\right)^3 e^{((bx+a)c)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(cot(e*x + d)^3*e^(b*c*x + a*c), x)
```

Sympy [F]

$$\int e^{c(a+bx)}\cot^3(d+ex)\,dx = e^{ac}\int e^{bcx}\cot^3{(d+ex)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)**3,x)
[Out] exp(a*c)*Integral(exp(b*c*x)*cot(d + e*x)**3, x)
```

Maxima [F]

$$\int e^{c(a+bx)}\cot^3(d+ex)\,dx = \int \cot{(ex+d)}^3\,e^{((bx+a)c)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="maxima")
[Out] -(4*e*cos(2*e*x + 2*d)^2*e^(b*c*x + a*c) + b*c*e^(b*c*x + a*c)*sin(2*e*x +
2*d) + 4*e*e^(b*c*x + a*c)*sin(2*e*x + 2*d)^2 - 2*e*cos(2*e*x + 2*d)*e^(b*c
*x + a*c) - (b*c*e^(b*c*x + a*c)*sin(2*e*x + 2*d) + 2*e*cos(2*e*x + 2*d)*e^
(b*c*x + a*c))*cos(4*e*x + 4*d) + 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) +
(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(4*e*x + 4*d)^2 + 4*(b^2*c^2*e^4*e
^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d)^2 + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e
^(a*c))*sin(4*e*x + 4*d)^2 - 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(4*
e*x + 4*d)*sin(2*e*x + 2*d) + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(2
*e*x + 2*d)^2 + 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) - 2*(b^2*c^2*e^4*e^(
a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) - 4*(b^2*c^2*e^4*e
^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d))*integrate(1/4*e^(b*c*x)*sin(e*x +
 d)/(e^4*cos(e*x + d)^2 + e^4*sin(e*x + d)^2 + 2*e^4*cos(e*x + d) + e^4), x
) - 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e
^(a*c))*cos(4*e*x + 4*d)^2 + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*
e*x + 2*d)^2 + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(4*e*x + 4*d)^2 - 4
*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) +
4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(2*e*x + 2*d)^2 + 2*(b^2*c^2*e^4
*e^(a*c) - 2*e^6*e^(a*c) - 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*e*
x + 2*d))*cos(4*e*x + 4*d) - 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*
e*x + 2*d))*integrate(1/4*e^(b*c*x)*sin(e*x + d)/(e^4*cos(e*x + d)^2 + e^4*
sin(e*x + d)^2 - 2*e^4*cos(e*x + d) + e^4), x) + (b*c*cos(2*e*x + 2*d)*e^(b
*c*x + a*c) - b*c*e^(b*c*x + a*c) - 2*e*e^(b*c*x + a*c)*sin(2*e*x + 2*d))*s
in(4*e*x + 4*d))/(e^2*cos(4*e*x + 4*d)^2 + 4*e^2*cos(2*e*x + 2*d)^2 + e^2*s
in(4*e*x + 4*d)^2 - 4*e^2*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*e^2*sin(2*e
*x + 2*d)^2 - 4*e^2*cos(2*e*x + 2*d) + e^2 - 2*(2*e^2*cos(2*e*x + 2*d) - e^
2)*cos(4*e*x + 4*d))
```

Giac [F]

$$\int e^{c(a+bx)} \cot^3(d+ex)\,dx = \int \cot\left(ex+d\right)^3 e^{((bx+a)c)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(cot(e*x + d)^3*e^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \cot^3(d+ex)\,dx = \int \cot(d+e\,x)^3\,\mathrm{e}^{c\,(a+b\,x)}\,dx$$

```
[In] int(cot(d + e*x)^3*exp(c*(a + b*x)),x)
```

```
[Out] int(cot(d + e*x)^3*exp(c*(a + b*x)), x)
```

3.25 $\int F^{a+bx} \tan \left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 76

$$\int F^{a+bx} \tan \left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right) \, dx = \frac{iF^{a+bx}}{b\log(F)}-\frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)}{b\log(F)}$$

```
[Out] I*F^(b*x+a)/b/ln(F)-2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(F)/
d],I*exp(I*(d*x+c)))/b/ln(F)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4552, 4527, 2225, 2283}

$$\int F^{a+bx} \tan \left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right) \, dx = \frac{iF^{a+bx}}{b\log(F)}-\frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)}{b\log(F)}$$

```
[In] Int[F^(a + b*x)*Tan[Pi/4 + (-c - d*x)/2],x]
```

```
[Out] (I*F^(a + b*x))/(b*Log[F]) - ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, ((-I)*
b*Log[F])/d, 1 - (I*b*Log[F])/d, I*E^(I*(c + d*x))])/(b*Log[F])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4552

```
Int[(F_)^((c_.)*(u_))*(G_)[v_]^(n_.), x_Symbol] :> Int[F^(c*ExpandToSum[u,
x])*G[ExpandToSum[v, x]]^n, x] /; FreeQ[{F, c, n}, x] && TrigQ[G] && Linear
Q[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int F^{a+bx}\tan\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)dx\\
&= -\left(i\int\left(-F^{a+bx}+\frac{2F^{a+bx}}{1+e^{2i\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)}}\right)dx\right)\\
&= i\int F^{a+bx}\,dx-2i\int\frac{F^{a+bx}}{1+e^{2i\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)}}\,dx\\
&= \frac{iF^{a+bx}}{b\log(F)}-\frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)}{b\log(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.75

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx$$
$$=\frac{F^{a+bx}\left(be^{i(c+dx)}\,\text{Hypergeometric2F1}\left(1,1-\frac{ib\log(F)}{d},2-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)\log(F)+\text{Hypergeometric2F1}\left(\right.\right.}{b\log(F)(id+b\log(F))}$$

```
[In] Integrate[F^(a + b*x)*Tan[Pi/4 + (-c - d*x)/2],x]
```

```
[Out] (F^(a + b*x)*(b*E^(I*(c + d*x))*Hypergeometric2F1[1, 1 - (I*b*Log[F])/d, 2
- (I*b*Log[F])/d, I*E^(I*(c + d*x))]*Log[F] + Hypergeometric2F1[1, ((-I)*b*
Log[F])/d, 1 - (I*b*Log[F])/d, I*E^(I*(c + d*x))]*(d - I*b*Log[F])))/(b*Log
[F]*(I*d + b*Log[F]))
```

Maple [F]

$$\int F^{xb+a}\cot\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx$$

```
[In] int(F^(b*x+a)*cot(1/2*c+1/4*Pi+1/2*d*x),x)
```

```
[Out] int(F^(b*x+a)*cot(1/2*c+1/4*Pi+1/2*d*x),x)
```

Fricas [F]

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx=\int F^{bx+a}\cot\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\,dx$$

```
[In] integrate(F^(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x, algorithm="fricas")
```

```
[Out] integral(F^(b*x + a)*cot(1/4*pi + 1/2*d*x + 1/2*c), x)
```

Sympy [F]

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx=\int F^{a+bx}\cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right)dx$$

```
[In] integrate(F**(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x)
```

```
[Out] Integral(F**(a + b*x)*cot(c/2 + d*x/2 + pi/4), x)
```

Maxima [F]

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx=\int F^{bx+a}\cot\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\,dx$$

```
[In] integrate(F^(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x, algorithm="maxima")
[Out] integrate(F^(b*x + a)*cot(1/4*pi + 1/2*d*x + 1/2*c), x)
```

Giac [F]

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx=\int F^{bx+a}\cot\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\,dx$$

```
[In] integrate(F^(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x, algorithm="giac")
[Out] integrate(F^(b*x + a)*cot(1/4*pi + 1/2*d*x + 1/2*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx=\int F^{a+b\,x}\cot\left(\frac{\Pi}{4}+\frac{c}{2}+\frac{d\,x}{2}\right)\,dx$$

```
[In] int(F^(a + b*x)*cot(Pi/4 + c/2 + (d*x)/2),x)
[Out] int(F^(a + b*x)*cot(Pi/4 + c/2 + (d*x)/2), x)
```

3.26 $\int F^{c(a+bx)} \sec^n(d+ex)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int F^{c(a+bx)} \sec^n(d+ex)\, dx = \frac{\left(1+e^{2i(d+ex)}\right)^n F^{ac+bcx} \text{Hypergeometric2F1}\left(n, \frac{en-ibc\log(F)}{2e}, \frac{1}{2}\left(2+n-\frac{ibc\log(F)}{e}\right), -e^{2i(d+ex)}\right) \sec^n(d+ex}{ien+bc\log(F)}$$

```
[Out] (1+exp(2*I*(e*x+d)))^n*F^(b*c*x+a*c)*hypergeom([n, 1/2*(e*n-I*b*c*ln(F))/e]
,[1+1/2*n-1/2*I*b*c*ln(F)/e],-exp(2*I*(e*x+d)))*sec(e*x+d)^n/(b*c*ln(F)+I*e
*n)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4539, 2291}

$$\int F^{c(a+bx)} \sec^n(d+ex)\, dx = \frac{\left(1+e^{2i(d+ex)}\right)^n F^{ac+bcx} \sec^n(d+ex) \text{Hypergeometric2F1}\left(n, \frac{en-ibc\log(F)}{2e}, \frac{1}{2}\left(n-\frac{ibc\log(F)}{e}+2\right), -e^{2i(d+ex)}\right.}{bc\log(F)+ien}$$

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x]^n,x]
```

```
[Out] ((1 + E^((2*I)*(d + e*x)))^n*F^(a*c + b*c*x)*Hypergeometric2F1[n, (e*n - I*
b*c*Log[F])/(2*e), (2 + n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]*Sec[
d + e*x]^n)/(I*e*n + b*c*Log[F])
```

Rule 2291

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 4539

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[(1 + E^(2*I*(d + e*x)))^n*(Sec[d + e*x]^n/E^(I*n*(d + e*x))), I
nt[SimplifyIntegrand[F^(c*(a + b*x))*(E^(I*n*(d + e*x))/(1 + E^(2*I*(d + e*
x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(e^{-in(d+ex)}\left(1+e^{2i(d+ex)}\right)^n \sec^n(d+ex)\right)\int e^{idn+ienx}\left(1+e^{2i(d+ex)}\right)^{-n} F^{ac+bcx}\,dx$$

$$= \frac{\left(1+e^{2i(d+ex)}\right)^n F^{ac+bcx}\,\text{Hypergeometric2F1}\left(n, \frac{en-ibc\log(F)}{2e}, \frac{1}{2}\left(2+n-\frac{ibc\log(F)}{e}\right), -e^{2i(d+ex)}\right)\sec^n(\ldots}{ien+bc\log(F)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}\sec^n(d+ex)\,dx =$$

$$-\frac{i\left(1+e^{2i(d+ex)}\right)^n F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(n, \frac{en-ibc\log(F)}{2e}, \frac{1}{2}\left(2+n-\frac{ibc\log(F)}{e}\right), -e^{2i(d+ex)}\right)\sec^n(d+\ldots}{en-ibc\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x]^n,x]
```

```
[Out] ((-I)*(1 + E^((2*I)*(d + e*x)))^n*F^(c*(a + b*x))*Hypergeometric2F1[n, (e*n
 - I*b*c*Log[F])/(2*e), (2 + n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]
*Sec[d + e*x]^n)/(e*n - I*b*c*Log[F])
```

Maple [F]

$$\int F^{c(xb+a)} \sec(ex+d)^n \, dx$$

```
[In] int(F^(c*(b*x+a))*sec(e*x+d)^n,x)
[Out] int(F^(c*(b*x+a))*sec(e*x+d)^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \sec^n(d+ex) \, dx = \int F^{(bx+a)c} \sec(ex+d)^n \, dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^n,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*sec(e*x + d)^n, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \sec^n(d+ex) \, dx = \int F^{c(a+bx)} \sec^n(d+ex) \, dx$$

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d)**n,x)
[Out] Integral(F**(c*(a + b*x))*sec(d + e*x)**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \sec^n(d+ex) \, dx = \int F^{(bx+a)c} \sec(ex+d)^n \, dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^n,x, algorithm="maxima")
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^n, x)
```

Giac [F]

$$\int F^{c(a+bx)} \sec^n(d+ex)\,dx = \int F^{(bx+a)c} \sec{(ex+d)}^n\;dx$$

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^n,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \sec^n(d+ex)\,dx = \int F^{c\,(a+b\,x)} \left(\frac{1}{\cos{(d+e\,x)}}\right)^n dx$$

```
[In] int(F^(c*(a + b*x))*(1/cos(d + e*x))^n,x)
[Out] int(F^(c*(a + b*x))*(1/cos(d + e*x))^n, x)
```

3.27 $\int F^{c(a+bx)} \csc^n(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 102

$$\int F^{c(a+bx)} \csc^n(d+ex)\,dx = -\frac{\left(1-e^{-2i(d+ex)}\right)^n F^{ac+bcx} \csc^n(d+ex)\,\text{Hypergeometric2F1}\left(n, \frac{en+ibc\log(F)}{2e}, \frac{1}{2}\left(2+n+\frac{ibc\log(F)}{e}\right), e^{-2i(d+\right.}{ien-bc\log(F)}$$

```
[Out] -(1-1/exp(2*I*(e*x+d)))^n*F^(b*c*x+a*c)*csc(e*x+d)^n*hypergeom([n, 1/2*(I*b
*c*ln(F)+e*n)/e],[1+1/2*n+1/2*I*b*c*ln(F)/e],exp(-2*I*(e*x+d)))/(I*e*n-b*c*
ln(F))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4540, 2291}

$$\int F^{c(a+bx)} \csc^n(d+ex)\,dx = -\frac{\left(1-e^{-2i(d+ex)}\right)^n F^{ac+bcx} \csc^n(d+ex)\,\text{Hypergeometric2F1}\left(n, \frac{en+ibc\log(F)}{2e}, \frac{1}{2}\left(n+\frac{ibc\log(F)}{e}+2\right), e^{-2i(d+\right.}{-bc\log(F)+ien}$$

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x]^n,x]
```

```
[Out] -(((1 - E^((-2*I)*(d + e*x)))^n*F^(a*c + b*c*x)*Csc[d + e*x]^n*Hypergeometr
ic2F1[n, (e*n + I*b*c*Log[F])/(2*e), (2 + n + (I*b*c*Log[F])/e)/2, E^((-2*I
)*(d + e*x))])/(I*e*n - b*c*Log[F]))
```

<u>Rule 2291</u>

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^(h*
(f + g*x))*H^(t*(r + s*x))*((a + b*F^(e*(c + d*x)))^p/((g*h*Log[G] + s*t*Lo
g[H])*((a + b*F^(e*(c + d*x)))/a)^p))*Hypergeometric2F1[-p, (g*h*Log[G] + s
*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]) + 1, Simpli
fy[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h,
r, s, t, p}, x] &&  !IntegerQ[p]
```

Rule 4540

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(1 - E^(-2*I*(d + e*x)))^n*(Csc[d + e*x]^n/E^((-I)*n*(d + e*x))
), Int[SimplifyIntegrand[F^(c*(a + b*x))*(1/(E^(I*n*(d + e*x))*(1 - E^(-2*I
*(d + e*x)))^n)), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[
n]
```

Rubi steps

$$\text{integral} = \left(e^{in(d+ex)}\left(1-e^{-2i(d+ex)}\right)^n \csc^n(d+ex)\right)\int e^{-idn-ienx}\left(1-e^{-2i(d+ex)}\right)^{-n}F^{ac+bcx}\,dx$$

$$= -\frac{\left(1-e^{-2i(d+ex)}\right)^n F^{ac+bcx}\csc^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(2+n+\frac{ibc\log(F)}{e}\right),\right.}{ien-bc\log(F)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\csc^n(d+ex)\,dx$$

$$= \frac{i\left(1-e^{-2i(d+ex)}\right)^n F^{c(a+bx)}\csc^n(d+ex)\,\text{Hypergeometric2F1}\left(n,\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(2+n+\frac{ibc\log(F)}{e}\right),e^{-2i(d+ex)}\right.}{en+ibc\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x]^n,x]
```

```
[Out] (I*(1 - E^((-2*I)*(d + e*x)))^n*F^(c*(a + b*x))*Csc[d + e*x]^n*Hypergeometr
ic2F1[n, (e*n + I*b*c*Log[F])/(2*e), (2 + n + (I*b*c*Log[F])/e)/2, E^((-2*I
)*(d + e*x))])/(e*n + I*b*c*Log[F])
```

Maple [F]

$$\int F^{c(xb+a)} \csc \left(ex+d\right)^{n} dx$$

```
[In] int(F^(c*(b*x+a))*csc(e*x+d)^n,x)
[Out] int(F^(c*(b*x+a))*csc(e*x+d)^n,x)
```

Fricas [F]

$$\int F^{c(a+bx)} \csc^n(d+ex)\,dx = \int F^{(bx+a)c} \csc\left(ex+d\right)^n\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^n,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*csc(e*x + d)^n, x)
```

Sympy [F]

$$\int F^{c(a+bx)} \csc^n(d+ex)\,dx = \int F^{c(a+bx)} \csc^n\left(d+ex\right) dx$$

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d)**n,x)
[Out] Integral(F**(c*(a + b*x))*csc(d + e*x)**n, x)
```

Maxima [F]

$$\int F^{c(a+bx)} \csc^n(d+ex)\,dx = \int F^{(bx+a)c} \csc\left(ex+d\right)^n\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^n,x, algorithm="maxima")
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^n, x)
```

Giac [F]

$$\int F^{c(a+bx)}\csc^n(d+ex)\,dx=\int F^{(bx+a)c}\csc\left(ex+d\right)^n\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^n,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\csc^n(d+ex)\,dx=\int F^{c\,(a+b\,x)}\left(\frac{1}{\sin\left(d+e\,x\right)}\right)^n\,dx$$

```
[In] int(F^(c*(a + b*x))*(1/sin(d + e*x))^n,x)
[Out] int(F^(c*(a + b*x))*(1/sin(d + e*x))^n, x)
```

3.28 $\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 139

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = -\frac{e^{-id}F^{ac}(fx)^m\Gamma(1+m,x(ie-bc\log(F)))(x(ie-bc\log(F)))^{-m}}{2(e+ibc\log(F))} - \frac{e^{id}F^{ac}(fx)^m\Gamma(1+m,-x(ie+bc\log(F)))(-x(ie+bc\log(F)))^{-m}}{2(e-ibc\log(F))}$$

```
[Out] -1/2*F^(a*c)*(f*x)^m*GAMMA(1+m,x*(I*e-b*c*ln(F)))/exp(I*d)/((x*(I*e-b*c*ln(
F)))^m)/(e+I*b*c*ln(F))-1/2*exp(I*d)*F^(a*c)*(f*x)^m*GAMMA(1+m,-x*(b*c*ln(F
)+I*e))/(e-I*b*c*ln(F))/((-x*(b*c*ln(F)+I*e))^m)
```

Rubi [F]

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx$$

```
[In] Int[F^(c*(a + b*x))*(f*x)^m*Sin[d + e*x],x]
[Out] Defer[Int][F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x], x]
```

Rubi steps

$$\text{integral} = \int F^{ac+bcx}(fx)^m \sin(d+ex)\,dx$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \frac{1}{2}F^{ac}(fx)^m(x(-ie-bc\log(F)))^{-m}\left(-ix\Gamma(1+m,iex-bcx\log(F))(ix(e+ibc\log(F)))^{-1-m}(-iex-bcx\log(F))^m(\cos(d)-i\sin(d))-\frac{\Gamma(1+m,-iex-bcx\log(F))(\cos(d)+i\sin(d))}{e-ibc\log(F)}\right)$$

```
[In] Integrate[F^(c*(a + b*x))*(f*x)^m*Sin[d + e*x],x]
```

```
[Out] (F^(a*c)*(f*x)^m*((-I)*x*Gamma[1 + m, I*e*x - b*c*x*Log[F]]*(I*x*(e + I*b*c
*Log[F]))^(-1 - m)*((-I)*e*x - b*c*x*Log[F])^m*(Cos[d] - I*Sin[d]) - (Gamma
[1 + m, (-I)*e*x - b*c*x*Log[F]]*(Cos[d] + I*Sin[d]))/(e - I*b*c*Log[F])))/
(2*(x*((-I)*e - b*c*Log[F]))^m)
```

Maple [F]

$$\int F^{c(xb+a)}(fx)^m \sin(ex+d)\,dx$$

```
[In] int(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \frac{(i\,bc\log(F)-e)e^{\left(ac\log(F)-m\log\left(-\frac{bc\log(F)-i\,e}{f}\right)-i\,d\right)}\Gamma(m+1,-bcx\log(F)+i\,ex)+(-i\,bc\log(F)-e)e^{\left(ac\log(F\right.}}{2\left(b^2c^2\log(F)^2+e^2\right)}$$

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*((I*b*c*log(F) - e)*e^(a*c*log(F) - m*log(-(b*c*log(F) - I*e)/f) - I*d)
*gamma(m + 1, -b*c*x*log(F) + I*e*x) + (-I*b*c*log(F) - e)*e^(a*c*log(F) -
m*log(-(b*c*log(F) + I*e)/f) + I*d)*gamma(m + 1, -b*c*x*log(F) - I*e*x))/(b
^2*c^2*log(F)^2 + e^2)
```

Sympy [F]

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx$$

```
[In] integrate(F**(c*(b*x+a))*(f*x)**m*sin(e*x+d),x)
[Out] Integral(F**(c*(a + b*x))*(f*x)**m*sin(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \int (fx)^m F^{(bx+a)c} \sin(ex+d)\,dx$$

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x, algorithm="maxima")
[Out] integrate((f*x)^m*F^((b*x + a)*c)*sin(e*x + d), x)
```

Giac [F]

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \int (fx)^m F^{(bx+a)c} \sin(ex+d)\,dx$$

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x, algorithm="giac")
[Out] integrate((f*x)^m*F^((b*x + a)*c)*sin(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \int F^{c\,(a+b\,x)} \sin(d+e\,x)\,(f\,x)^m\,dx$$

```
[In] int(F^(c*(a + b*x))*sin(d + e*x)*(f*x)^m,x)
[Out] int(F^(c*(a + b*x))*sin(d + e*x)*(f*x)^m, x)
```

3.29 $\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \operatorname{Int}\left(F^{ac+bcx}(fx)^m \csc(d+ex),x\right)$$

```
[Out] CannotIntegrate(F^(b*c*x+a*c)*(f*x)^m*csc(e*x+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx$$

```
[In] Int[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x],x]
```

```
[Out] Defer[Int][F^(a*c + b*c*x)*(f*x)^m*Csc[d + e*x], x]
```

Rubi steps

$$\text{integral} = \int F^{ac+bcx}(fx)^m \csc(d+ex)\,dx$$

Mathematica [N/A]

Not integrable

Time = 16.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx$$

```
[In] Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x],x]
[Out] Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{c(xb+a)}(fx)^m}{\sin(ex+d)}dx$$

```
[In] int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x)
[Out] int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)}\,dx$$

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x, algorithm="fricas")
[Out] integral((f*x)^m*F^(b*c*x + a*c)/sin(e*x + d), x)
```

Sympy [N/A]

Not integrable

Time = 2.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int \frac{F^{c(a+bx)}(fx)^m}{\sin(d+ex)}\,dx$$

```
[In] integrate(F**(c*(b*x+a))*(f*x)**m/sin(e*x+d),x)
[Out] Integral(F**(c*(a + b*x))*(f*x)**m/sin(d + e*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)}\,dx$$

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x, algorithm="maxima")
[Out] integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)}\,dx$$

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x, algorithm="giac")
[Out] integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d), x)
```

Mupad [N/A]

Not integrable

Time = 27.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}\,(f\,x)^m}{\sin(d+e\,x)}\,dx$$

```
[In] int((F^(c*(a + b*x))*(f*x)^m)/sin(d + e*x),x)
[Out] int((F^(c*(a + b*x))*(f*x)^m)/sin(d + e*x), x)
```

3.30 $\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \operatorname{Int}\left(F^{ac+bcx}(fx)^m \csc^2(d+ex),x\right)$$

```
[Out] CannotIntegrate(F^(b*c*x+a*c)*(f*x)^m*csc(e*x+d)^2,x)
```

Rubi [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx$$

```
[In] Int[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x]^2,x]
```

```
[Out] Defer[Int][F^(a*c + b*c*x)*(f*x)^m*Csc[d + e*x]^2, x]
```

Rubi steps

$$\text{integral} = \int F^{ac+bcx}(fx)^m \csc^2(d+ex)\,dx$$

Mathematica [N/A]

Not integrable

Time = 14.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx$$

```
[In] Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x]^2,x]
[Out] Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(xb+a)}(fx)^m}{\sin(ex+d)^2}dx$$

```
[In] int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x)
[Out] int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)^2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x, algorithm="fricas")
[Out] integral(-(f*x)^m*F^(b*c*x + a*c)/(cos(e*x + d)^2 - 1), x)
```

Sympy [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int \frac{F^{c(a+bx)}(fx)^m}{\sin^2(d+ex)}\,dx$$

```
[In] integrate(F**(c*(b*x+a))*(f*x)**m/sin(e*x+d)**2,x)
[Out] Integral(F**(c*(a + b*x))*(f*x)**m/sin(d + e*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)^2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)^2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x, algorithm="giac")
[Out] integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 28.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}\,(f\,x)^m}{\sin(d+e\,x)^2}\,dx$$

```
[In] int((F^(c*(a + b*x))*(f*x)^m)/sin(d + e*x)^2,x)
[Out] int((F^(c*(a + b*x))*(f*x)^m)/sin(d + e*x)^2, x)
```

3.31 $\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx$

Optimal result

Integrand size = 44, antiderivative size = 24

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx = F^{ac+bcx}(fx)^{-1+m}\sin(d+ex)$$

```
[Out] F^(b*c*x+a*c)*(f*x)^(-1+m)*sin(e*x+d)
```

Rubi [A] (verified)

Time = 4.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {12, 6873, 6874, 4556, 4555}

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx = (fx)^{m-1}\sin(d+ex)F^{ac+bcx}$$

```
[In] Int[f*F^(c*(a + b*x))*(f*x)^(-2 + m)*(e*x*Cos[d + e*x] + (-1 + m + b*c*x*Lo
g[F])*Sin[d + e*x]),x]
```

```
[Out] F^(a*c + b*c*x)*(f*x)^(-1 + m)*Sin[d + e*x]
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4555

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(
x_)], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*F^(c*(a + b*x))*Sin[d +
 e*x], x] + (-Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d +
 e*x], x], x] - Dist[b*c*(Log[F]/(f*(m + 1))), Int[(f*x)^(m + 1)*F^(c*(a +
b*x))*Sin[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m
, -1] || SumSimplerQ[m, 1])
```

Rule 4556

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^
(m_), x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*F^(c*(a + b*x))*Cos[d +
 e*x], x] + (Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Sin[d +
e*x], x], x] - Dist[b*c*(Log[F]/(f*(m + 1))), Int[(f*x)^(m + 1)*F^(c*(a + b
*x))*Cos[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m,
 -1] || SumSimplerQ[m, 1])
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= f\int F^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx \\
&= f\int F^{ac+bcx}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx \\
&= f\int \left(\frac{eF^{ac+bcx}(fx)^{-1+m}\cos(d+ex)}{f} \right.\\
&\qquad \left. +F^{ac+bcx}(fx)^{-2+m}(-1+m+bcx\log(F))\sin(d+ex)\right)\,dx \\
&= e\int F^{ac+bcx}(fx)^{-1+m}\cos(d+ex)\,dx \\
&\quad +f\int F^{ac+bcx}(fx)^{-2+m}(-1+m+bcx\log(F))\sin(d+ex)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{eF^{ac+bcx}(fx)^m\cos(d+ex)}{fm} + f\int \left(-F^{ac+bcx}(1-m)(fx)^{-2+m}\sin(d+ex) \right. \\
&\qquad \left. + \frac{bcF^{ac+bcx}(fx)^{-1+m}\log(F)\sin(d+ex)}{f}\right)dx \\
&\quad + \frac{e^2\int F^{ac+bcx}(fx)^m\sin(d+ex)\,dx}{fm} - \frac{(bce\log(F))\int F^{ac+bcx}(fx)^m\cos(d+ex)\,dx}{fm} \\
&= \frac{eF^{ac+bcx}(fx)^m\cos(d+ex)}{fm} - (f(1-m))\int F^{ac+bcx}(fx)^{-2+m}\sin(d+ex)\,dx \\
&\quad + \frac{e^2\int F^{ac+bcx}(fx)^m\sin(d+ex)\,dx}{fm} + (bc\log(F))\int F^{ac+bcx}(fx)^{-1+m}\sin(d+ex)\,dx \\
&\quad - \frac{(bce\log(F))\int F^{ac+bcx}(fx)^m\cos(d+ex)\,dx}{fm} \\
&= \frac{eF^{ac+bcx}(fx)^m\cos(d+ex)}{fm} + F^{ac+bcx}(fx)^{-1+m}\sin(d+ex) \\
&\quad + \frac{bcF^{ac+bcx}(fx)^m\log(F)\sin(d+ex)}{fm} - e\int F^{ac+bcx}(fx)^{-1+m}\cos(d+ex)\,dx \\
&\quad + \frac{e^2\int F^{ac+bcx}(fx)^m\sin(d+ex)\,dx}{fm} - (bc\log(F))\int F^{ac+bcx}(fx)^{-1+m}\sin(d+ex)\,dx \\
&\quad - 2\frac{(bce\log(F))\int F^{ac+bcx}(fx)^m\cos(d+ex)\,dx}{fm} \\
&\quad - \frac{(b^2c^2\log^2(F))\int F^{ac+bcx}(fx)^m\sin(d+ex)\,dx}{fm} \\
&= F^{ac+bcx}(fx)^{-1+m}\sin(d+ex)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$
\begin{aligned}
&\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex) + (-1+m+bcx\log(F))\sin(d+ex))\,dx \\
&= fF^{ac+bcx}x(fx)^{-2+m}\sin(d+ex)
\end{aligned}
$$

```
[In] Integrate[f*F^(c*(a + b*x))*(f*x)^(-2 + m)*(e*x*Cos[d + e*x] + (-1 + m + b*
c*x*Log[F])*Sin[d + e*x]),x]
```

```
[Out] f*F^(a*c + b*c*x)*x*(f*x)^(-2 + m)*Sin[d + e*x]
```

Maple [A] (verified)

Time = 5.93 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

method	result	size
parallelrisch	$\frac{F^{c(xb+a)}(fx)^{m}\sin(ex+d)}{fx}$	28
risch	$\frac{F^{c(xb+a)}\sin(ex+d)x^{m}f^{m}\mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(ifx)m(\mathrm{csgn}(ifx)-\mathrm{csgn}(ix))(-\mathrm{csgn}(ifx)+\mathrm{csgn}(if))}{2}}}{xf}$	69

```
[In] int(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*ln(F))*sin(e*x
+d)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*F^(c*(b*x+a))/x*(f*x)^m*sin(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx$$
$$=(fx)^{m-2}F^{bcx+ac}fx\sin(ex+d)$$

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*log(F))*
sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] (f*x)^(m - 2)*F^(b*c*x + a*c)*f*x*sin(e*x + d)
```

Sympy [F]

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx$$
$$=f\left(\int\left(-F^{ac+bcx}(fx)^{m-2}\sin(d+ex)\right)dx+\int F^{ac+bcx}m(fx)^{m-2}\sin(d+ex)\,dx\right.$$
$$\left.+\int F^{ac+bcx}ex(fx)^{m-2}\cos(d+ex)\,dx+\int F^{ac+bcx}bcx(fx)^{m-2}\log(F)\sin(d+ex)\,dx\right)$$

```
[In] integrate(f*F**(c*(b*x+a))*(f*x)**(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*ln(F))
*sin(e*x+d)),x)
```

```
[Out] f*(Integral(-F**(a*c + b*c*x)*(f*x)**(m - 2)*sin(d + e*x), x) + Integral(F*
*(a*c + b*c*x)*m*(f*x)**(m - 2)*sin(d + e*x), x) + Integral(F**(a*c + b*c*x
)*e*x*(f*x)**(m - 2)*cos(d + e*x), x) + Integral(F**(a*c + b*c*x)*b*c*x*(f*
x)**(m - 2)*log(F)*sin(d + e*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx = \frac{F^{ac}f^{m-1}e^{(bcx\log(F)+m\log(x))}\sin(ex+d)}{x}$$

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*log(F))*
sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] F^(a*c)*f^(m - 1)*e^(b*c*x*log(F) + m*log(x))*sin(e*x + d)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6346 vs. 2(24) = 48.

Time = 0.61 (sec) , antiderivative size = 6346, normalized size of antiderivative = 264.42

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx = \text{Too large to display}$$

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*log(F))*
sin(e*x+d)),x, algorithm="giac")
```

```
[Out] (x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*
abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x -
2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2
*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(
x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*flo
or(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4
*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c -
 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*lo
g(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*
sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1
/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*
sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x -
 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^
2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*p
i*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x
```

```
)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*fl
oor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2
*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f)
- 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4
*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m -
1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi
*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F
) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs
(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x
+ pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn
(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*p
i*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*flo
or(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*
pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) -
 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*
a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) +
m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*
pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4
*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1
) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x
 + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sg
n(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*
pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) -
x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*a
bs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1
/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2
*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*
tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x
) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floo
r(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi
*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) +
 m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4
*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/
4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x
+ pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn
(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*p
i*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 +
 x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*
abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x -
2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2
*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(
x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*flo
or(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4
```

*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F))
+ m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/
4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1
/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c +
1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*abs(F)^(a*c)*e^(b
*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*
b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/
4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(
f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(
F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*
abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs
(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4
*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*p
i*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*ta
n(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x)
+ 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(
-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*a
*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m
*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*p
i*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*
pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1)
- 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x +
pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x
) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*
sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x
*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*ab
s(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/
4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*
pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*t
an(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a
*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x))
- 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor
(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi
*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1
/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*
sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*lo
g(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b
*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*
m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) -
1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^
2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*l
og(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*
sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m
*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1

/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1/4
*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)
^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*
tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x
) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floo
r(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*
pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) +
 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*s
gn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x)) + x*abs(F)^(a*c)*e
^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*
pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) +
 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*s
gn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*b*c*x*s
gn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*
sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/
4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2 - x*abs(F)^(a*c)*e^(b*c*x*
log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x
*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*
m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) -
1/4*pi*a*c + 1/2*d) + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*ab
s(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m
*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) -
1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(
f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) - x*abs(F
)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))
*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(
x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*flo
or(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*p
i*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F))
+ m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/
4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1
/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) +
 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1
/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*lo
g(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*
sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1
/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*
sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*abs(F)^(a*c)*e^(b
*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*
b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/
4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(
f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(
F) - 1/4*pi*a*c - 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(
f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2

*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*abs(F)^(a*c)*e^(b*c*x
*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*
x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi
*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1
/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*a
bs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*
m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) -
1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn
(f) - 1/2*pi*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(
F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x))
)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*p
i*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x
)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*fl
oor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2
*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f)
- 1/2*pi*sgn(x)) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x
)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*fl
oor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2
*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f)
- 1/2*pi*sgn(x)) + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x
)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) - x*
abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs
(x)))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d))*f/(tan(1/4*pi*b*c*x*sgn(
F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn
(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*s
gn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4
*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/
4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c +
1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*x*s
gn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*
sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/
4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) -
1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) +
1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x)
+ 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c
+ 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(
f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e
*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(
x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4
*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*p
i*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*ta
n(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4
*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/

```
4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c +
1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*x*s
gn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*
sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/
4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/
4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4
*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1)
+ 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*
sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*
x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi
*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) -
 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2 + tan(1/4*pi*b*c*x*sgn(
F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn
(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*s
gn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*p
i*a*c + 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4
*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m -
1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi
*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + tan(1/4*pi*b*c*x
*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*
m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2*pi*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) -
1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor
(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi
*m - 1/2*e*x - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1
/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*
a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2
*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1
/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x - 2
*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2
+ tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn
(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x - 2*pi*fl
oor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2 + tan(
1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + tan(1/4*pi*a*c*sgn(F) - 1/4*pi*
a*c - 1/2*d)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 29.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx$$
$$=\frac{F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)\,\left(f\,x\right)^m}{f\,x}$$

```
[In] int(F^(c*(a + b*x))*f*(f*x)^(m - 2)*(sin(d + e*x)*(m + b*c*x*log(F) - 1) +
e*x*cos(d + e*x)),x)
```

```
[Out] (F^(c*(a + b*x))*sin(d + e*x)*(f*x)^m)/(f*x)
```

3.32 $\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$

Optimal result

Integrand size = 42, antiderivative size = 23

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx = fF^{c(a+bx)}x(fx)^m\sin(d+ex)$$

```
[Out] f*F^(c*(b*x+a))*x*(f*x)^m*sin(e*x+d)
```

Rubi [F]

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx = \int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$$

```
[In] Int[f*F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (1 + m + b*c*x*Log[F])*Si
n[d + e*x]),x]
```

```
[Out] e*Defer[Int][F^(a*c + b*c*x)*(f*x)^(1 + m)*Cos[d + e*x], x] + f*(1 + m)*Def
er[Int][F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x], x] + b*c*Log[F]*Defer[Int][F^
(a*c + b*c*x)*(f*x)^(1 + m)*Sin[d + e*x], x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= f\int F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx\\
&= f\int F^{ac+bcx}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx\\
&= f\int\left(\frac{eF^{ac+bcx}(fx)^{1+m}\cos(d+ex)}{f}+F^{ac+bcx}(fx)^m(1+m+bcx\log(F))\sin(d+ex)\right)dx\\
&= e\int F^{ac+bcx}(fx)^{1+m}\cos(d+ex)\,dx+f\int F^{ac+bcx}(fx)^m(1+m+bcx\log(F))\sin(d\\
&\quad+ex)\,dx\\
&= e\int F^{ac+bcx}(fx)^{1+m}\cos(d+ex)\,dx+f\int\Bigg(F^{ac+bcx}(1+m)(fx)^m\sin(d+ex)\\
&\quad+\frac{bcF^{ac+bcx}(fx)^{1+m}\log(F)\sin(d+ex)}{f}\Bigg)dx\\
&= e\int F^{ac+bcx}(fx)^{1+m}\cos(d+ex)\,dx+(f(1+m))\int F^{ac+bcx}(fx)^m\sin(d+ex)\,dx\\
&\quad+(bc\log(F))\int F^{ac+bcx}(fx)^{1+m}\sin(d+ex)\,dx
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$
\begin{aligned}
&\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx\\
&= fF^{ac+bcx}x(fx)^m\sin(d+ex)
\end{aligned}
$$

```
[In] Integrate[f*F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (1 + m + b*c*x*Log[
F])*Sin[d + e*x]),x]
```

```
[Out] f*F^(a*c + b*c*x)*x*(f*x)^m*Sin[d + e*x]
```

Maple [A] (verified)

Time = 5.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result
parallelrisch	$f\,F^{c(xb+a)}x(fx)^{m}\sin\left(ex+d\right)$
risch	$-\frac{i x^{m} f^{m} F^{c(xb+a)} x f \left(\mathrm{e}^{iex}\mathrm{e}^{id}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}(ifx)^{3}m}{2}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ifx)^{2}\,\mathrm{csgn}(if)m}{2}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ifx)^{2}\,\mathrm{csgn}(ix)m}{2}}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}(ifx)\,\mathrm{csgn}(if)\,\mathrm{csgn}(ix)m}{2}}\right.}{2}$

[In] int(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*ln(F))*sin(e*x+d)),x ,method=_RETURNVERBOSE)

[Out] f*F^(c*(b*x+a))*x*(f*x)^m*sin(e*x+d)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx = (fx)^m\,F^{bcx+ac}fx\sin\left(ex+d\right)$$

[In] integrate(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*log(F))*sin(e*x+d)),x, algorithm="fricas")

[Out] (f*x)^m*F^(b*c*x + a*c)*f*x*sin(e*x + d)

Sympy [F]

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx = f\left(\int F^{ac+bcx}(fx)^m\sin\left(d+ex\right)dx+\int F^{ac+bcx}m(fx)^m\sin\left(d+ex\right)dx + \int F^{ac+bcx}ex(fx)^m\cos\left(d+ex\right)dx+\int F^{ac+bcx}bcx(fx)^m\log\left(F\right)\sin\left(d+ex\right)dx\right)$$

[In] integrate(f*F**(c*(b*x+a))*(f*x)**m*(e*x*cos(e*x+d)+(1+m+b*c*x*ln(F))*sin(e*x+d)),x)

[Out] f*(Integral(F**(a*c + b*c*x)*(f*x)**m*sin(d + e*x), x) + Integral(F**(a*c + b*c*x)*m*(f*x)**m*sin(d + e*x), x) + Integral(F**(a*c + b*c*x)*e*x*(f*x)**m*cos(d + e*x), x) + Integral(F**(a*c + b*c*x)*b*c*x*(f*x)**m*log(F)*sin(d + e*x), x))

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx = F^{ac}f^{m+1}xe^{(bcx\log(F)+m\log(x))}\sin\left(ex+d\right)$$

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*log(F))*sin(e*
x+d)),x, algorithm="maxima")
```

```
[Out] F^(a*c)*f^(m + 1)*x*e^(b*c*x*log(F) + m*log(x))*sin(e*x + d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4746 vs. 2(23) = 46.

Time = 0.53 (sec) , antiderivative size = 4746, normalized size of antiderivative = 206.35

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx = \text{Too large to display}$$

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*log(F))*sin(e*
x+d)),x, algorithm="giac")
```

```
[Out] (x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c
*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*p
i*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F
) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(
f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi
*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) - x*abs(F)^(a*c
)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/
4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1
/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c
*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*
sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*
tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log
(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + p
i*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x)
 - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floo
r(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*p
i*m - 1/2*e*x)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c
*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*l
og(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4
```

*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2 *tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan (1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1

/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*
tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*
a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x))
)*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn
(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/
4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c -
1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(
1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)*tan(1/4*pi*a*c
*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)
^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi
*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1
/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)*tan(1/4*pi*a*c*sgn(F
) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x
*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x
*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*
m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F)
- 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f)
+ 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x) + x*abs(F)^(a*c)*e^(b*c*x*log(abs(
F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*f
loor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/
2*pi*m + 1/2*e*x)*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*
sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1
/2*e*x)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan
(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*
a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*
log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/
4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m -
1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) - x*abs(F)^(a*c)*e^
(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi
*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*p
i*m*sgn(x) - 1/2*pi*m + 1/2*e*x)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d
)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*p
i*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) +
1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)*tan(1/4*pi*a*c*sgn(
F) - 1/4*pi*a*c + 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(ab
s(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(
f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e
*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*abs(F)^(a*c)*e^(b*c*x
*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x
+ pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sg
n(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) -
x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*a*c*
sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)

+ x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*
c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*
pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)*tan(1/4*pi*a*c*sgn(F) -
1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)
*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)*
tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log
(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d
)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*l
og(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x +
pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(
x) - 1/2*pi*m + 1/2*e*x) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(
f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f)
- 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x
) + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*
a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*
log(abs(f)*abs(x)))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d))*f/(tan(1/4
*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1)
+ 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*
x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi
*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) -
1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(
1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b
*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4
*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F
) - 1/4*pi*a*c + 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*f
loor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/
2*pi*m + 1/2*e*x)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/
4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m -
1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*
x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi
*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) -
1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(
1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a
*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*
d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/
4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*t
an(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x)
+ 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*e*x)^2 + tan(1/4
*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1)
+ 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi*a*c*
sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + p
i*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x)

```
 - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + ta
n(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x)
+ 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*e*x)^2*tan(1/4*pi
*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*
x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*s
gn(x) - 1/2*pi*m - 1/2*e*x)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2
 + tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/
4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-
1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m
 + 1/2*e*x)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sg
n(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2
*e*x)^2 + tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + tan(1/4*pi*a*c*sg
n(F) - 1/4*pi*a*c - 1/2*d)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 28.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$$
$$=F^{c\,(a+b\,x)}\,f\,x\,\sin\left(d+e\,x\right)\,\left(f\,x\right)^m$$

```
[In] int(F^(c*(a + b*x))*f*(f*x)^m*(sin(d + e*x)*(m + b*c*x*log(F) + 1) + e*x*co
s(d + e*x)),x)
```

```
[Out] F^(c*(a + b*x))*f*x*sin(d + e*x)*(f*x)^m
```

3.33 $\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 22

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = F^{ac+bcx}(fx)^m\sin(d+ex)$$

```
[Out] F^(b*c*x+a*c)*(f*x)^m*sin(e*x+d)
```

Rubi [A] (verified)

Time = 2.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {16, 6873, 6874, 4555}

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = (fx)^m\sin(d+ex)F^{ac+bcx}$$

```
[In] Int[(F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (m + b*c*x*Log[F])*Sin[d + e*x]))/x,x]
```

```
[Out] F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 4555

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(
x_)], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*F^(c*(a + b*x))*Sin[d +
 e*x], x] + (-Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d +
 e*x], x], x] - Dist[b*c*(Log[F]/(f*(m + 1))), Int[(f*x)^(m + 1)*F^(c*(a +
b*x))*Sin[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m
, -1] || SumSimplerQ[m, 1])
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= f\int F^{c(a+bx)}(fx)^{-1+m}(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))\,dx\\
&= f\int F^{ac+bcx}(fx)^{-1+m}(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))\,dx\\
&= f\int\left(\frac{eF^{ac+bcx}(fx)^m\cos(d+ex)}{f}+F^{ac+bcx}(fx)^{-1+m}(m+bcx\log(F))\sin(d+ex)\right)dx\\
&= e\int F^{ac+bcx}(fx)^m\cos(d+ex)\,dx+f\int F^{ac+bcx}(fx)^{-1+m}(m+bcx\log(F))\sin(d+ex)\,dx\\
&= e\int F^{ac+bcx}(fx)^m\cos(d+ex)\,dx\\
&\quad+f\int\left(F^{ac+bcx}m(fx)^{-1+m}\sin(d+ex)+\frac{bcF^{ac+bcx}(fx)^m\log(F)\sin(d+ex)}{f}\right)dx\\
&= e\int F^{ac+bcx}(fx)^m\cos(d+ex)\,dx+(fm)\int F^{ac+bcx}(fx)^{-1+m}\sin(d+ex)\,dx\\
&\quad+(bc\log(F))\int F^{ac+bcx}(fx)^m\sin(d+ex)\,dx\\
&= F^{ac+bcx}(fx)^m\sin(d+ex)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx$$
$$=F^{ac+bcx}(fx)^m\sin(d+ex)$$

```
[In] Integrate[(F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (m + b*c*x*Log[F])*S
in[d + e*x]))/x,x]
```

```
[Out] F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x]
```

Maple [A] (verified)

Time = 5.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

method	result
parallelrisch	$F^{c(xb+a)}(fx)^m\sin(ex+d)$
risch	$-\frac{ix^m f^m F^{c(xb+a)}\left(\mathrm{e}^{iex}\mathrm{e}^{id}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}(ifx)^3 m}{2}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ifx)^2\,\mathrm{csgn}(if)m}{2}}\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}(ifx)^2\,\mathrm{csgn}(ix)m}{2}}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}(ifx)\,\mathrm{csgn}(if)\,\mathrm{csgn}(ix)m}{2}} -}{2}$

```
[In] int(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*ln(F))*sin(e*x+d))/x,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] F^(c*(b*x+a))*(f*x)^m*sin(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx$$
$$=(fx)^m F^{bcx+ac}\sin(ex+d)$$

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*log(F))*sin(e*x+d)
)/x,x, algorithm="fricas")
```

```
[Out] (f*x)^m*F^(b*c*x + a*c)*sin(e*x + d)
```

Sympy [F]

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = \int \frac{F^{c(a+bx)}(fx)^m\,(bcx\log(F)\sin(d+ex)+ex\cos(d+ex)+m\sin(d+ex))}{x}\,dx$$

```
[In] integrate(F**(c*(b*x+a))*(f*x)**m*(e*x*cos(e*x+d)+(m+b*c*x*ln(F))*sin(e*x+d
))/x,x)
```

```
[Out] Integral(F**(c*(a + b*x))*(f*x)**m*(b*c*x*log(F)*sin(d + e*x) + e*x*cos(d +
 e*x) + m*sin(d + e*x))/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = F^{ac}f^m e^{(bcx\log(F)+m\log(x))}\sin(ex+d)$$

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*log(F))*sin(e*x+d)
)/x,x, algorithm="maxima")
```

```
[Out] F^(a*c)*f^m*e^(b*c*x*log(F) + m*log(x))*sin(e*x + d)
```

Giac [F]

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = \int \frac{(ex\cos(ex+d)+(bcx\log(F)+m)\sin(ex+d))(fx)^m\,F^{(bx+a)c}}{x}\,dx$$

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*log(F))*sin(e*x+d)
)/x,x, algorithm="giac")
```

```
[Out] integrate((e*x*cos(e*x + d) + (b*c*x*log(F) + m)*sin(e*x + d))*(f*x)^m*F^((
b*x + a)*c)/x, x)
```

Mupad [B] (verification not implemented)

Time = 27.98 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx$$
$$=F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)\,\left(f\,x\right)^m$$

```
[In] int((F^(c*(a + b*x))*(f*x)^m*(sin(d + e*x)*(m + b*c*x*log(F)) + e*x*cos(d +
 e*x)))/x,x)
```

```
[Out] F^(c*(a + b*x))*sin(d + e*x)*(f*x)^m
```

3.34 $\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 17

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx=F^{c(a+bx)}x\sin(d+ex)$$

```
[Out] F^(c*(b*x+a))*x*sin(e*x+d)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 327 vs. 2(17) = 34.

Time = 1.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 19.24, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {6873, 6874, 4518, 4554, 4517, 4553}

$$\begin{aligned}
&\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx\\
&=\frac{b^2c^2x\log^2(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{e^2x\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}\\
&\quad+\frac{bc\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{bce^2\log(F)\sin(d+ex)F^{ac+bcx}}{\left(b^2c^2\log^2(F)+e^2\right)^2}\\
&\quad-\frac{e\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{b^2c^2e\log^2(F)\cos(d+ex)F^{ac+bcx}}{\left(b^2c^2\log^2(F)+e^2\right)^2}\\
&\quad+\frac{e^3\cos(d+ex)F^{ac+bcx}}{\left(b^2c^2\log^2(F)+e^2\right)^2}-\frac{b^3c^3\log^3(F)\sin(d+ex)F^{ac+bcx}}{\left(b^2c^2\log^2(F)+e^2\right)^2}
\end{aligned}$$

```
[In] Int[F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (1 + b*c*x*Log[F])*Sin[d + e*x]),x]
```

```
[Out] (e^3*F^(a*c + b*c*x)*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)^2 + (b^2*c^2*e*
F^(a*c + b*c*x)*Cos[d + e*x]*Log[F]^2)/(e^2 + b^2*c^2*Log[F]^2)^2 - (e*F^(a
*c + b*c*x)*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) - (b*c*e^2*F^(a*c + b*c*
x)*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)^2 - (b^3*c^3*F^(a*c + b*c*
x)*Log[F]^3*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)^2 + (e^2*F^(a*c + b*c*x)
*x*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) + (b*c*F^(a*c + b*c*x)*Log[F]*Sin
[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) + (b^2*c^2*F^(a*c + b*c*x)*x*Log[F]^2*S
in[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int F^{ac+bcx}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx$$

$$= \int \left(eF^{ac+bcx}x\cos(d+ex)+F^{ac+bcx}(1+bcx\log(F))\sin(d+ex)\right)\,dx$$

$$= e\int F^{ac+bcx}x\cos(d+ex)\,dx+\int F^{ac+bcx}(1+bcx\log(F))\sin(d+ex)\,dx$$

$$= \frac{bceF^{ac+bcx}x\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{e^2F^{ac+bcx}x\sin(d+ex)}{e^2+b^2c^2\log^2(F)} - e\int \left(\frac{bcF^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{eF^{ac+bcx}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}\right)\,dx + \int \left(F^{ac+bcx}\sin(d+ex)+bcF^{ac+bcx}x\log(F)\sin(d+ex)\right)\,dx$$

$$= \frac{bceF^{ac+bcx}x\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{e^2F^{ac+bcx}x\sin(d+ex)}{e^2+b^2c^2\log^2(F)} + (bc\log(F))\int F^{ac+bcx}x\sin(d+ex)\,dx-\frac{e^2\int F^{ac+bcx}\sin(d+ex)\,dx}{e^2+b^2c^2\log^2(F)} - \frac{(bce\log(F))\int F^{ac+bcx}\cos(d+ex)\,dx}{e^2+b^2c^2\log^2(F)}+\int F^{ac+bcx}\sin(d+ex)\,dx$$

$$= \frac{e^3F^{ac+bcx}\cos(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2}-\frac{b^2c^2eF^{ac+bcx}\cos(d+ex)\log^2(F)}{\left(e^2+b^2c^2\log^2(F)\right)^2}-\frac{eF^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)} - \frac{2bce^2F^{ac+bcx}\log(F)\sin(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2}+\frac{e^2F^{ac+bcx}x\sin(d+ex)}{e^2+b^2c^2\log^2(F)} + \frac{bcF^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{b^2c^2F^{ac+bcx}x\log^2(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)} - (bc\log(F))\int \left(-\frac{eF^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{bcF^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}\right)\,dx$$

$$= \frac{e^3F^{ac+bcx}\cos(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2}-\frac{b^2c^2eF^{ac+bcx}\cos(d+ex)\log^2(F)}{\left(e^2+b^2c^2\log^2(F)\right)^2}-\frac{eF^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)} - \frac{2bce^2F^{ac+bcx}\log(F)\sin(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2}+\frac{e^2F^{ac+bcx}x\sin(d+ex)}{e^2+b^2c^2\log^2(F)} + \frac{bcF^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{b^2c^2F^{ac+bcx}x\log^2(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)} + \frac{(bce\log(F))\int F^{ac+bcx}\cos(d+ex)\,dx}{e^2+b^2c^2\log^2(F)}-\frac{\left(b^2c^2\log^2(F)\right)\int F^{ac+bcx}\sin(d+ex)\,dx}{e^2+b^2c^2\log^2(F)}$$

$$
\begin{aligned}
&= \frac{e^3 F^{ac+bcx}\cos(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2} + \frac{b^2c^2eF^{ac+bcx}\cos(d+ex)\log^2(F)}{\left(e^2+b^2c^2\log^2(F)\right)^2} \\
&\quad - \frac{eF^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)} - \frac{bce^2F^{ac+bcx}\log(F)\sin(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2} \\
&\quad - \frac{b^3c^3F^{ac+bcx}\log^3(F)\sin(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2} + \frac{e^2F^{ac+bcx}x\sin(d+ex)}{e^2+b^2c^2\log^2(F)} \\
&\quad + \frac{bcF^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)} + \frac{b^2c^2F^{ac+bcx}x\log^2(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$
\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx = F^{ac+bcx}x\sin(d+ex)
$$

```
[In] Integrate[F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (1 + b*c*x*Log[F])*Sin[d + e*
x]),x]
[Out] F^(a*c + b*c*x)*x*Sin[d + e*x]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result
risch	$F^{c(xb+a)}x\sin(ex+d)$
parallelrisch	$F^{c(xb+a)}x\sin(ex+d)$
norman	$\frac{2x\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$
parts	$\frac{\frac{e\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}-\frac{e\,\mathrm{e}^{c(xb+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2bc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}+\frac{bc\ln(F)\left(\frac{ex\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}\right.}{}$

```
[In] int(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*ln(F))*sin(e*x+d)),x,method=_RET
URNVERBOSE)
[Out] F^(c*(b*x+a))*x*sin(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx = F^{bcx+ac}x\sin\left(ex+d\right)$$

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*log(F))*sin(e*x+d)),x, alg
orithm="fricas")
```

```
[Out] F^(b*c*x + a*c)*x*sin(e*x + d)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx = F^{ac+bcx}x\sin\left(d+ex\right)$$

```
[In] integrate(F**(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*ln(F))*sin(e*x+d)),x)
```

```
[Out] F**(a*c + b*c*x)*x*sin(d + e*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. $2(17) = 34$.

Time = 0.33 (sec) , antiderivative size = 1382, normalized size of antiderivative = 81.29

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*log(F))*sin(e*x+d)),x, alg
orithm="maxima")
```

```
[Out] 1/2*((F^(a*c)*b^2*c^2*log(F)^2*sin(d) + 2*F^(a*c)*b*c*e*cos(d)*log(F) - F^(
a*c)*e^2*sin(d) - (F^(a*c)*b^3*c^3*log(F)^3*sin(d) + F^(a*c)*b^2*c^2*e*cos(
d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(d) + F^(a*c)*e^3*cos(d))*x)*F^(b*c
*x)*cos(e*x + 2*d) - (F^(a*c)*b^2*c^2*log(F)^2*sin(d) - 2*F^(a*c)*b*c*e*cos
(d)*log(F) - F^(a*c)*e^2*sin(d) - (F^(a*c)*b^3*c^3*log(F)^3*sin(d) - F^(a*c
)*b^2*c^2*e*cos(d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(d) - F^(a*c)*e^3*c
os(d))*x)*F^(b*c*x)*cos(e*x) - (F^(a*c)*b^2*c^2*cos(d)*log(F)^2 - 2*F^(a*c)
*b*c*e*log(F)*sin(d) - F^(a*c)*e^2*cos(d) - (F^(a*c)*b^3*c^3*cos(d)*log(F)^
3 - F^(a*c)*b^2*c^2*e*log(F)^2*sin(d) + F^(a*c)*b*c*e^2*cos(d)*log(F) - F^(
a*c)*e^3*sin(d))*x)*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b^2*c^2*cos(d)*log(
```

```
F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(d) - F^(a*c)*e^2*cos(d) - (F^(a*c)*b^3*c^
3*cos(d)*log(F)^3 + F^(a*c)*b^2*c^2*e*log(F)^2*sin(d) + F^(a*c)*b*c*e^2*cos
(d)*log(F) + F^(a*c)*e^3*sin(d))*x)*F^(b*c*x)*sin(e*x))*b*c*log(F)/(b^4*c^4
*cos(d)^2*log(F)^4 + b^4*c^4*log(F)^4*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^4
+ 2*(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2)*e^2) - 1/2*((F^
(a*c)*b^2*c^2*cos(d)*log(F)^2 - 2*F^(a*c)*b*c*e*log(F)*sin(d) - F^(a*c)*e^2
*cos(d) - (F^(a*c)*b^3*c^3*cos(d)*log(F)^3 - F^(a*c)*b^2*c^2*e*log(F)^2*sin
(d) + F^(a*c)*b*c*e^2*cos(d)*log(F) - F^(a*c)*e^3*sin(d))*x)*F^(b*c*x)*cos(
e*x + 2*d) + (F^(a*c)*b^2*c^2*cos(d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(
d) - F^(a*c)*e^2*cos(d) - (F^(a*c)*b^3*c^3*cos(d)*log(F)^3 + F^(a*c)*b^2*c^
2*e*log(F)^2*sin(d) + F^(a*c)*b*c*e^2*cos(d)*log(F) + F^(a*c)*e^3*sin(d))*x
)*F^(b*c*x)*cos(e*x) + (F^(a*c)*b^2*c^2*log(F)^2*sin(d) + 2*F^(a*c)*b*c*e*c
os(d)*log(F) - F^(a*c)*e^2*sin(d) - (F^(a*c)*b^3*c^3*log(F)^3*sin(d) + F^(a
*c)*b^2*c^2*e*cos(d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(d) + F^(a*c)*e^3
*cos(d))*x)*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b^2*c^2*log(F)^2*sin(d) - 2
*F^(a*c)*b*c*e*cos(d)*log(F) - F^(a*c)*e^2*sin(d) - (F^(a*c)*b^3*c^3*log(F)
^3*sin(d) - F^(a*c)*b^2*c^2*e*cos(d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(
d) - F^(a*c)*e^3*cos(d))*x)*F^(b*c*x)*sin(e*x))*e/(b^4*c^4*cos(d)^2*log(F)^
4 + b^4*c^4*log(F)^4*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^4 + 2*(b^2*c^2*cos(
d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2)*e^2) - 1/2*((F^(a*c)*b*c*log(F)*
sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2*d) - (F^(a*c)*b*c*log(F)*s
in(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) - (F^(a*c)*b*c*cos(d)*log(F) -
 F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b*c*cos(d)*log(F) +
F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))/(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*
log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 1941, normalized size of antiderivative = 114.18

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*log(F))*sin(e*x+d)),x, alg
orithm="giac")
```

```
[Out] -1/4*((pi*b^2*c^2*x*log(F)*sgn(F) - pi*b^2*c^2*x*log(F) - 2*I*b^2*c^2*x*log
(F)*log(abs(F)) - I*pi*b*c*e*x*sgn(F) + I*pi*b*c*e*x + 2*b*c*e*x*log(F) - 2
*b*c*e*x*log(abs(F)) + pi*b*c*sgn(F) - pi*b*c - 2*I*e^2*x + 2*I*b*c*log(F)
- 2*I*b*c*log(abs(F)) + 4*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/
2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(pi^2*b^2*c^2*sgn(F) + 2*I*
pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) +
 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e + 4*I*b*c*e*log(a
bs(F)) - 2*e^2) - (pi*b^2*c^2*x*log(F)*sgn(F) - pi*b^2*c^2*x*log(F) + 2*I*b
```

^2*c^2*x*log(F)*log(abs(F)) - I*pi*b*c*e*x*sgn(F) + I*pi*b*c*e*x + 2*b*c*e*
x*log(F) + 2*b*c*e*x*log(abs(F)) + pi*b*c*sgn(F) - pi*b*c - 2*I*e^2*x - 2*I
*b*c*log(F) + 2*I*b*c*log(abs(F)))*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c
*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(pi^2*b^2*c^2*sgn(F)
 - 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^2*c^2*log(ab
s(F)) + 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e - 4*I*b*c*
e*log(abs(F)) - 2*e^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*I*((-
I*pi*b^2*c^2*x*log(F)*sgn(F) + I*pi*b^2*c^2*x*log(F) - 2*b^2*c^2*x*log(F)*l
og(abs(F)) - pi*b*c*e*x*sgn(F) + pi*b*c*e*x - 2*I*b*c*e*x*log(F) + 2*I*b*c*
e*x*log(abs(F)) - I*pi*b*c*sgn(F) + I*pi*b*c - 2*e^2*x + 2*b*c*log(F) - 2*b
*c*log(abs(F)) - 4*I*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*p
i*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^
2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^
2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e + 4*I*b*c*e*log(abs(F)
) - 2*e^2) - (I*pi*b^2*c^2*x*log(F)*sgn(F) - I*pi*b^2*c^2*x*log(F) - 2*b^2*
c^2*x*log(F)*log(abs(F)) + pi*b*c*e*x*sgn(F) - pi*b*c*e*x + 2*I*b*c*e*x*log
(F) + 2*I*b*c*e*x*log(abs(F)) + I*pi*b*c*sgn(F) - I*pi*b*c + 2*e^2*x + 2*b*
c*log(F) - 2*b*c*log(abs(F)))*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(pi^2*b^2*c^2*sgn(F) - 2*
I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^2*c^2*log(abs(F))
 + 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e - 4*I*b*c*e*log
(abs(F)) - 2*e^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*((pi*b^2*c
^2*x*log(F)*sgn(F) - pi*b^2*c^2*x*log(F) - 2*I*b^2*c^2*x*log(F)*log(abs(F))
 + I*pi*b*c*e*x*sgn(F) - I*pi*b*c*e*x - 2*b*c*e*x*log(F) + 2*b*c*e*x*log(ab
s(F)) + pi*b*c*sgn(F) - pi*b*c - 2*I*e^2*x + 2*I*b*c*log(F) - 2*I*b*c*log(a
bs(F)) - 4*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(
F) - 1/2*I*pi*a*c - I*e*x - I*d)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(
abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(
abs(F))^2 + 2*pi*b*c*e*sgn(F) - 2*pi*b*c*e - 4*I*b*c*e*log(abs(F)) - 2*e^2)
 - (pi*b^2*c^2*x*log(F)*sgn(F) - pi*b^2*c^2*x*log(F) + 2*I*b^2*c^2*x*log(F)
*log(abs(F)) + I*pi*b*c*e*x*sgn(F) - I*pi*b*c*e*x - 2*b*c*e*x*log(F) - 2*b*
c*e*x*log(abs(F)) + pi*b*c*sgn(F) - pi*b*c - 2*I*e^2*x - 2*I*b*c*log(F) + 2
*I*b*c*log(abs(F)))*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a
*c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(pi^2*b^2*c^2*sgn(F) - 2*I*pi*b^2*c
^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c
^2*log(abs(F))^2 + 2*pi*b*c*e*sgn(F) - 2*pi*b*c*e + 4*I*b*c*e*log(abs(F)) -
 2*e^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*I*((I*pi*b^2*c^2*x*l
og(F)*sgn(F) - I*pi*b^2*c^2*x*log(F) + 2*b^2*c^2*x*log(F)*log(abs(F)) - pi*
b*c*e*x*sgn(F) + pi*b*c*e*x - 2*I*b*c*e*x*log(F) + 2*I*b*c*e*x*log(abs(F))
+ I*pi*b*c*sgn(F) - I*pi*b*c + 2*e^2*x - 2*b*c*log(F) + 2*b*c*log(abs(F)) -
 4*I*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1
/2*I*pi*a*c - I*e*x - I*d)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(abs(F)
)*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F)
)^2 + 2*pi*b*c*e*sgn(F) - 2*pi*b*c*e - 4*I*b*c*e*log(abs(F)) - 2*e^2) - (-I
*pi*b^2*c^2*x*log(F)*sgn(F) + I*pi*b^2*c^2*x*log(F) + 2*b^2*c^2*x*log(F)*lo

```
g(abs(F)) + pi*b*c*e*x*sgn(F) - pi*b*c*e*x + 2*I*b*c*e*x*log(F) + 2*I*b*c*e
*x*log(abs(F)) - I*pi*b*c*sgn(F) + I*pi*b*c - 2*e^2*x - 2*b*c*log(F) + 2*b*
c*log(abs(F)))*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(pi^2*b^2*c^2*sgn(F) - 2*I*pi*b^2*c^2*lo
g(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*lo
g(abs(F))^2 + 2*pi*b*c*e*sgn(F) - 2*pi*b*c*e + 4*I*b*c*e*log(abs(F)) - 2*e^
2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 27.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx=F^{c\,(a+b\,x)}\,x\,\sin\left(d+e\,x\right)$$

```
[In] int(F^(c*(a + b*x))*(sin(d + e*x)*(b*c*x*log(F) + 1) + e*x*cos(d + e*x)),x)
[Out] F^(c*(a + b*x))*x*sin(d + e*x)
```

3.35 $\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 16

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx = F^{c(a+bx)}\sin(d+ex)$$

```
[Out] F^(c*(b*x+a))*sin(e*x+d)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2326}

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx = \sin(d+ex)F^{c(a+bx)}$$

```
[In] Int[F^(c*(a + b*x))*(e*Cos[d + e*x] + b*c*Log[F]*Sin[d + e*x]),x]
[Out] F^(c*(a + b*x))*Sin[d + e*x]
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\text{integral} = F^{c(a+bx)}\sin(d+ex)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx=F^{c(a+bx)}\sin(d+ex)$$

```
[In] Integrate[F^(c*(a + b*x))*(e*Cos[d + e*x] + b*c*Log[F]*Sin[d + e*x]),x]
[Out] F^(c*(a + b*x))*Sin[d + e*x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result
risch	$F^{c(xb+a)}\sin(ex+d)$
parallelrisch	$F^{c(xb+a)}\sin(ex+d)$
norman	$\frac{2\,e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$
parts	$\frac{\frac{ebc\ln(F)e^{c(xb+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2e^2e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}-\frac{ebc\ln(F)e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}+\frac{ebc\ln(F)e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\cdots\right.}{e^2+b^2c^2\ln(F)^2}$

```
[In] int(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*ln(F)*sin(e*x+d)),x,method=_RETURNVERBO
SE)
[Out] F^(c*(b*x+a))*sin(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx=F^{bcx+ac}\sin(ex+d)$$

```
[In] integrate(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*log(F)*sin(e*x+d)),x, algorithm="
fricas")
[Out] F^(b*c*x + a*c)*sin(e*x + d)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx=F^{ac+bcx}\sin{(d+ex)}$$

```
[In] integrate(F**(c*(b*x+a))*(e*cos(e*x+d)+b*c*ln(F)*sin(e*x+d)),x)
```

```
[Out] F**(a*c + b*c*x)*sin(d + e*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 392, normalized size of antiderivative = 24.50

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx=$$
$$-\frac{((F^{ac}bc\log{(F)}\sin{(d)}+F^{ac}e\cos{(d)})F^{bcx}\cos{(ex+2d)}-(F^{ac}bc\log{(F)}\sin{(d)}-F^{ac}e\cos{(d)})F^{bcx}\cos{(}}{2\left(b^2c^2\cos{(d)}^2\log{(F)}^2+\right.}$$
$$+\frac{((F^{ac}bc\cos{(d)}\log{(F)}-F^{ac}e\sin{(d)})F^{bcx}\cos{(ex+2d)}+(F^{ac}bc\cos{(d)}\log{(F)}+F^{ac}e\sin{(d)})F^{bcx}\cos{(}}{2\left(b^2c^2\cos{(d)}^2\log{(F)}^2+b^2c^2\right.}$$

```
[In] integrate(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*log(F)*sin(e*x+d)),x, algorithm="
maxima")
```

```
[Out] -1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2*
d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) - (F
^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^
(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))*b*c*log(F)/
(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)
^2)*e^2) + 1/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*co
s(e*x + 2*d) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos
(e*x) + (F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x +
2*d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x))*e
/(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d
)^2)*e^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 639, normalized size of antiderivative = 39.94

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*log(F)*sin(e*x+d)),x, algorithm="
giac")
```

```
[Out] -I*((b*c*log(F) - I*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi
*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c +
 4*b*c*log(abs(F)) + 4*I*e) - (b*c*log(F) - I*e)*e^(-1/2*I*pi*b*c*x*sgn(F)
+ 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*
pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F
)) + a*c*log(abs(F))) - ((-I*b*c*log(F) - e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2
*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c
*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e) + (-I*b*c*log(F) - e)*e^(
-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*
c - I*e*x - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I
*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*((b*c*log(F) + I*e)*e^(1/2
*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c -
I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e) -
 (b*c*log(F) + I*e)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a
*c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c +
4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - ((I*b
*c*log(F) - e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn
(F) - 1/2*I*pi*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*l
og(abs(F)) - 4*I*e) + (I*b*c*log(F) - e)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*
pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*s
gn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c
*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 26.96 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx = F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)$$

```
[In] int(F^(c*(a + b*x))*(e*cos(d + e*x) + b*c*sin(d + e*x)*log(F)),x)
```

```
[Out] F^(c*(a + b*x))*sin(d + e*x)
```

3.36 $\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 20

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx = \frac{F^{ac+bcx}\sin(d+ex)}{x}$$

```
[Out] F^(b*c*x+a*c)*sin(e*x+d)/x
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {6873, 6874, 4555}

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx = \frac{\sin(d+ex)F^{ac+bcx}}{x}$$

```
[In] Int[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-1 + b*c*x*Log[F])*Sin[d + e*x]))
/x^2,x]
```

```
[Out] (F^(a*c + b*c*x)*Sin[d + e*x])/x
```

Rule 4555

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(
x_)], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*F^(c*(a + b*x))*Sin[d +
 e*x], x] + (-Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d +
 e*x], x], x] - Dist[b*c*(Log[F]/(f*(m + 1))), Int[(f*x)^(m + 1)*F^(c*(a +
b*x))*Sin[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m
, -1] || SumSimplerQ[m, 1])
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{F^{ac+bcx}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx \\
&= \int \left(\frac{eF^{ac+bcx}\cos(d+ex)}{x}+\frac{F^{ac+bcx}(-1+bcx\log(F))\sin(d+ex)}{x^2}\right)dx \\
&= e\int \frac{F^{ac+bcx}\cos(d+ex)}{x}\,dx+\int \frac{F^{ac+bcx}(-1+bcx\log(F))\sin(d+ex)}{x^2}\,dx \\
&= e\int \frac{F^{ac+bcx}\cos(d+ex)}{x}\,dx+\int \left(-\frac{F^{ac+bcx}\sin(d+ex)}{x^2}+\frac{bcF^{ac+bcx}\log(F)\sin(d+ex)}{x}\right)dx \\
&= e\int \frac{F^{ac+bcx}\cos(d+ex)}{x}\,dx+(bc\log(F))\int \frac{F^{ac+bcx}\sin(d+ex)}{x}\,dx-\int \frac{F^{ac+bcx}\sin(d+ex)}{x^2}\,dx \\
&= \frac{F^{ac+bcx}\sin(d+ex)}{x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$
\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx=\frac{F^{ac+bcx}\sin(d+ex)}{x}
$$

```
[In] Integrate[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-1 + b*c*x*Log[F])*Sin[d +
e*x]))/x^2,x]
```

```
[Out] (F^(a*c + b*c*x)*Sin[d + e*x])/x
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{F^{c(xb+a)}\sin(ex+d)}{x}$	20
parallelrisch	$\frac{F^{c(xb+a)}\sin(ex+d)}{x}$	20
norman	$\frac{2\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}\right)x}$	40

```
[In] int(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*ln(F))*sin(e*x+d))/x^2,x,method
=_RETURNVERBOSE)
[Out] 1/x*F^(c*(b*x+a))*sin(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx=\frac{F^{bcx+ac}\sin\left(ex+d\right)}{x}$$

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*log(F))*sin(e*x+d))/x^2,x
, algorithm="fricas")
[Out] F^(b*c*x + a*c)*sin(e*x + d)/x
```

Sympy [F]

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx$$
$$=\int \frac{F^{c(a+bx)}(bcx\log\left(F\right)\sin\left(d+ex\right)+ex\cos\left(d+ex\right)-\sin\left(d+ex\right))}{x^2}\,dx$$

```
[In] integrate(F**(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*ln(F))*sin(e*x+d))/x**2,
x)
[Out] Integral(F**(c*(a + b*x))*(b*c*x*log(F)*sin(d + e*x) + e*x*cos(d + e*x) - s
in(d + e*x))/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 564, normalized size of antiderivative = 28.20

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*log(F))*sin(e*x+d))/x^2,x
, algorithm="maxima")
```

```
[Out] -1/4*F^(a*c)*b*c*(I*conjugate(gamma(-1, -(b*c*log(F) + I*e)*x)) - I*conjuga
te(gamma(-1, -(b*c*log(F) - I*e)*x)) - I*gamma(-1, -(b*c*log(F) + I*e)*x) +
 I*gamma(-1, -(b*c*log(F) - I*e)*x))*cos(d)*log(F) - 1/4*F^(a*c)*b*c*(conju
gate(gamma(-1, -(b*c*log(F) + I*e)*x)) + conjugate(gamma(-1, -(b*c*log(F) -
 I*e)*x)) + gamma(-1, -(b*c*log(F) + I*e)*x) + gamma(-1, -(b*c*log(F) - I*e
)*x))*log(F)*sin(d) - 1/4*(F^(a*c)*(I*Ei((b*c*log(F) + I*e)*x) - I*Ei((b*c*
log(F) - I*e)*x) - I*conjugate(Ei((b*c*log(F) + I*e)*x)) + I*conjugate(Ei((
b*c*log(F) - I*e)*x)))*cos(d) - F^(a*c)*(Ei((b*c*log(F) + I*e)*x) + Ei((b*c
*log(F) - I*e)*x) + conjugate(Ei((b*c*log(F) + I*e)*x)) + conjugate(Ei((b*c
*log(F) - I*e)*x)))*sin(d))*b*c*log(F) + 1/4*(F^(a*c)*(Ei((b*c*log(F) + I*e
)*x) + Ei((b*c*log(F) - I*e)*x) + conjugate(Ei((b*c*log(F) + I*e)*x)) + con
jugate(Ei((b*c*log(F) - I*e)*x)))*cos(d) - F^(a*c)*(-I*Ei((b*c*log(F) + I*e
)*x) + I*Ei((b*c*log(F) - I*e)*x) + I*conjugate(Ei((b*c*log(F) + I*e)*x)) -
 I*conjugate(Ei((b*c*log(F) - I*e)*x)))*sin(d))*e - 1/4*(F^(a*c)*(conjugate
(gamma(-1, -(b*c*log(F) + I*e)*x)) + conjugate(gamma(-1, -(b*c*log(F) - I*e
)*x)) + gamma(-1, -(b*c*log(F) + I*e)*x) + gamma(-1, -(b*c*log(F) - I*e)*x)
)*cos(d) + F^(a*c)*(-I*conjugate(gamma(-1, -(b*c*log(F) + I*e)*x)) + I*conj
ugate(gamma(-1, -(b*c*log(F) - I*e)*x)) + I*gamma(-1, -(b*c*log(F) + I*e)*x
) - I*gamma(-1, -(b*c*log(F) - I*e)*x))*sin(d))*e
```

Giac [F]

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx = \int \frac{(ex\cos(ex+d)+(bcx\log(F)-1)\sin(ex+d))F^{(bx+a)c}}{x^2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*log(F))*sin(e*x+d))/x^2,x
, algorithm="giac")
```

```
[Out] integrate((e*x*cos(e*x + d) + (b*c*x*log(F) - 1)*sin(e*x + d))*F^((b*x + a)
*c)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 27.72 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx=\frac{F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)}{x}$$

```
[In] int((F^(c*(a + b*x))*(sin(d + e*x)*(b*c*x*log(F) - 1) + e*x*cos(d + e*x)))/
x^2,x)
```

```
[Out] (F^(c*(a + b*x))*sin(d + e*x))/x
```

3.37 $\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 20

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx=\frac{F^{ac+bcx}\sin(d+ex)}{x^2}$$

```
[Out] F^(b*c*x+a*c)*sin(e*x+d)/x^2
```

Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6873, 6874, 4556, 4555}

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx=\frac{\sin(d+ex)F^{ac+bcx}}{x^2}$$

```
[In] Int[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-2 + b*c*x*Log[F])*Sin[d + e*x]))
/x^3,x]
```

```
[Out] (F^(a*c + b*c*x)*Sin[d + e*x])/x^2
```

Rule 4555

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(
x_)], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*F^(c*(a + b*x))*Sin[d +
 e*x], x] + (-Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d +
 e*x], x], x] - Dist[b*c*(Log[F]/(f*(m + 1))), Int[(f*x)^(m + 1)*F^(c*(a +
b*x))*Sin[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m
, -1] || SumSimplerQ[m, 1])
```

Rule 4556

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^
(m_), x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*F^(c*(a + b*x))*Cos[d +
 e*x], x] + (Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Sin[d +
e*x], x], x] - Dist[b*c*(Log[F]/(f*(m + 1))), Int[(f*x)^(m + 1)*F^(c*(a + b
*x))*Cos[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m,
 -1] || SumSimplerQ[m, 1])
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{F^{ac+bcx}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx \\
&= \int \left(\frac{eF^{ac+bcx}\cos(d+ex)}{x^2}+\frac{F^{ac+bcx}(-2+bcx\log(F))\sin(d+ex)}{x^3}\right)dx \\
&= e\int \frac{F^{ac+bcx}\cos(d+ex)}{x^2}\,dx+\int \frac{F^{ac+bcx}(-2+bcx\log(F))\sin(d+ex)}{x^3}\,dx \\
&= -\frac{eF^{ac+bcx}\cos(d+ex)}{x}-e^2\int \frac{F^{ac+bcx}\sin(d+ex)}{x}\,dx \\
&\quad +(bce\log(F))\int \frac{F^{ac+bcx}\cos(d+ex)}{x}\,dx \\
&\quad +\int \left(-\frac{2F^{ac+bcx}\sin(d+ex)}{x^3}+\frac{bcF^{ac+bcx}\log(F)\sin(d+ex)}{x^2}\right)dx \\
&= -\frac{eF^{ac+bcx}\cos(d+ex)}{x}-2\int \frac{F^{ac+bcx}\sin(d+ex)}{x^3}\,dx-e^2\int \frac{F^{ac+bcx}\sin(d+ex)}{x}\,dx \\
&\quad +(bc\log(F))\int \frac{F^{ac+bcx}\sin(d+ex)}{x^2}\,dx+(bce\log(F))\int \frac{F^{ac+bcx}\cos(d+ex)}{x}\,dx
\end{aligned}$$

$$= -\frac{eF^{ac+bcx}\cos(d+ex)}{x} + \frac{F^{ac+bcx}\sin(d+ex)}{x^2} - \frac{bcF^{ac+bcx}\log(F)\sin(d+ex)}{x} - e\int \frac{F^{ac+bcx}\cos(d+ex)}{x^2}\,dx - e^2\int \frac{F^{ac+bcx}\sin(d+ex)}{x}\,dx - (bc\log(F))\int \frac{F^{ac+bcx}\sin(d+ex)}{x^2}\,dx + 2\left((bce\log(F))\int \frac{F^{ac+bcx}\cos(d+ex)}{x}\,dx\right) + \left(b^2c^2\log^2(F)\right)\int \frac{F^{ac+bcx}\sin(d+ex)}{x}\,dx$$

$$= \frac{F^{ac+bcx}\sin(d+ex)}{x^2}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex) + (-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx = \frac{F^{ac+bcx}\sin(d+ex)}{x^2}$$

```
[In] Integrate[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-2 + b*c*x*Log[F])*Sin[d + e*x]))/x^3,x]
[Out] (F^(a*c + b*c*x)*Sin[d + e*x])/x^2
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\sin(ex+d)F^{c(xb+a)}}{x^2}$	20
parallelrisch	$\frac{\sin(ex+d)F^{c(xb+a)}}{x^2}$	20
norman	$\frac{2\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)x^2}$	40

```
[In] int(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*ln(F))*sin(e*x+d))/x^3,x,method=_RETURNVERBOSE)
[Out] sin(e*x+d)*F^(c*(b*x+a))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx = \frac{F^{bcx+ac}\sin(ex+d)}{x^2}$$

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*log(F))*sin(e*x+d))/x^3,x
, algorithm="fricas")
```

```
[Out] F^(b*c*x + a*c)*sin(e*x + d)/x^2
```

Sympy [F]

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx = \int \frac{F^{c(a+bx)}(bcx\log(F)\sin(d+ex)+ex\cos(d+ex)-2\sin(d+ex))}{x^3}\,dx$$

```
[In] integrate(F**(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*ln(F))*sin(e*x+d))/x**3,
x)
```

```
[Out] Integral(F**(c*(a + b*x))*(b*c*x*log(F)*sin(d + e*x) + e*x*cos(d + e*x) - 2
*sin(d + e*x))/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.08 (sec) , antiderivative size = 1069, normalized size of antiderivative = 53.45

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*log(F))*sin(e*x+d))/x^3,x
, algorithm="maxima")
```

```
[Out] -1/2*F^(a*c)*b^2*c^2*(-I*conjugate(gamma(-2, -(b*c*log(F) + I*e)*x)) + I*co
njugate(gamma(-2, -(b*c*log(F) - I*e)*x)) + I*gamma(-2, -(b*c*log(F) + I*e)
*x) - I*gamma(-2, -(b*c*log(F) - I*e)*x))*cos(d)*log(F)^2 + 1/2*F^(a*c)*b^2
*c^2*(conjugate(gamma(-2, -(b*c*log(F) + I*e)*x)) + conjugate(gamma(-2, -(b
*c*log(F) - I*e)*x)) + gamma(-2, -(b*c*log(F) + I*e)*x) + gamma(-2, -(b*c*l
og(F) - I*e)*x))*log(F)^2*sin(d) + 1/4*(F^(a*c)*b*c*(I*conjugate(gamma(-1,
```

```
-(b*c*log(F) + I*e)*x)) - I*conjugate(gamma(-1, -(b*c*log(F) - I*e)*x)) - I
*gamma(-1, -(b*c*log(F) + I*e)*x) + I*gamma(-1, -(b*c*log(F) - I*e)*x))*cos
(d)*log(F) + F^(a*c)*b*c*(conjugate(gamma(-1, -(b*c*log(F) + I*e)*x)) + con
jugate(gamma(-1, -(b*c*log(F) - I*e)*x)) + gamma(-1, -(b*c*log(F) + I*e)*x)
 + gamma(-1, -(b*c*log(F) - I*e)*x))*log(F)*sin(d) + (F^(a*c)*(conjugate(ga
mma(-1, -(b*c*log(F) + I*e)*x)) + conjugate(gamma(-1, -(b*c*log(F) - I*e)*x
)) + gamma(-1, -(b*c*log(F) + I*e)*x) + gamma(-1, -(b*c*log(F) - I*e)*x))*c
os(d) + F^(a*c)*(-I*conjugate(gamma(-1, -(b*c*log(F) + I*e)*x)) + I*conjuga
te(gamma(-1, -(b*c*log(F) - I*e)*x)) + I*gamma(-1, -(b*c*log(F) + I*e)*x) -
 I*gamma(-1, -(b*c*log(F) - I*e)*x))*sin(d))*e)*b*c*log(F) - 1/2*(F^(a*c)*(
I*conjugate(gamma(-2, -(b*c*log(F) + I*e)*x)) - I*conjugate(gamma(-2, -(b*c
*log(F) - I*e)*x)) - I*gamma(-2, -(b*c*log(F) + I*e)*x) + I*gamma(-2, -(b*c
*log(F) - I*e)*x))*cos(d) + F^(a*c)*(conjugate(gamma(-2, -(b*c*log(F) + I*e
)*x)) + conjugate(gamma(-2, -(b*c*log(F) - I*e)*x)) + gamma(-2, -(b*c*log(F
) + I*e)*x) + gamma(-2, -(b*c*log(F) - I*e)*x))*sin(d))*e^2 + 1/4*(F^(a*c)*
b*c*(conjugate(gamma(-1, -(b*c*log(F) + I*e)*x)) + conjugate(gamma(-1, -(b*
c*log(F) - I*e)*x)) + gamma(-1, -(b*c*log(F) + I*e)*x) + gamma(-1, -(b*c*lo
g(F) - I*e)*x))*cos(d)*log(F) - F^(a*c)*b*c*(I*conjugate(gamma(-1, -(b*c*lo
g(F) + I*e)*x)) - I*conjugate(gamma(-1, -(b*c*log(F) - I*e)*x)) - I*gamma(-
1, -(b*c*log(F) + I*e)*x) + I*gamma(-1, -(b*c*log(F) - I*e)*x))*log(F)*sin(
d) - (F^(a*c)*(I*conjugate(gamma(-1, -(b*c*log(F) + I*e)*x)) - I*conjugate(
gamma(-1, -(b*c*log(F) - I*e)*x)) - I*gamma(-1, -(b*c*log(F) + I*e)*x) + I*
gamma(-1, -(b*c*log(F) - I*e)*x))*cos(d) + F^(a*c)*(conjugate(gamma(-1, -(b
*c*log(F) + I*e)*x)) + conjugate(gamma(-1, -(b*c*log(F) - I*e)*x)) + gamma(
-1, -(b*c*log(F) + I*e)*x) + gamma(-1, -(b*c*log(F) - I*e)*x))*sin(d))*e)*e
 + (F^(a*c)*b*c*(conjugate(gamma(-2, -(b*c*log(F) + I*e)*x)) + conjugate(ga
mma(-2, -(b*c*log(F) - I*e)*x)) + gamma(-2, -(b*c*log(F) + I*e)*x) + gamma(
-2, -(b*c*log(F) - I*e)*x))*cos(d)*log(F) + F^(a*c)*b*c*(-I*conjugate(gamma
(-2, -(b*c*log(F) + I*e)*x)) + I*conjugate(gamma(-2, -(b*c*log(F) - I*e)*x)
) + I*gamma(-2, -(b*c*log(F) + I*e)*x) - I*gamma(-2, -(b*c*log(F) - I*e)*x)
)*log(F)*sin(d))*e
```

Giac [F]

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx = \int \frac{(ex\cos{(ex+d)}+(bcx\log{(F)}-2)\sin{(ex+d)})F^{(bx+a)c}}{x^3}\,dx$$

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*log(F))*sin(e*x+d))/x^3,x
, algorithm="giac")
```

```
[Out] integrate((e*x*cos(e*x + d) + (b*c*x*log(F) - 2)*sin(e*x + d))*F^((b*x + a)
*c)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 27.84 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx=\frac{F^{c\,(a+b\,x)}\,\sin\left(d+e\,x\right)}{x^2}$$

```
[In] int((F^(c*(a + b*x))*(sin(d + e*x)*(b*c*x*log(F) - 2) + e*x*cos(d + e*x)))/
x^3,x)
```

```
[Out] (F^(c*(a + b*x))*sin(d + e*x))/x^2
```

3.38 $\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 63

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx=-\frac{de^{a+bx}\cos(2c+2dx)}{b^2+4d^2}+\frac{be^{a+bx}\sin(2c+2dx)}{2(b^2+4d^2)}$$

```
[Out] -d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+1/2*b*exp(b*x+a)*sin(2*d*x+2*c)/(b
^2+4*d^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4557, 12, 4517}

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx=\frac{be^{a+bx}\sin(2c+2dx)}{2(b^2+4d^2)}-\frac{de^{a+bx}\cos(2c+2dx)}{b^2+4d^2}$$

```
[In] Int[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x],x]
```

```
[Out] -((d*E^(a + b*x)*Cos[2*c + 2*d*x])/(b^2 + 4*d^2)) + (b*E^(a + b*x)*Sin[2*c
+ 2*d*x])/(2*(b^2 + 4*d^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
```

```
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{2}e^{a+bx}\sin(2c+2dx)\,dx \\ &= \frac{1}{2}\int e^{a+bx}\sin(2c+2dx)\,dx \\ &= -\frac{de^{a+bx}\cos(2c+2dx)}{b^2+4d^2}+\frac{be^{a+bx}\sin(2c+2dx)}{2\,(b^2+4d^2)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx=\frac{e^{a+bx}(-2d\cos(2(c+dx))+b\sin(2(c+dx)))}{2\,(b^2+4d^2)}$$

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x],x]
[Out] (E^(a + b*x)*(-2*d*Cos[2*(c + d*x)] + b*Sin[2*(c + d*x)]))/(2*(b^2 + 4*d^2)
)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$\frac{e^{xb+a}(b\sin(2dx+2c)-2d\cos(2dx+2c))}{2b^2+8d^2}$	45
risch	$-\frac{ie^{xb+a}(4id\cos(2dx+2c)-2ib\sin(2dx+2c))}{4(2id+b)(2id-b)}$	55
default	$-\frac{d\,e^{xb+a}\cos(2dx+2c)}{b^2+4d^2}+\frac{b\,e^{xb+a}\sin(2dx+2c)}{2b^2+8d^2}$	60
norman	$\frac{-\frac{d\,e^{xb+a}}{b^2+4d^2}+\frac{2b\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2+4d^2}-\frac{2b\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{b^2+4d^2}+\frac{6d\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{b^2+4d^2}-\frac{d\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{b^2+4d^2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$	160

```
[In] int(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] exp(b*x+a)*(b*sin(2*d*x+2*c)-2*d*cos(2*d*x+2*c))/(2*b^2+8*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx = \frac{b\cos(dx+c)\,e^{(bx+a)}\sin(dx+c)-\left(2\,d\cos(dx+c)^2-d\right)e^{(bx+a)}}{b^2+4\,d^2}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x, algorithm="fricas")
[Out] (b*cos(d*x + c)*e^(b*x + a)*sin(d*x + c) - (2*d*cos(d*x + c)^2 - d)*e^(b*x
+ a))/(b^2 + 4*d^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 325, normalized size of antiderivative = 5.16

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx = \begin{cases} xe^a\sin(c)\cos(c) & \text{for } b=0 \\ \frac{ixe^ae^{-2idx}\sin^2(c+dx)}{4}+\frac{xe^ae^{-2idx}\sin(c+dx)\cos(c+dx)}{2}-\frac{ixe^ae^{-2idx}\cos^2(c+dx)}{4}+\frac{ie^ae^{-2idx}\sin(c+dx)\cos(c+dx)}{4d} & \text{for } b=- \\ -\frac{ixe^ae^{2idx}\sin^2(c+dx)}{4}+\frac{xe^ae^{2idx}\sin(c+dx)\cos(c+dx)}{2}+\frac{ixe^ae^{2idx}\cos^2(c+dx)}{4}-\frac{ie^ae^{2idx}\sin(c+dx)\cos(c+dx)}{4d} & \text{for } b=2i \\ \frac{be^ae^{bx}\sin(c+dx)\cos(c+dx)}{b^2+4d^2}+\frac{de^ae^{bx}\sin^2(c+dx)}{b^2+4d^2}-\frac{de^ae^{bx}\cos^2(c+dx)}{b^2+4d^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)*cos(c), Eq(b, 0) & Eq(d, 0)), (I*x*exp(a)*exp(-2
*I*d*x)*sin(c + d*x)**2/4 + x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)*cos(c + d*x
)/2 - I*x*exp(a)*exp(-2*I*d*x)*cos(c + d*x)**2/4 + I*exp(a)*exp(-2*I*d*x)*s
in(c + d*x)*cos(c + d*x)/(4*d), Eq(b, -2*I*d)), (-I*x*exp(a)*exp(2*I*d*x)*s
in(c + d*x)**2/4 + x*exp(a)*exp(2*I*d*x)*sin(c + d*x)*cos(c + d*x)/2 + I*x*
exp(a)*exp(2*I*d*x)*cos(c + d*x)**2/4 - I*exp(a)*exp(2*I*d*x)*sin(c + d*x)*
cos(c + d*x)/(4*d), Eq(b, 2*I*d)), (b*exp(a)*exp(b*x)*sin(c + d*x)*cos(c +
d*x)/(b**2 + 4*d**2) + d*exp(a)*exp(b*x)*sin(c + d*x)**2/(b**2 + 4*d**2) -
d*exp(a)*exp(b*x)*cos(c + d*x)**2/(b**2 + 4*d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx = -\frac{(2\,d\cos{(2\,dx+2\,c)} - b\sin{(2\,dx+2\,c)})e^{(bx+a)}}{2\,(b^2+4\,d^2)}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*d*cos(2*d*x + 2*c) - b*sin(2*d*x + 2*c))*e^(b*x + a)/(b^2 + 4*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx = -\frac{1}{2}\left(\frac{2\,d\cos{(2\,dx+2\,c)}}{b^2+4\,d^2} - \frac{b\sin{(2\,dx+2\,c)}}{b^2+4\,d^2}\right)e^{(bx+a)}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x + 2*c)/(b^2 + 4*d^2)
)*e^(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 27.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx = -\frac{e^{a+b\,x}\,(2\,d\,\cos{(2\,c+2\,d\,x)} - b\,\sin{(2\,c+2\,d\,x)})}{2\,(b^2+4\,d^2)}$$

```
[In] int(cos(c + d*x)*exp(a + b*x)*sin(c + d*x),x)
```

```
[Out] -(exp(a + b*x)*(2*d*cos(2*c + 2*d*x) - b*sin(2*c + 2*d*x)))/(2*(b^2 + 4*d^2
))
```

3.39 $\int e^{a+bx} \cos(c+dx) \sin^2(c+dx) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int e^{a+bx} \cos(c+dx) \sin^2(c+dx) \, dx = \frac{be^{a+bx} \cos(c+dx)}{4 (b^2+d^2)} - \frac{be^{a+bx} \cos(3c+3dx)}{4 (b^2+9d^2)} + \frac{de^{a+bx} \sin(c+dx)}{4 (b^2+d^2)} - \frac{3de^{a+bx} \sin(3c+3dx)}{4 (b^2+9d^2)}$$

```
[Out] 1/4*b*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-1/4*b*exp(b*x+a)*cos(3*d*x+3*c)/(b^2+
9*d^2)+1/4*d*exp(b*x+a)*sin(d*x+c)/(b^2+d^2)-3/4*d*exp(b*x+a)*sin(3*d*x+3*c
)/(b^2+9*d^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4557, 4518}

$$\int e^{a+bx} \cos(c+dx) \sin^2(c+dx) \, dx = \frac{de^{a+bx} \sin(c+dx)}{4 (b^2+d^2)} - \frac{3de^{a+bx} \sin(3c+3dx)}{4 (b^2+9d^2)} + \frac{be^{a+bx} \cos(c+dx)}{4 (b^2+d^2)} - \frac{be^{a+bx} \cos(3c+3dx)}{4 (b^2+9d^2)}$$

```
[In] Int[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^2,x]
```

```
[Out] (b*E^(a + b*x)*Cos[c + d*x])/(4*(b^2 + d^2)) - (b*E^(a + b*x)*Cos[3*c + 3*d
*x])/(4*(b^2 + 9*d^2)) + (d*E^(a + b*x)*Sin[c + d*x])/(4*(b^2 + d^2)) - (3*
d*E^(a + b*x)*Sin[3*c + 3*d*x])/(4*(b^2 + 9*d^2))
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{4}e^{a+bx}\cos(c+dx)-\frac{1}{4}e^{a+bx}\cos(3c+3dx)\right)\,dx\\
&= \frac{1}{4}\int e^{a+bx}\cos(c+dx)\,dx-\frac{1}{4}\int e^{a+bx}\cos(3c+3dx)\,dx\\
&= \frac{be^{a+bx}\cos(c+dx)}{4\,(b^2+d^2)}-\frac{be^{a+bx}\cos(3c+3dx)}{4\,(b^2+9d^2)}+\frac{de^{a+bx}\sin(c+dx)}{4\,(b^2+d^2)}-\frac{3de^{a+bx}\sin(3c+3dx)}{4\,(b^2+9d^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx=\frac{1}{4}e^{a+bx}\left(\frac{b\cos(c+dx)+d\sin(c+dx)}{b^2+d^2}-\frac{b\cos(3(c+dx))+3d\sin(3(c+dx))}{b^2+9d^2}\right)$$

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^2,x]
```

```
[Out] (E^(a + b*x)*((b*Cos[c + d*x] + d*Sin[c + d*x])/(b^2 + d^2) - (b*Cos[3*(c +
 d*x)] + 3*d*Sin[3*(c + d*x)])/(b^2 + 9*d^2)))/4
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

method	result
parallelrisch	$-\frac{3\,\mathrm{e}^{xb+a}\left(\frac{\left(b^{3}+b\,d^{2}\right)\cos\left(3dx+3c\right)}{3}+\left(b^{2}d+d^{3}\right)\sin\left(3dx+3c\right)-\frac{\left(b^{2}+9d^{2}\right)\left(\cos\left(dx+c\right)b+d\sin\left(dx+c\right)\right)}{3}\right)}{4\left(b^{4}+10b^{2}d^{2}+9d^{4}\right)}$
default	$\frac{b\,\mathrm{e}^{xb+a}\cos\left(dx+c\right)}{4b^{2}+4d^{2}}-\frac{b\,\mathrm{e}^{xb+a}\cos\left(3dx+3c\right)}{4\left(b^{2}+9d^{2}\right)}+\frac{d\,\mathrm{e}^{xb+a}\sin\left(dx+c\right)}{4b^{2}+4d^{2}}-\frac{3d\,\mathrm{e}^{xb+a}\sin\left(3dx+3c\right)}{4\left(b^{2}+9d^{2}\right)}$
risch	$-\frac{\mathrm{e}^{xb+a}\left(\left(-2b^{3}-18b\,d^{2}\right)\cos\left(dx+c\right)-2d\left(b^{2}+9d^{2}\right)\sin\left(dx+c\right)+\left(2b^{3}+2b\,d^{2}\right)\cos\left(3dx+3c\right)+6d\left(b^{2}+d^{2}\right)\sin\left(3dx+3c\right)\right)}{8\left(3id+b\right)\left(id+b\right)\left(id-b\right)\left(3id-b\right)}$
norman	$\frac{\frac{2b\,d^{2}\mathrm{e}^{xb+a}}{b^{4}+10b^{2}d^{2}+9d^{4}}-\frac{2b\,d^{2}\mathrm{e}^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{b^{4}+10b^{2}d^{2}+9d^{4}}+\frac{2b\left(2b^{2}+3d^{2}\right)\mathrm{e}^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{b^{4}+10b^{2}d^{2}+9d^{4}}-\frac{2b\left(2b^{2}+3d^{2}\right)\mathrm{e}^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{b^{4}+10b^{2}d^{2}+9d^{4}}-\frac{4b^{2}d\,\mathrm{e}^{xb+a}\tan\left(\right.}{b^{4}+10b^{2}d^{2}+9}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$

```
[In] int(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*exp(b*x+a)*(1/3*(b^3+b*d^2)*cos(3*d*x+3*c)+(b^2*d+d^3)*sin(3*d*x+3*c)-
1/3*(b^2+9*d^2)*(cos(d*x+c)*b+d*sin(d*x+c)))/(b^4+10*b^2*d^2+9*d^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx$$
$$=\frac{\left(b^2d+3\,d^3-3\,(b^2d+d^3)\cos{(dx+c)}^2\right)e^{(bx+a)}\sin{(dx+c)}-\left((b^3+bd^2)\cos{(dx+c)}^3-(b^3+3\,bd^2)\cos{(\right.}{b^4+10\,b^2d^2+9\,d^4}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*d + 3*d^3 - 3*(b^2*d + d^3)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c)
- ((b^3 + b*d^2)*cos(d*x + c)^3 - (b^3 + 3*b*d^2)*cos(d*x + c))*e^(b*x + a)
)/(b^4 + 10*b^2*d^2 + 9*d^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 1030, normalized size of antiderivative = 8.66

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)**2,x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)**2*cos(c), Eq(b, 0) & Eq(d, 0)), (I*x*exp(a)*exp
(-3*I*d*x)*sin(c + d*x)**3/8 + 3*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**2*cos
(c + d*x)/8 - 3*I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - x
*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**3/8 + exp(a)*exp(-3*I*d*x)*sin(c + d*x)
**3/(24*d) - exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) + I*ex
p(a)*exp(-3*I*d*x)*cos(c + d*x)**3/(8*d), Eq(b, -3*I*d)), (I*x*exp(a)*exp(-
I*d*x)*sin(c + d*x)**3/8 + x*exp(a)*exp(-I*d*x)*sin(c + d*x)**2*cos(c + d*x
)/8 + I*x*exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + x*exp(a)*exp(
-I*d*x)*cos(c + d*x)**3/8 + exp(a)*exp(-I*d*x)*sin(c + d*x)**3/(8*d) - exp(
a)*exp(-I*d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) + I*exp(a)*exp(-I*d*x)*co
s(c + d*x)**3/(8*d), Eq(b, -I*d)), (-I*x*exp(a)*exp(I*d*x)*sin(c + d*x)**3/
8 + x*exp(a)*exp(I*d*x)*sin(c + d*x)**2*cos(c + d*x)/8 - I*x*exp(a)*exp(I*d
*x)*sin(c + d*x)*cos(c + d*x)**2/8 + x*exp(a)*exp(I*d*x)*cos(c + d*x)**3/8
+ exp(a)*exp(I*d*x)*sin(c + d*x)**3/(8*d) - exp(a)*exp(I*d*x)*sin(c + d*x)*
cos(c + d*x)**2/(4*d) - I*exp(a)*exp(I*d*x)*cos(c + d*x)**3/(8*d), Eq(b, I*
d)), (-I*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**3/8 + 3*x*exp(a)*exp(3*I*d*x)*
sin(c + d*x)**2*cos(c + d*x)/8 + 3*I*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)*cos
(c + d*x)**2/8 - x*exp(a)*exp(3*I*d*x)*cos(c + d*x)**3/8 + exp(a)*exp(3*I*d
*x)*sin(c + d*x)**3/(24*d) - exp(a)*exp(3*I*d*x)*sin(c + d*x)*cos(c + d*x)*
*2/(4*d) - I*exp(a)*exp(3*I*d*x)*cos(c + d*x)**3/(8*d), Eq(b, 3*I*d)), (b**
3*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)/(b**4 + 10*b**2*d**2 + 9*d**
4) + b**2*d*exp(a)*exp(b*x)*sin(c + d*x)**3/(b**4 + 10*b**2*d**2 + 9*d**4)
- 2*b**2*d*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**2/(b**4 + 10*b**2*d**
2 + 9*d**4) + 3*b*d**2*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)/(b**4 +
 10*b**2*d**2 + 9*d**4) + 2*b*d**2*exp(a)*exp(b*x)*cos(c + d*x)**3/(b**4 +
10*b**2*d**2 + 9*d**4) + 3*d**3*exp(a)*exp(b*x)*sin(c + d*x)**3/(b**4 + 10*
b**2*d**2 + 9*d**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(107) = 214$.

Time = 0.24 (sec) , antiderivative size = 538, normalized size of antiderivative = 4.52

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx = -\frac{\left(b^3\cos(3c)e^a + bd^2\cos(3c)e^a + 3b^2de^a\sin(3c) + 3d^3e^a\sin(3c)\right)\cos(3dx)e^{(bx)} + \left(b^3\cos(3c)e^a + bd\right.}{}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((b^3*cos(3*c)*e^a + b*d^2*cos(3*c)*e^a + 3*b^2*d*e^a*sin(3*c) + 3*d^3
*e^a*sin(3*c))*cos(3*d*x)*e^(b*x) + (b^3*cos(3*c)*e^a + b*d^2*cos(3*c)*e^a
- 3*b^2*d*e^a*sin(3*c) - 3*d^3*e^a*sin(3*c))*cos(3*d*x + 6*c)*e^(b*x) - (b^
3*cos(3*c)*e^a + 9*b*d^2*cos(3*c)*e^a - b^2*d*e^a*sin(3*c) - 9*d^3*e^a*sin(
3*c))*cos(d*x + 4*c)*e^(b*x) - (b^3*cos(3*c)*e^a + 9*b*d^2*cos(3*c)*e^a + b
^2*d*e^a*sin(3*c) + 9*d^3*e^a*sin(3*c))*cos(d*x - 2*c)*e^(b*x) + (3*b^2*d*c
os(3*c)*e^a + 3*d^3*cos(3*c)*e^a - b^3*e^a*sin(3*c) - b*d^2*e^a*sin(3*c))*e
^(b*x)*sin(3*d*x) + (3*b^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c)*e^a + b^3*e^a*si
n(3*c) + b*d^2*e^a*sin(3*c))*e^(b*x)*sin(3*d*x + 6*c) - (b^2*d*cos(3*c)*e^a
 + 9*d^3*cos(3*c)*e^a + b^3*e^a*sin(3*c) + 9*b*d^2*e^a*sin(3*c))*e^(b*x)*si
n(d*x + 4*c) - (b^2*d*cos(3*c)*e^a + 9*d^3*cos(3*c)*e^a - b^3*e^a*sin(3*c)
- 9*b*d^2*e^a*sin(3*c))*e^(b*x)*sin(d*x - 2*c))/(b^4*cos(3*c)^2 + b^4*sin(3
*c)^2 + 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 + 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)
^2)*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx = -\frac{1}{4}\left(\frac{b\cos(3\,dx+3\,c)}{b^2+9\,d^2} + \frac{3\,d\sin(3\,dx+3\,c)}{b^2+9\,d^2}\right)e^{(bx+a)} + \frac{1}{4}\left(\frac{b\cos(dx+c)}{b^2+d^2} + \frac{d\sin(dx+c)}{b^2+d^2}\right)e^{(bx+a)}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(b*cos(3*d*x + 3*c)/(b^2 + 9*d^2) + 3*d*sin(3*d*x + 3*c)/(b^2 + 9*d^2)
)*e^(b*x + a) + 1/4*(b*cos(d*x + c)/(b^2 + d^2) + d*sin(d*x + c)/(b^2 + d^2
))*e^(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 27.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.39

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx = \frac{e^{a+b\,x}\,(\cos(d\,x)-\sin(d\,x)\ 1\mathrm{i})\ (\cos(c)-\sin(c)\ 1\mathrm{i})}{8\ (b-d\,1\mathrm{i})} - \frac{e^{a+b\,x}\,(\cos(3\,d\,x)+\sin(3\,d\,x)\ 1\mathrm{i})\ (\cos(3\,c)+\sin(3\,c)\ 1\mathrm{i})\ 1\mathrm{i}}{8\ (-3\,d+b\,1\mathrm{i})} + \frac{e^{a+b\,x}\,(\cos(d\,x)+\sin(d\,x)\ 1\mathrm{i})\ (\cos(c)+\sin(c)\ 1\mathrm{i})\ 1\mathrm{i}}{8\ (-d+b\,1\mathrm{i})} - \frac{e^{a+b\,x}\,(\cos(3\,d\,x)-\sin(3\,d\,x)\ 1\mathrm{i})\ (\cos(3\,c)-\sin(3\,c)\ 1\mathrm{i})}{8\ (b-d\,3\mathrm{i})}$$

```
[In] int(cos(c + d*x)*exp(a + b*x)*sin(c + d*x)^2,x)
```

```
[Out] (exp(a + b*x)*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i))/(8*(b - d*1i))
 - (exp(a + b*x)*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*1i)/
(8*(b*1i - 3*d)) + (exp(a + b*x)*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*
1i)*1i)/(8*(b*1i - d)) - (exp(a + b*x)*(cos(3*d*x) - sin(3*d*x)*1i)*(cos(3*
c) - sin(3*c)*1i))/(8*(b - d*3i))
```

3.40 $\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 129

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx = -\frac{de^{a+bx}\cos(2c+2dx)}{2(b^2+4d^2)}+\frac{de^{a+bx}\cos(4c+4dx)}{2(b^2+16d^2)}+\frac{be^{a+bx}\sin(2c+2dx)}{4(b^2+4d^2)}-\frac{be^{a+bx}\sin(4c+4dx)}{8(b^2+16d^2)}$$

```
[Out] -1/2*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+1/2*d*exp(b*x+a)*cos(4*d*x+4*c
)/(b^2+16*d^2)+1/4*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)-1/8*b*exp(b*x+a)
*sin(4*d*x+4*c)/(b^2+16*d^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4557, 4517}

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx = \frac{be^{a+bx}\sin(2c+2dx)}{4(b^2+4d^2)}-\frac{be^{a+bx}\sin(4c+4dx)}{8(b^2+16d^2)}-\frac{de^{a+bx}\cos(2c+2dx)}{2(b^2+4d^2)}+\frac{de^{a+bx}\cos(4c+4dx)}{2(b^2+16d^2)}$$

```
[In] Int[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^3,x]
```

```
[Out] -1/2*(d*E^(a + b*x)*Cos[2*c + 2*d*x])/(b^2 + 4*d^2) + (d*E^(a + b*x)*Cos[4*
c + 4*d*x])/(2*(b^2 + 16*d^2)) + (b*E^(a + b*x)*Sin[2*c + 2*d*x])/(4*(b^2 +
 4*d^2)) - (b*E^(a + b*x)*Sin[4*c + 4*d*x])/(8*(b^2 + 16*d^2))
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{4}e^{a+bx}\sin(2c+2dx) - \frac{1}{8}e^{a+bx}\sin(4c+4dx)\right) dx$$

$$= -\left(\frac{1}{8}\int e^{a+bx}\sin(4c+4dx)\,dx\right) + \frac{1}{4}\int e^{a+bx}\sin(2c+2dx)\,dx$$

$$= -\frac{de^{a+bx}\cos(2c+2dx)}{2(b^2+4d^2)} + \frac{de^{a+bx}\cos(4c+4dx)}{2(b^2+16d^2)} + \frac{be^{a+bx}\sin(2c+2dx)}{4(b^2+4d^2)} - \frac{be^{a+bx}\sin(4c+4dx)}{8(b^2+16d^2)}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx = \frac{1}{8}e^{a+bx}\left(\frac{2(-2d\cos(2(c+dx))+b\sin(2(c+dx)))}{b^2+4d^2} + \frac{4d\cos(4(c+dx))-b\sin(4(c+dx))}{b^2+16d^2}\right)$$

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*((2*(-2*d*Cos[2*(c + d*x)] + b*Sin[2*(c + d*x)]))/(b^2 + 4*d^2
) + (4*d*Cos[4*(c + d*x)] - b*Sin[4*(c + d*x)])/(b^2 + 16*d^2)))/8
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

method	result
parallelrisch	$-\frac{\left(\left(b^3+4b\,d^2\right)\sin(4dx+4c)+\left(-4b^2d-16d^3\right)\cos(4dx+4c)-2\left(b^2+16d^2\right)\left(b\sin(2dx+2c)-2d\cos(2dx+2c)\right)\right)e^{xb+a}}{8b^4+160b^2d^2+512d^4}$
default	$-\frac{d\,e^{xb+a}\cos(2dx+2c)}{2(b^2+4d^2)}+\frac{d\,e^{xb+a}\cos(4dx+4c)}{2b^2+32d^2}+\frac{b\,e^{xb+a}\sin(2dx+2c)}{4b^2+16d^2}-\frac{b\,e^{xb+a}\sin(4dx+4c)}{8(b^2+16d^2)}$
risch	$\frac{ie^{xb+a}\left(-8id\left(b^2+4d^2\right)\cos(4dx+4c)-i\left(-2b^3-8b\,d^2\right)\sin(4dx+4c)+8id\left(b^2+16d^2\right)\cos(2dx+2c)-i\left(4b^3+64b\,d^2\right)\sin(2dx+2c)\right)}{16(4id+b)(2id+b)(2id-b)(4id-b)}$
norman	$\frac{-\frac{6d^3e^{xb+a}}{b^4+20b^2d^2+64d^4}-\frac{6d^3e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{b^4+20b^2d^2+64d^4}+\frac{12b\,d^2e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^4+20b^2d^2+64d^4}-\frac{12b\,d^2e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{b^4+20b^2d^2+64d^4}+\frac{4b\left(2b^2+11d^2\right)e^{xb+a}\tan\left(\frac{dx}{2}+\ldots\right.}{b^4+20b^2d^2+64d^4}}{\left(1\ldots\right.}$

[In] int(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] -((b^3+4*b*d^2)*sin(4*d*x+4*c)+(-4*b^2*d-16*d^3)*cos(4*d*x+4*c)-2*(b^2+16*d^2)*(b*sin(2*d*x+2*c)-2*d*cos(2*d*x+2*c)))*exp(b*x+a)/(8*b^4+160*b^2*d^2+512*d^4)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx =$$
$$-\frac{\left(\left(b^3+4\,bd^2\right)\cos\left(dx+c\right)^3-\left(b^3+10\,bd^2\right)\cos\left(dx+c\right)\right)e^{(bx+a)}\sin\left(dx+c\right)-\left(4\left(b^2d+4\,d^3\right)\cos\left(dx+c\right)\ldots\right.}{b^4+20\,b^2d^2+64\,d^4}$$

[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x, algorithm="fricas")

[Out] -(((b^3 + 4*b*d^2)*cos(d*x + c)^3 - (b^3 + 10*b*d^2)*cos(d*x + c))*e^(b*x + a)*sin(d*x + c) - (4*(b^2*d + 4*d^3)*cos(d*x + c)^4 + b^2*d + 10*d^3 - (5*b^2*d + 32*d^3)*cos(d*x + c)^2)*e^(b*x + a))/(b^4 + 20*b^2*d^2 + 64*d^4)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.29 (sec) , antiderivative size = 1353, normalized size of antiderivative = 10.49

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)**3,x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)**3*cos(c), Eq(b, 0) & Eq(d, 0)), (I*x*exp(a)*exp
(-4*I*d*x)*sin(c + d*x)**4/16 + x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos(
c + d*x)/4 - 3*I*x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/8 -
 x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 + I*x*exp(a)*exp(-4*
I*d*x)*cos(c + d*x)**4/16 - exp(a)*exp(-4*I*d*x)*sin(c + d*x)**4/(24*d) + 1
1*I*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) + 5*I*exp(a)*e
xp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) + exp(a)*exp(-4*I*d*x)*cos
(c + d*x)**4/(24*d), Eq(b, -4*I*d)), (I*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)
**4/8 + x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + x*exp(a)*ex
p(-2*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - I*x*exp(a)*exp(-2*I*d*x)*cos(c
 + d*x)**4/8 + exp(a)*exp(-2*I*d*x)*sin(c + d*x)**4/(16*d) - exp(a)*exp(-2*
I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/(4*d) + I*exp(a)*exp(-2*I*d*x)*sin(c
 + d*x)*cos(c + d*x)**3/(6*d) + exp(a)*exp(-2*I*d*x)*cos(c + d*x)**4/(48*d)
, Eq(b, -2*I*d)), (-I*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**4/8 + x*exp(a)*ex
p(2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + x*exp(a)*exp(2*I*d*x)*sin(c + d
*x)*cos(c + d*x)**3/4 + I*x*exp(a)*exp(2*I*d*x)*cos(c + d*x)**4/8 + exp(a)*
exp(2*I*d*x)*sin(c + d*x)**4/(16*d) - exp(a)*exp(2*I*d*x)*sin(c + d*x)**2*c
os(c + d*x)**2/(4*d) - I*exp(a)*exp(2*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(
6*d) + exp(a)*exp(2*I*d*x)*cos(c + d*x)**4/(48*d), Eq(b, 2*I*d)), (-I*x*exp
(a)*exp(4*I*d*x)*sin(c + d*x)**4/16 + x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**3
*cos(c + d*x)/4 + 3*I*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2
/8 - x*exp(a)*exp(4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - I*x*exp(a)*exp(
4*I*d*x)*cos(c + d*x)**4/16 - exp(a)*exp(4*I*d*x)*sin(c + d*x)**4/(24*d) -
11*I*exp(a)*exp(4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) - 5*I*exp(a)*e
xp(4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) + exp(a)*exp(4*I*d*x)*cos(c
 + d*x)**4/(24*d), Eq(b, 4*I*d)), (b**3*exp(a)*exp(b*x)*sin(c + d*x)**3*cos
(c + d*x)/(b**4 + 20*b**2*d**2 + 64*d**4) + b**2*d*exp(a)*exp(b*x)*sin(c +
d*x)**4/(b**4 + 20*b**2*d**2 + 64*d**4) - 3*b**2*d*exp(a)*exp(b*x)*sin(c +
d*x)**2*cos(c + d*x)**2/(b**4 + 20*b**2*d**2 + 64*d**4) + 10*b*d**2*exp(a)*
exp(b*x)*sin(c + d*x)**3*cos(c + d*x)/(b**4 + 20*b**2*d**2 + 64*d**4) + 6*b
*d**2*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**3/(b**4 + 20*b**2*d**2 + 6
4*d**4) + 10*d**3*exp(a)*exp(b*x)*sin(c + d*x)**4/(b**4 + 20*b**2*d**2 + 64
*d**4) - 12*d**3*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**2/(b**4 + 20
*b**2*d**2 + 64*d**4) - 6*d**3*exp(a)*exp(b*x)*cos(c + d*x)**4/(b**4 + 20*b
**2*d**2 + 64*d**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(117) = 234$.

Time = 0.22 (sec) , antiderivative size = 550, normalized size of antiderivative = 4.26

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx = \frac{\left(4\,b^2d\cos\left(4\,c\right)e^a + 16\,d^3\cos\left(4\,c\right)e^a - b^3e^a\sin\left(4\,c\right) - 4\,bd^2e^a\sin\left(4\,c\right)\right)\cos\left(4\,dx\right)e^{(bx)} + \left(4\,b^2d\cos\left(4\,c\right)e^a + \right.}{}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 4*b*
d^2*e^a*sin(4*c))*cos(4*d*x)*e^(b*x) + (4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(4
*c)*e^a + b^3*e^a*sin(4*c) + 4*b*d^2*e^a*sin(4*c))*cos(4*d*x + 8*c)*e^(b*x)
 - 2*(2*b^2*d*cos(4*c)*e^a + 32*d^3*cos(4*c)*e^a + b^3*e^a*sin(4*c) + 16*b*
d^2*e^a*sin(4*c))*cos(2*d*x + 6*c)*e^(b*x) - 2*(2*b^2*d*cos(4*c)*e^a + 32*d
^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 16*b*d^2*e^a*sin(4*c))*cos(2*d*x - 2*c
)*e^(b*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*cos(4*c)*e^a + 4*b^2*d*e^a*sin(4*c)
 + 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*co
s(4*c)*e^a - 4*b^2*d*e^a*sin(4*c) - 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x
+ 8*c) + 2*(b^3*cos(4*c)*e^a + 16*b*d^2*cos(4*c)*e^a - 2*b^2*d*e^a*sin(4*c)
 - 32*d^3*e^a*sin(4*c))*e^(b*x)*sin(2*d*x + 6*c) + 2*(b^3*cos(4*c)*e^a + 16
*b*d^2*cos(4*c)*e^a + 2*b^2*d*e^a*sin(4*c) + 32*d^3*e^a*sin(4*c))*e^(b*x)*s
in(2*d*x - 2*c))/(b^4*cos(4*c)^2 + b^4*sin(4*c)^2 + 64*(cos(4*c)^2 + sin(4*
c)^2)*d^4 + 20*(b^2*cos(4*c)^2 + b^2*sin(4*c)^2)*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx = \frac{1}{8}\left(\frac{4\,d\cos\left(4\,dx+4\,c\right)}{b^2+16\,d^2} - \frac{b\sin\left(4\,dx+4\,c\right)}{b^2+16\,d^2}\right)e^{(bx+a)} - \frac{1}{4}\left(\frac{2\,d\cos\left(2\,dx+2\,c\right)}{b^2+4\,d^2} - \frac{b\sin\left(2\,dx+2\,c\right)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*d*cos(4*d*x + 4*c)/(b^2 + 16*d^2) - b*sin(4*d*x + 4*c)/(b^2 + 16*d^2
))*e^(b*x + a) - 1/4*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x + 2*
c)/(b^2 + 4*d^2))*e^(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 28.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.38

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx$$
$$= -\frac{e^{a+b\,x}\,(\cos(2\,d\,x)-\sin(2\,d\,x)\;1\mathrm{i})\;(\cos(2\,c)-\sin(2\,c)\;1\mathrm{i})}{8\;(2\,d+b\,1\mathrm{i})} + \frac{e^{a+b\,x}\,(\cos(4\,d\,x)-\sin(4\,d\,x)\;1\mathrm{i})\;(\cos(4\,c)-\sin(4\,c)\;1\mathrm{i})}{16\;(4\,d+b\,1\mathrm{i})} - \frac{e^{a+b\,x}\,(\cos(2\,d\,x)+\sin(2\,d\,x)\;1\mathrm{i})\;(\cos(2\,c)+\sin(2\,c)\;1\mathrm{i})\;1\mathrm{i}}{8\;(b+d\,2\mathrm{i})} + \frac{e^{a+b\,x}\,(\cos(4\,d\,x)+\sin(4\,d\,x)\;1\mathrm{i})\;(\cos(4\,c)+\sin(4\,c)\;1\mathrm{i})\;1\mathrm{i}}{16\;(b+d\,4\mathrm{i})}$$

```
[In] int(cos(c + d*x)*exp(a + b*x)*sin(c + d*x)^3,x)
```

```
[Out] (exp(a + b*x)*(cos(4*d*x) - sin(4*d*x)*1i)*(cos(4*c) - sin(4*c)*1i))/(16*(b
*1i + 4*d)) - (exp(a + b*x)*(cos(2*d*x) - sin(2*d*x)*1i)*(cos(2*c) - sin(2*
c)*1i))/(8*(b*1i + 2*d)) - (exp(a + b*x)*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(
2*c) + sin(2*c)*1i)*1i)/(8*(b + d*2i)) + (exp(a + b*x)*(cos(4*d*x) + sin(4*
d*x)*1i)*(cos(4*c) + sin(4*c)*1i)*1i)/(16*(b + d*4i))
```

3.41 $\int e^{a+bx} \cos^2(c+dx) \sin(c+dx) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int e^{a+bx} \cos^2(c+dx) \sin(c+dx) \, dx = -\frac{de^{a+bx} \cos(c+dx)}{4(b^2+d^2)} - \frac{3de^{a+bx} \cos(3c+3dx)}{4(b^2+9d^2)} + \frac{be^{a+bx} \sin(c+dx)}{4(b^2+d^2)} + \frac{be^{a+bx} \sin(3c+3dx)}{4(b^2+9d^2)}$$

```
[Out] -1/4*d*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-3/4*d*exp(b*x+a)*cos(3*d*x+3*c)/(b^2
+9*d^2)+1/4*b*exp(b*x+a)*sin(d*x+c)/(b^2+d^2)+1/4*b*exp(b*x+a)*sin(3*d*x+3*
c)/(b^2+9*d^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4557, 4517}

$$\int e^{a+bx} \cos^2(c+dx) \sin(c+dx) \, dx = \frac{be^{a+bx} \sin(c+dx)}{4(b^2+d^2)} + \frac{be^{a+bx} \sin(3c+3dx)}{4(b^2+9d^2)} - \frac{de^{a+bx} \cos(c+dx)}{4(b^2+d^2)} - \frac{3de^{a+bx} \cos(3c+3dx)}{4(b^2+9d^2)}$$

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x],x]
```

```
[Out] -1/4*(d*E^(a + b*x)*Cos[c + d*x])/(b^2 + d^2) - (3*d*E^(a + b*x)*Cos[3*c +
3*d*x])/(4*(b^2 + 9*d^2)) + (b*E^(a + b*x)*Sin[c + d*x])/(4*(b^2 + d^2)) +
(b*E^(a + b*x)*Sin[3*c + 3*d*x])/(4*(b^2 + 9*d^2))
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{4}e^{a+bx}\sin(c+dx)+\frac{1}{4}e^{a+bx}\sin(3c+3dx)\right)\,dx\\ &= \frac{1}{4}\int e^{a+bx}\sin(c+dx)\,dx+\frac{1}{4}\int e^{a+bx}\sin(3c+3dx)\,dx\\ &= -\frac{de^{a+bx}\cos(c+dx)}{4\left(b^2+d^2\right)}-\frac{3de^{a+bx}\cos(3c+3dx)}{4\left(b^2+9d^2\right)}+\frac{be^{a+bx}\sin(c+dx)}{4\left(b^2+d^2\right)}+\frac{be^{a+bx}\sin(3c+3dx)}{4\left(b^2+9d^2\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

$$\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx=\frac{1}{4}e^{a+bx}\left(\frac{-d\cos(c+dx)+b\sin(c+dx)}{b^2+d^2}+\frac{-3d\cos(3(c+dx))+b\sin(3(c+dx))}{b^2+9d^2}\right)$$

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x],x]
[Out] (E^(a + b*x)*((-(d*Cos[c + d*x]) + b*Sin[c + d*x])/(b^2 + d^2) + (-3*d*Cos[
3*(c + d*x)] + b*Sin[3*(c + d*x)])/(b^2 + 9*d^2)))/4
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{\left(\left(-3 b^{2} d-3 d^{3}\right) \cos (3 d x+3 c)+\left(b^{3}+b\, d^{2}\right) \sin (3 d x+3 c)+\left(b^{2}+9 d^{2}\right) \left(b \sin (d x+c)-d \cos (d x+c)\right)\right) {\mathrm e}^{x b+a}}{4 b^{4}+40 b^{2} d^{2}+36 d^{4}}$
default	$-\frac{d\,{\mathrm e}^{x b+a} \cos (d x+c)}{4 \left(b^{2}+d^{2}\right)}-\frac{3 d\,{\mathrm e}^{x b+a} \cos (3 d x+3 c)}{4 \left(b^{2}+9 d^{2}\right)}+\frac{b\,{\mathrm e}^{x b+a} \sin (d x+c)}{4 b^{2}+4 d^{2}}+\frac{b\,{\mathrm e}^{x b+a} \sin (3 d x+3 c)}{4 b^{2}+36 d^{2}}$
risch	$-\frac{i {\mathrm e}^{x b+a} \left(-2 i d \left(b^{2}+9 d^{2}\right) \cos (d x+c)+i \left(2 b^{3}+18 b\, d^{2}\right) \sin (d x+c)-6 i d \left(b^{2}+d^{2}\right) \cos (3 d x+3 c)-i \left(-2 b^{3}-2 b\, d^{2}\right) \sin (3 d x+3 c)\right)}{8 (3 i d+b) (i d+b) (i d-b) (3 i d-b)}$
norman	$\frac{\frac{d \left(b^{2}+3 d^{2}\right) {\mathrm e}^{x b+a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{b^{4}+10 b^{2} d^{2}+9 d^{4}}+\frac{d \left(11 b^{2}+9 d^{2}\right) {\mathrm e}^{x b+a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{b^{4}+10 b^{2} d^{2}+9 d^{4}}-\frac{d \left(b^{2}+3 d^{2}\right) {\mathrm e}^{x b+a}}{b^{4}+10 b^{2} d^{2}+9 d^{4}}-\frac{4 b \left(b^{2}-d^{2}\right) {\mathrm e}^{x b+a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{b^{4}+10 b^{2} d^{2}+9 d^{4}}+\frac{2 b \left(b^{2}+3 d^{2}\right) \cdots}{b^{4}+\cdots}}{\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$

```
[In] int(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] ((-3*b^2*d-3*d^3)*cos(3*d*x+3*c)+(b^3+b*d^2)*sin(3*d*x+3*c)+(b^2+9*d^2)*(b*
sin(d*x+c)-d*cos(d*x+c)))*exp(b*x+a)/(4*b^4+40*b^2*d^2+36*d^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int e^{a+bx} \cos^2(c+dx) \sin(c+dx)\, dx$$
$$= \frac{\left(2\, bd^2 + (b^3 + bd^2) \cos\left(dx+c\right)^2\right) e^{(bx+a)} \sin\left(dx+c\right) + \left(2\, b^2 d \cos\left(dx+c\right) - 3\left(b^2 d + d^3\right) \cos\left(dx+c\right)^3\right) e^{(bx \cdots}}{b^4 + 10\, b^2 d^2 + 9\, d^4}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] ((2*b*d^2 + (b^3 + b*d^2)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c) + (2*b^2
*d*cos(d*x + c) - 3*(b^2*d + d^3)*cos(d*x + c)^3)*e^(b*x + a))/(b^4 + 10*b^
2*d^2 + 9*d^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.49 (sec) , antiderivative size = 1040, normalized size of antiderivative = 8.74

$$\int e^{a+bx} \cos^2(c+dx) \sin(c+dx)\, dx = \text{Too large to display}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), (-x*exp(a)*exp(
-3*I*d*x)*sin(c + d*x)**3/8 + 3*I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**2*co
s(c + d*x)/8 + 3*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - I*
x*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**3/8 + I*exp(a)*exp(-3*I*d*x)*sin(c + d
*x)**3/(24*d) + I*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) +
 exp(a)*exp(-3*I*d*x)*cos(c + d*x)**3/(24*d), Eq(b, -3*I*d)), (x*exp(a)*exp
(-I*d*x)*sin(c + d*x)**3/8 - I*x*exp(a)*exp(-I*d*x)*sin(c + d*x)**2*cos(c +
 d*x)/8 + x*exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - I*x*exp(a)*
exp(-I*d*x)*cos(c + d*x)**3/8 - I*exp(a)*exp(-I*d*x)*sin(c + d*x)**3/(8*d)
- I*exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c + d*x)**2/(4*d) - 3*exp(a)*exp(-I
*d*x)*cos(c + d*x)**3/(8*d), Eq(b, -I*d)), (x*exp(a)*exp(I*d*x)*sin(c + d*x
)**3/8 + I*x*exp(a)*exp(I*d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + x*exp(a)*ex
p(I*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + I*x*exp(a)*exp(I*d*x)*cos(c + d*x
)**3/8 + I*exp(a)*exp(I*d*x)*sin(c + d*x)**3/(8*d) + I*exp(a)*exp(I*d*x)*si
n(c + d*x)*cos(c + d*x)**2/(4*d) - 3*exp(a)*exp(I*d*x)*cos(c + d*x)**3/(8*d
), Eq(b, I*d)), (-x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**3/8 - 3*I*x*exp(a)*ex
p(3*I*d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + 3*x*exp(a)*exp(3*I*d*x)*sin(c +
 d*x)*cos(c + d*x)**2/8 + I*x*exp(a)*exp(3*I*d*x)*cos(c + d*x)**3/8 - I*exp
(a)*exp(3*I*d*x)*sin(c + d*x)**3/(24*d) - I*exp(a)*exp(3*I*d*x)*sin(c + d*x
)*cos(c + d*x)**2/(4*d) + exp(a)*exp(3*I*d*x)*cos(c + d*x)**3/(24*d), Eq(b,
 3*I*d)), (b**3*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**2/(b**4 + 10*b**
2*d**2 + 9*d**4) + 2*b**2*d*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)/(b
**4 + 10*b**2*d**2 + 9*d**4) - b**2*d*exp(a)*exp(b*x)*cos(c + d*x)**3/(b**4
 + 10*b**2*d**2 + 9*d**4) + 2*b*d**2*exp(a)*exp(b*x)*sin(c + d*x)**3/(b**4
+ 10*b**2*d**2 + 9*d**4) + 3*b*d**2*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*
x)**2/(b**4 + 10*b**2*d**2 + 9*d**4) - 3*d**3*exp(a)*exp(b*x)*cos(c + d*x)*
*3/(b**4 + 10*b**2*d**2 + 9*d**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(107) = 214$.

Time = 0.23 (sec) , antiderivative size = 538, normalized size of antiderivative = 4.52

$$\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx =$$

$$-\frac{(3\,b^2d\cos{(3\,c)}\,e^a+3\,d^3\cos{(3\,c)}\,e^a-b^3e^a\sin{(3\,c)}-bd^2e^a\sin{(3\,c)})\cos{(3\,dx)}\,e^{(bx)}+(3\,b^2d\cos{(3\,c)}\,e^a+3}{}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*((3*b^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c)*e^a - b^3*e^a*sin(3*c) - b*d^2
*e^a*sin(3*c))*cos(3*d*x)*e^(b*x) + (3*b^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c)*
e^a + b^3*e^a*sin(3*c) + b*d^2*e^a*sin(3*c))*cos(3*d*x + 6*c)*e^(b*x) + (b^
2*d*cos(3*c)*e^a + 9*d^3*cos(3*c)*e^a + b^3*e^a*sin(3*c) + 9*b*d^2*e^a*sin(
```

```
3*c))*cos(d*x + 4*c)*e^(b*x) + (b^2*d*cos(3*c)*e^a + 9*d^3*cos(3*c)*e^a - b
^3*e^a*sin(3*c) - 9*b*d^2*e^a*sin(3*c))*cos(d*x - 2*c)*e^(b*x) - (b^3*cos(3
*c)*e^a + b*d^2*cos(3*c)*e^a + 3*b^2*d*e^a*sin(3*c) + 3*d^3*e^a*sin(3*c))*e
^(b*x)*sin(3*d*x) - (b^3*cos(3*c)*e^a + b*d^2*cos(3*c)*e^a - 3*b^2*d*e^a*si
n(3*c) - 3*d^3*e^a*sin(3*c))*e^(b*x)*sin(3*d*x + 6*c) - (b^3*cos(3*c)*e^a +
 9*b*d^2*cos(3*c)*e^a - b^2*d*e^a*sin(3*c) - 9*d^3*e^a*sin(3*c))*e^(b*x)*si
n(d*x + 4*c) - (b^3*cos(3*c)*e^a + 9*b*d^2*cos(3*c)*e^a + b^2*d*e^a*sin(3*c
) + 9*d^3*e^a*sin(3*c))*e^(b*x)*sin(d*x - 2*c))/(b^4*cos(3*c)^2 + b^4*sin(3
*c)^2 + 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 + 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)
^2)*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx = -\frac{1}{4}\left(\frac{3\,d\cos\left(3\,dx+3\,c\right)}{b^2+9\,d^2}-\frac{b\sin\left(3\,dx+3\,c\right)}{b^2+9\,d^2}\right)e^{(bx+a)} - \frac{1}{4}\left(\frac{d\cos\left(dx+c\right)}{b^2+d^2}-\frac{b\sin\left(dx+c\right)}{b^2+d^2}\right)e^{(bx+a)}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -1/4*(3*d*cos(3*d*x + 3*c)/(b^2 + 9*d^2) - b*sin(3*d*x + 3*c)/(b^2 + 9*d^2)
)*e^(b*x + a) - 1/4*(d*cos(d*x + c)/(b^2 + d^2) - b*sin(d*x + c)/(b^2 + d^2
))*e^(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 28.84 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.40

$$\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx = -\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1{}\mathrm{i}\right)}{8\,\left(d+b\,1{}\mathrm{i}\right)} - \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(d\,x\right)+\sin\left(d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(c\right)+\sin\left(c\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{8\,\left(b+d\,1{}\mathrm{i}\right)} - \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)-\sin\left(3\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(3\,c\right)-\sin\left(3\,c\right)\,1{}\mathrm{i}\right)}{8\,\left(3\,d+b\,1{}\mathrm{i}\right)} - \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)+\sin\left(3\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(3\,c\right)+\sin\left(3\,c\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{8\,\left(b+d\,3{}\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^2*exp(a + b*x)*sin(c + d*x),x)
```

```
[Out] - (exp(a + b*x)*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i))/(8*(b*1i + d
)) - (exp(a + b*x)*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*1i)/(8*(b
+ d*1i)) - (exp(a + b*x)*(cos(3*d*x) - sin(3*d*x)*1i)*(cos(3*c) - sin(3*c)*
1i))/(8*(b*1i + 3*d)) - (exp(a + b*x)*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c
) + sin(3*c)*1i)*1i)/(8*(b + d*3i))
```

3.42 $\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx=\frac{e^{a+bx}}{8b}-\frac{be^{a+bx}\cos(4c+4dx)}{8\,(b^2+16d^2)}-\frac{de^{a+bx}\sin(4c+4dx)}{2\,(b^2+16d^2)}$$

```
[Out] 1/8*exp(b*x+a)/b-1/8*b*exp(b*x+a)*cos(4*d*x+4*c)/(b^2+16*d^2)-1/2*d*exp(b*x
+a)*sin(4*d*x+4*c)/(b^2+16*d^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4557, 2225, 4518}

$$\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx=-\frac{de^{a+bx}\sin(4c+4dx)}{2\,(b^2+16d^2)}-\frac{be^{a+bx}\cos(4c+4dx)}{8\,(b^2+16d^2)}+\frac{e^{a+bx}}{8b}$$

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^2,x]
```

```
[Out] E^(a + b*x)/(8*b) - (b*E^(a + b*x)*Cos[4*c + 4*d*x])/(8*(b^2 + 16*d^2)) - (
d*E^(a + b*x)*Sin[4*c + 4*d*x])/(2*(b^2 + 16*d^2))
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
```

```
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{8}e^{a+bx} - \frac{1}{8}e^{a+bx}\cos(4c+4dx)\right)\,dx\\
&= \frac{1}{8}\int e^{a+bx}\,dx - \frac{1}{8}\int e^{a+bx}\cos(4c+4dx)\,dx\\
&= \frac{e^{a+bx}}{8b} - \frac{be^{a+bx}\cos(4c+4dx)}{8\,(b^2+16d^2)} - \frac{de^{a+bx}\sin(4c+4dx)}{2\,(b^2+16d^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\begin{aligned}
&\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx\\
&= \frac{e^{a+bx}(b^2+16d^2-b^2\cos(4(c+dx))-4bd\sin(4(c+dx)))}{8\,(b^3+16bd^2)}
\end{aligned}$$

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^2,x]
[Out] (E^(a + b*x)*(b^2 + 16*d^2 - b^2*Cos[4*(c + d*x)] - 4*b*d*Sin[4*(c + d*x)])
)/(8*(b^3 + 16*b*d^2))
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

method	result
parallelrisch	$-\frac{e^{xb+a}\left(4bd\sin(4dx+4c)+b^{2}\cos(4dx+4c)-b^{2}-16d^{2}\right)}{8(b^{2}+16d^{2})b}$
risch	$\frac{e^{xb+a}\left(-2b^{2}-32d^{2}+2b^{2}\cos(4dx+4c)+8bd\sin(4dx+4c)\right)}{16b(4id+b)(4id-b)}$
default	$\frac{e^{xb+a}}{8b}-\frac{b\,e^{xb+a}\cos(4dx+4c)}{8(b^{2}+16d^{2})}-\frac{d\,e^{xb+a}\sin(4dx+4c)}{2(b^{2}+16d^{2})}$
norman	$\frac{-\frac{4d\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^{2}+16d^{2}}+\frac{28d\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{b^{2}+16d^{2}}-\frac{28d\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{b^{2}+16d^{2}}+\frac{4d\,e^{xb+a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{b^{2}+16d^{2}}+\frac{2d^{2}e^{xb+a}}{\left(b^{2}+16d^{2}\right)b}+\frac{2d^{2}e^{xb+a}\tan\left(\right.}{\left(b^{2}+16d^{2}\right.}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}}$

[In] int(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] -1/8*exp(b*x+a)*(4*b*d*sin(4*d*x+4*c)+b^2*cos(4*d*x+4*c)-b^2-16*d^2)/(b^2+1
6*d^2)/b

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx = -\frac{2\left(2\,bd\cos\left(dx+c\right)^{3}-bd\cos\left(dx+c\right)\right)e^{(bx+a)}\sin\left(dx+c\right)+\left(b^{2}\cos\left(dx+c\right)^{4}-b^{2}\cos\left(dx+c\right)^{2}-2\,d^{2}\right)e}{b^{3}+16\,bd^{2}}$$

[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x, algorithm="fricas")

[Out] -(2*(2*b*d*cos(d*x + c)^3 - b*d*cos(d*x + c))*e^(b*x + a)*sin(d*x + c) + (b
^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*d^2)*e^(b*x + a))/(b^3 + 16*b*d^
2)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.58 (sec) , antiderivative size = 850, normalized size of antiderivative = 10.76

$$\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx$$

$$= \begin{cases} xe^a\sin^2(c)\cos^2(c) \\ \left(\frac{x\sin^4(c+dx)}{8}+\frac{x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{x\cos^4(c+dx)}{8}+\frac{\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{\sin(c+dx)\cos^3(c+dx)}{8d}\right)e^a \\ -\frac{xe^ae^{-4idx}\sin^4(c+dx)}{16}+\frac{ixe^ae^{-4idx}\sin^3(c+dx)\cos(c+dx)}{4}+\frac{3xe^ae^{-4idx}\sin^2(c+dx)\cos^2(c+dx)}{8}-\frac{ixe^ae^{-4idx}\sin(c+dx)\cos^3(c+dx)}{4} \\ -\frac{xe^ae^{4idx}\sin^4(c+dx)}{16}-\frac{ixe^ae^{4idx}\sin^3(c+dx)\cos(c+dx)}{4}+\frac{3xe^ae^{4idx}\sin^2(c+dx)\cos^2(c+dx)}{8}+\frac{ixe^ae^{4idx}\sin(c+dx)\cos^3(c+dx)}{4}-x \\ \frac{b^2e^ae^{bx}\sin^2(c+dx)\cos^2(c+dx)}{b^3+16bd^2}+\frac{2bde^ae^{bx}\sin^3(c+dx)\cos(c+dx)}{b^3+16bd^2}-\frac{2bde^ae^{bx}\sin(c+dx)\cos^3(c+dx)}{b^3+16bd^2}+\frac{2d^2e^ae^{bx}\sin^4(c+dx)}{b^3+16bd^2}+\frac{4d^2e^ae}{} \end{cases}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**2*sin(d*x+c)**2,x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c +
d*x)**4/8 + x*sin(c + d*x)**2*cos(c + d*x)**2/4 + x*cos(c + d*x)**4/8 + sin
(c + d*x)**3*cos(c + d*x)/(8*d) - sin(c + d*x)*cos(c + d*x)**3/(8*d))*exp(a
), Eq(b, 0)), (-x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**4/16 + I*x*exp(a)*exp(
-4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + 3*x*exp(a)*exp(-4*I*d*x)*sin(c +
 d*x)**2*cos(c + d*x)**2/8 - I*x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c +
d*x)**3/4 - x*exp(a)*exp(-4*I*d*x)*cos(c + d*x)**4/16 + I*exp(a)*exp(-4*I*d
*x)*sin(c + d*x)**4/(24*d) + 5*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos(c +
 d*x)/(48*d) - 5*exp(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) +
 I*exp(a)*exp(-4*I*d*x)*cos(c + d*x)**4/(24*d), Eq(b, -4*I*d)), (-x*exp(a)*
exp(4*I*d*x)*sin(c + d*x)**4/16 - I*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**3*c
os(c + d*x)/4 + 3*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/8 +
 I*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - x*exp(a)*exp(4*I*
d*x)*cos(c + d*x)**4/16 - I*exp(a)*exp(4*I*d*x)*sin(c + d*x)**4/(24*d) + 5*
exp(a)*exp(4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) - 5*exp(a)*exp(4*I*
d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) - I*exp(a)*exp(4*I*d*x)*cos(c + d*
x)**4/(24*d), Eq(b, 4*I*d)), (b**2*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c +
d*x)**2/(b**3 + 16*b*d**2) + 2*b*d*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c +
d*x)/(b**3 + 16*b*d**2) - 2*b*d*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**
3/(b**3 + 16*b*d**2) + 2*d**2*exp(a)*exp(b*x)*sin(c + d*x)**4/(b**3 + 16*b*
d**2) + 4*d**2*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**2/(b**3 + 16*b
*d**2) + 2*d**2*exp(a)*exp(b*x)*cos(c + d*x)**4/(b**3 + 16*b*d**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(70) = 140.

Time = 0.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.99

$$\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx = -\frac{(b^2\cos(4c)\,e^a + 4\,bde^a\sin(4c))\cos(4\,dx)\,e^{(bx)} + (b^2\cos(4c)\,e^a - 4\,bde^a\sin(4c))\cos(4\,dx+8\,c)\,e^{(bx)} +}{}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*((b^2*cos(4*c)*e^a + 4*b*d*e^a*sin(4*c))*cos(4*d*x)*e^(b*x) + (b^2*co
s(4*c)*e^a - 4*b*d*e^a*sin(4*c))*cos(4*d*x + 8*c)*e^(b*x) + (4*b*d*cos(4*c)
*e^a - b^2*e^a*sin(4*c))*e^(b*x)*sin(4*d*x) + (4*b*d*cos(4*c)*e^a + b^2*e^a
*sin(4*c))*e^(b*x)*sin(4*d*x + 8*c) - 2*(b^2*cos(4*c)^2*e^a + b^2*e^a*sin(4
*c)^2 + 16*(cos(4*c)^2*e^a + e^a*sin(4*c)^2)*d^2)*e^(b*x))/(b^3*cos(4*c)^2
+ b^3*sin(4*c)^2 + 16*(b*cos(4*c)^2 + b*sin(4*c)^2)*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx = -\frac{1}{8}\left(\frac{b\cos(4\,dx+4\,c)}{b^2+16\,d^2} + \frac{4\,d\sin(4\,dx+4\,c)}{b^2+16\,d^2}\right)e^{(bx+a)} + \frac{e^{(bx+a)}}{8\,b}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*(b*cos(4*d*x + 4*c)/(b^2 + 16*d^2) + 4*d*sin(4*d*x + 4*c)/(b^2 + 16*d^
2))*e^(b*x + a) + 1/8*e^(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx = \frac{e^{a+b\,x}\,\left(b^2+16\,d^2-b^2\,\cos(4\,c+4\,d\,x)-4\,b\,d\,\sin(4\,c+4\,d\,x)\right)}{8\,b\,\left(b^2+16\,d^2\right)}$$

```
[In] int(cos(c + d*x)^2*exp(a + b*x)*sin(c + d*x)^2,x)
```

```
[Out] (exp(a + b*x)*(b^2 + 16*d^2 - b^2*cos(4*c + 4*d*x) - 4*b*d*sin(4*c + 4*d*x)
))/(8*b*(b^2 + 16*d^2))
```

3.43 $\int e^{a+bx} \cos^2(c+dx) \sin^3(c+dx)\, dx$

Optimal result

Integrand size = 24, antiderivative size = 183

$$\int e^{a+bx} \cos^2(c+dx) \sin^3(c+dx)\, dx = -\frac{de^{a+bx}\cos(c+dx)}{8(b^2+d^2)} - \frac{3de^{a+bx}\cos(3c+3dx)}{16(b^2+9d^2)} + \frac{5de^{a+bx}\cos(5c+5dx)}{16(b^2+25d^2)} + \frac{be^{a+bx}\sin(c+dx)}{8(b^2+d^2)} + \frac{be^{a+bx}\sin(3c+3dx)}{16(b^2+9d^2)} - \frac{be^{a+bx}\sin(5c+5dx)}{16(b^2+25d^2)}$$

```
[Out] -1/8*d*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*cos(3*d*x+3*c)/(b^
2+9*d^2)+5/16*d*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*b*exp(b*x+a)*sin
(d*x+c)/(b^2+d^2)+1/16*b*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-1/16*b*exp(b
*x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4557, 4517}

$$\int e^{a+bx} \cos^2(c+dx) \sin^3(c+dx)\, dx = \frac{be^{a+bx}\sin(c+dx)}{8(b^2+d^2)} + \frac{be^{a+bx}\sin(3c+3dx)}{16(b^2+9d^2)} - \frac{be^{a+bx}\sin(5c+5dx)}{16(b^2+25d^2)} - \frac{de^{a+bx}\cos(c+dx)}{8(b^2+d^2)} - \frac{3de^{a+bx}\cos(3c+3dx)}{16(b^2+9d^2)} + \frac{5de^{a+bx}\cos(5c+5dx)}{16(b^2+25d^2)}$$

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^3,x]
```

```
[Out] -1/8*(d*E^(a + b*x)*Cos[c + d*x])/(b^2 + d^2) - (3*d*E^(a + b*x)*Cos[3*c +
3*d*x])/(16*(b^2 + 9*d^2)) + (5*d*E^(a + b*x)*Cos[5*c + 5*d*x])/(16*(b^2 +
25*d^2)) + (b*E^(a + b*x)*Sin[c + d*x])/(8*(b^2 + d^2)) + (b*E^(a + b*x)*Si
n[3*c + 3*d*x])/(16*(b^2 + 9*d^2)) - (b*E^(a + b*x)*Sin[5*c + 5*d*x])/(16*(
b^2 + 25*d^2))
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{8}e^{a+bx}\sin(c+dx)+\frac{1}{16}e^{a+bx}\sin(3c+3dx)-\frac{1}{16}e^{a+bx}\sin(5c+5dx)\right)\,dx\\
&= \frac{1}{16}\int e^{a+bx}\sin(3c+3dx)\,dx-\frac{1}{16}\int e^{a+bx}\sin(5c+5dx)\,dx+\frac{1}{8}\int e^{a+bx}\sin(c+dx)\,dx\\
&= -\frac{de^{a+bx}\cos(c+dx)}{8\left(b^2+d^2\right)}-\frac{3de^{a+bx}\cos(3c+3dx)}{16\left(b^2+9d^2\right)}+\frac{5de^{a+bx}\cos(5c+5dx)}{16\left(b^2+25d^2\right)}\\
&\quad+\frac{be^{a+bx}\sin(c+dx)}{8\left(b^2+d^2\right)}+\frac{be^{a+bx}\sin(3c+3dx)}{16\left(b^2+9d^2\right)}-\frac{be^{a+bx}\sin(5c+5dx)}{16\left(b^2+25d^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.60

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx=\frac{1}{16}e^{a+bx}\left(\frac{2(-d\cos(c+dx)+b\sin(c+dx))}{b^2+d^2}+\frac{-3d\cos(3(c+dx))+b\sin(3(c+dx))}{b^2+9d^2}+\frac{5d\cos(5(c+dx))-b\sin(5(c+dx))}{b^2+25d^2}\right)$$

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*((2*(-(d*Cos[c + d*x]) + b*Sin[c + d*x]))/(b^2 + d^2) + (-3*d*
Cos[3*(c + d*x)] + b*Sin[3*(c + d*x)])/(b^2 + 9*d^2) + (5*d*Cos[5*(c + d*x)
] - b*Sin[5*(c + d*x)])/(b^2 + 25*d^2)))/16
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

method	result
default	$-\frac{d\,\mathrm{e}^{xb+a}\cos(dx+c)}{8(b^2+d^2)}-\frac{3d\,\mathrm{e}^{xb+a}\cos(3dx+3c)}{16(b^2+9d^2)}+\frac{5d\,\mathrm{e}^{xb+a}\cos(5dx+5c)}{16(b^2+25d^2)}+\frac{b\,\mathrm{e}^{xb+a}\sin(dx+c)}{8b^2+8d^2}+\frac{b\,\mathrm{e}^{xb+a}\sin(3dx+3c)}{16b^2+144d^2}-\frac{b}{}$
parallelrisch	$\frac{\mathrm{e}^{xb+a}\left(3\left(-\frac{1}{2}b^4d-13d^3b^2-\frac{25}{2}d^5\right)\cos(3dx+3c)+5\left(\frac{1}{2}b^4d+5d^3b^2+\frac{9}{2}d^5\right)\cos(5dx+5c)+\left(\frac{1}{2}b^5+13d^2b^3+\frac{25}{2}d^4b\right)\sin(3dx+3c)+(b^2+\right.}{8b^6+280b^4d^2+2072b^2d^4+1800d^6}$
risch	$\frac{i\mathrm{e}^{xb+a}\left(i\left(4b^5+136d^2b^3+900d^4b\right)\sin(dx+c)-4id\left(b^4+34b^2d^2+225d^4\right)\cos(dx+c)+10id\left(b^4+10b^2d^2+9d^4\right)\cos(5dx+5c)-i\left(2b^5+2\right.\right.}{32(5id+b)(3id+b)(id+b)(id-b)(3i}$

```
[In] int(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*cos(3*d*x+3*c)/(b^
2+9*d^2)+5/16*d*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*b*exp(b*x+a)*sin
(d*x+c)/(b^2+d^2)+1/16*b*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-1/16*b*exp(b
*x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx$$
$$=\frac{\left(2\,b^3d^2+26\,bd^4-\left(b^5+10\,b^3d^2+9\,bd^4\right)\cos\left(dx+c\right)^4+\left(b^5+14\,b^3d^2+13\,bd^4\right)\cos\left(dx+c\right)^2\right)e^{(bx+a)}\sin\left(dx\right.}{b^6+35}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] ((2*b^3*d^2 + 26*b*d^4 - (b^5 + 10*b^3*d^2 + 9*b*d^4)*cos(d*x + c)^4 + (b^5
 + 14*b^3*d^2 + 13*b*d^4)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c) + (5*(b^
4*d + 10*b^2*d^3 + 9*d^5)*cos(d*x + c)^5 - (7*b^4*d + 82*b^2*d^3 + 75*d^5)*
cos(d*x + c)^3 + 2*(b^4*d + 13*b^2*d^3)*cos(d*x + c))*e^(b*x + a))/(b^6 + 3
5*b^4*d^2 + 259*b^2*d^4 + 225*d^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.82 (sec) , antiderivative size = 2958, normalized size of antiderivative = 16.16

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**2*sin(d*x+c)**3,x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)**3*cos(c)**2, Eq(b, 0) & Eq(d, 0)), (-x*exp(a)*e
xp(-5*I*d*x)*sin(c + d*x)**5/32 + 5*I*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**
4*cos(c + d*x)/32 + 5*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**
2/16 - 5*I*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/16 - 5*x*
exp(a)*exp(-5*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/32 + I*x*exp(a)*exp(-5*I*
d*x)*cos(c + d*x)**5/32 + 31*I*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**5/(960*d)
 + 25*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(192*d) - I*exp(a)*
exp(-5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(6*d) - exp(a)*exp(-5*I*d*x)*
sin(c + d*x)**2*cos(c + d*x)**3/(3*d) + 41*I*exp(a)*exp(-5*I*d*x)*sin(c + d
*x)*cos(c + d*x)**4/(192*d) + 47*exp(a)*exp(-5*I*d*x)*cos(c + d*x)**5/(960*
d), Eq(b, -5*I*d)), (-x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**5/32 + 3*I*x*exp
(a)*exp(-3*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/32 + x*exp(a)*exp(-3*I*d*x)*
sin(c + d*x)**3*cos(c + d*x)**2/16 + I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*
*2*cos(c + d*x)**3/16 + 3*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)*
*4/32 - I*x*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**5/32 - 7*I*exp(a)*exp(-3*I*d
*x)*sin(c + d*x)**5/(192*d) - 9*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**4*cos(c
+ d*x)/(64*d) + I*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(6*d
) - exp(a)*exp(-3*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) + 25*I*exp(a
)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(64*d) + 23*exp(a)*exp(-3*I*d*
x)*cos(c + d*x)**5/(192*d), Eq(b, -3*I*d)), (x*exp(a)*exp(-I*d*x)*sin(c + d
*x)**5/16 - I*x*exp(a)*exp(-I*d*x)*sin(c + d*x)**4*cos(c + d*x)/16 + x*exp(
a)*exp(-I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/8 - I*x*exp(a)*exp(-I*d*x)*s
in(c + d*x)**2*cos(c + d*x)**3/8 + x*exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c
+ d*x)**4/16 - I*x*exp(a)*exp(-I*d*x)*cos(c + d*x)**5/16 - 5*I*exp(a)*exp(-
I*d*x)*sin(c + d*x)**5/(96*d) + exp(a)*exp(-I*d*x)*sin(c + d*x)**4*cos(c +
d*x)/(96*d) - I*exp(a)*exp(-I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(6*d) -
exp(a)*exp(-I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 7*I*exp(a)*exp(-
I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(96*d) - 13*exp(a)*exp(-I*d*x)*cos(c +
d*x)**5/(96*d), Eq(b, -I*d)), (x*exp(a)*exp(I*d*x)*sin(c + d*x)**5/16 + I*x
*exp(a)*exp(I*d*x)*sin(c + d*x)**4*cos(c + d*x)/16 + x*exp(a)*exp(I*d*x)*si
n(c + d*x)**3*cos(c + d*x)**2/8 + I*x*exp(a)*exp(I*d*x)*sin(c + d*x)**2*cos
(c + d*x)**3/8 + x*exp(a)*exp(I*d*x)*sin(c + d*x)*cos(c + d*x)**4/16 + I*x*
exp(a)*exp(I*d*x)*cos(c + d*x)**5/16 + 5*I*exp(a)*exp(I*d*x)*sin(c + d*x)**
5/(96*d) + exp(a)*exp(I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(96*d) + I*exp(a)
*exp(I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(6*d) - exp(a)*exp(I*d*x)*sin(c
```

```
 + d*x)**2*cos(c + d*x)**3/(3*d) + 7*I*exp(a)*exp(I*d*x)*sin(c + d*x)*cos(c
 + d*x)**4/(96*d) - 13*exp(a)*exp(I*d*x)*cos(c + d*x)**5/(96*d), Eq(b, I*d)
), (-x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**5/32 - 3*I*x*exp(a)*exp(3*I*d*x)*s
in(c + d*x)**4*cos(c + d*x)/32 + x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**3*cos(
c + d*x)**2/16 - I*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/16
 + 3*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/32 + I*x*exp(a)*exp
(3*I*d*x)*cos(c + d*x)**5/32 + 7*I*exp(a)*exp(3*I*d*x)*sin(c + d*x)**5/(192
*d) - 9*exp(a)*exp(3*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(64*d) - I*exp(a)*
exp(3*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(6*d) - exp(a)*exp(3*I*d*x)*si
n(c + d*x)**2*cos(c + d*x)**3/(3*d) - 25*I*exp(a)*exp(3*I*d*x)*sin(c + d*x)
*cos(c + d*x)**4/(64*d) + 23*exp(a)*exp(3*I*d*x)*cos(c + d*x)**5/(192*d), E
q(b, 3*I*d)), (-x*exp(a)*exp(5*I*d*x)*sin(c + d*x)**5/32 - 5*I*x*exp(a)*exp
(5*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/32 + 5*x*exp(a)*exp(5*I*d*x)*sin(c +
 d*x)**3*cos(c + d*x)**2/16 + 5*I*x*exp(a)*exp(5*I*d*x)*sin(c + d*x)**2*cos
(c + d*x)**3/16 - 5*x*exp(a)*exp(5*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/32 -
 I*x*exp(a)*exp(5*I*d*x)*cos(c + d*x)**5/32 - 31*I*exp(a)*exp(5*I*d*x)*sin(
c + d*x)**5/(960*d) + 25*exp(a)*exp(5*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(
192*d) + I*exp(a)*exp(5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(6*d) - exp(
a)*exp(5*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 41*I*exp(a)*exp(5*I
*d*x)*sin(c + d*x)*cos(c + d*x)**4/(192*d) + 47*exp(a)*exp(5*I*d*x)*cos(c +
 d*x)**5/(960*d), Eq(b, 5*I*d)), (b**5*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(
c + d*x)**2/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 2*b**4*d*exp
(a)*exp(b*x)*sin(c + d*x)**4*cos(c + d*x)/(b**6 + 35*b**4*d**2 + 259*b**2*d
**4 + 225*d**6) - 3*b**4*d*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**3/
(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 2*b**3*d**2*exp(a)*exp(b
*x)*sin(c + d*x)**5/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 18*b
**3*d**2*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)**2/(b**6 + 35*b**4*d*
*2 + 259*b**2*d**4 + 225*d**6) + 6*b**3*d**2*exp(a)*exp(b*x)*sin(c + d*x)*c
os(c + d*x)**4/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 26*b**2*d
**3*exp(a)*exp(b*x)*sin(c + d*x)**4*cos(c + d*x)/(b**6 + 35*b**4*d**2 + 259
*b**2*d**4 + 225*d**6) - 30*b**2*d**3*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c
 + d*x)**3/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) - 6*b**2*d**3*e
xp(a)*exp(b*x)*cos(c + d*x)**5/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d
**6) + 26*b*d**4*exp(a)*exp(b*x)*sin(c + d*x)**5/(b**6 + 35*b**4*d**2 + 259
*b**2*d**4 + 225*d**6) + 65*b*d**4*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c +
d*x)**2/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 30*b*d**4*exp(a)
*exp(b*x)*sin(c + d*x)*cos(c + d*x)**4/(b**6 + 35*b**4*d**2 + 259*b**2*d**4
 + 225*d**6) - 75*d**5*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**3/(b**
6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) - 30*d**5*exp(a)*exp(b*x)*cos(
c + d*x)**5/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. 2(165) = 330.

Time = 0.27 (sec) , antiderivative size = 1148, normalized size of antiderivative = 6.27

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*((5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos(5*c)*e^a + 45*d^5*cos(5*c)*e^a
 - b^5*e^a*sin(5*c) - 10*b^3*d^2*e^a*sin(5*c) - 9*b*d^4*e^a*sin(5*c))*cos(5
*d*x)*e^(b*x) + (5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos(5*c)*e^a + 45*d^5*co
s(5*c)*e^a + b^5*e^a*sin(5*c) + 10*b^3*d^2*e^a*sin(5*c) + 9*b*d^4*e^a*sin(5
*c))*cos(5*d*x + 10*c)*e^(b*x) - (3*b^4*d*cos(5*c)*e^a + 78*b^2*d^3*cos(5*c
)*e^a + 75*d^5*cos(5*c)*e^a + b^5*e^a*sin(5*c) + 26*b^3*d^2*e^a*sin(5*c) +
25*b*d^4*e^a*sin(5*c))*cos(3*d*x + 8*c)*e^(b*x) - (3*b^4*d*cos(5*c)*e^a + 7
8*b^2*d^3*cos(5*c)*e^a + 75*d^5*cos(5*c)*e^a - b^5*e^a*sin(5*c) - 26*b^3*d^
2*e^a*sin(5*c) - 25*b*d^4*e^a*sin(5*c))*cos(3*d*x - 2*c)*e^(b*x) - 2*(b^4*d
*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e^a + 225*d^5*cos(5*c)*e^a + b^5*e^a*si
n(5*c) + 34*b^3*d^2*e^a*sin(5*c) + 225*b*d^4*e^a*sin(5*c))*cos(d*x + 6*c)*e
^(b*x) - 2*(b^4*d*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e^a + 225*d^5*cos(5*c)
*e^a - b^5*e^a*sin(5*c) - 34*b^3*d^2*e^a*sin(5*c) - 225*b*d^4*e^a*sin(5*c))
*cos(d*x - 4*c)*e^(b*x) - (b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)*e^a + 9*b
*d^4*cos(5*c)*e^a + 5*b^4*d*e^a*sin(5*c) + 50*b^2*d^3*e^a*sin(5*c) + 45*d^5
*e^a*sin(5*c))*e^(b*x)*sin(5*d*x) - (b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)
*e^a + 9*b*d^4*cos(5*c)*e^a - 5*b^4*d*e^a*sin(5*c) - 50*b^2*d^3*e^a*sin(5*c
) - 45*d^5*e^a*sin(5*c))*e^(b*x)*sin(5*d*x + 10*c) + (b^5*cos(5*c)*e^a + 26
*b^3*d^2*cos(5*c)*e^a + 25*b*d^4*cos(5*c)*e^a - 3*b^4*d*e^a*sin(5*c) - 78*b
^2*d^3*e^a*sin(5*c) - 75*d^5*e^a*sin(5*c))*e^(b*x)*sin(3*d*x + 8*c) + (b^5*
cos(5*c)*e^a + 26*b^3*d^2*cos(5*c)*e^a + 25*b*d^4*cos(5*c)*e^a + 3*b^4*d*e^
a*sin(5*c) + 78*b^2*d^3*e^a*sin(5*c) + 75*d^5*e^a*sin(5*c))*e^(b*x)*sin(3*d
*x - 2*c) + 2*(b^5*cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a + 225*b*d^4*cos(5
*c)*e^a - b^4*d*e^a*sin(5*c) - 34*b^2*d^3*e^a*sin(5*c) - 225*d^5*e^a*sin(5*
c))*e^(b*x)*sin(d*x + 6*c) + 2*(b^5*cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a
+ 225*b*d^4*cos(5*c)*e^a + b^4*d*e^a*sin(5*c) + 34*b^2*d^3*e^a*sin(5*c) + 2
25*d^5*e^a*sin(5*c))*e^(b*x)*sin(d*x - 4*c))/(b^6*cos(5*c)^2 + b^6*sin(5*c)
^2 + 225*(cos(5*c)^2 + sin(5*c)^2)*d^6 + 259*(b^2*cos(5*c)^2 + b^2*sin(5*c)
^2)*d^4 + 35*(b^4*cos(5*c)^2 + b^4*sin(5*c)^2)*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.85

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx = \frac{1}{16}\left(\frac{5\,d\cos\left(5\,dx+5\,c\right)}{b^2+25\,d^2}-\frac{b\sin\left(5\,dx+5\,c\right)}{b^2+25\,d^2}\right)e^{(bx+a)} - \frac{1}{16}\left(\frac{3\,d\cos\left(3\,dx+3\,c\right)}{b^2+9\,d^2}-\frac{b\sin\left(3\,dx+3\,c\right)}{b^2+9\,d^2}\right)e^{(bx+a)} - \frac{1}{8}\left(\frac{d\cos\left(dx+c\right)}{b^2+d^2}-\frac{b\sin\left(dx+c\right)}{b^2+d^2}\right)e^{(bx+a)}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*d*cos(5*d*x + 5*c)/(b^2 + 25*d^2) - b*sin(5*d*x + 5*c)/(b^2 + 25*d^
2))*e^(b*x + a) - 1/16*(3*d*cos(3*d*x + 3*c)/(b^2 + 9*d^2) - b*sin(3*d*x +
3*c)/(b^2 + 9*d^2))*e^(b*x + a) - 1/8*(d*cos(d*x + c)/(b^2 + d^2) - b*sin(d
*x + c)/(b^2 + d^2))*e^(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 28.58 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.39

$$\begin{aligned}&\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx\\&= -\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1\mathrm{i}\right)}{16\,\left(d+b\,1\mathrm{i}\right)}\\&\quad-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(d\,x\right)+\sin\left(d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(c\right)+\sin\left(c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,\left(b+d\,1\mathrm{i}\right)}\\&\quad-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)-\sin\left(3\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,c\right)-\sin\left(3\,c\right)\,1\mathrm{i}\right)}{32\,\left(3\,d+b\,1\mathrm{i}\right)}\\&\quad+\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(5\,d\,x\right)-\sin\left(5\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(5\,c\right)-\sin\left(5\,c\right)\,1\mathrm{i}\right)}{32\,\left(5\,d+b\,1\mathrm{i}\right)}\\&\quad-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)+\sin\left(3\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,c\right)+\sin\left(3\,c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{32\,\left(b+d\,3\mathrm{i}\right)}\\&\quad+\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(5\,d\,x\right)+\sin\left(5\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(5\,c\right)+\sin\left(5\,c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{32\,\left(b+d\,5\mathrm{i}\right)}\end{aligned}$$

```
[In] int(cos(c + d*x)^2*exp(a + b*x)*sin(c + d*x)^3,x)
```

```
[Out] (exp(a + b*x)*(cos(5*d*x) - sin(5*d*x)*1i)*(cos(5*c) - sin(5*c)*1i))/(32*(b
*1i + 5*d)) - (exp(a + b*x)*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*1
i)/(16*(b + d*1i)) - (exp(a + b*x)*(cos(3*d*x) - sin(3*d*x)*1i)*(cos(3*c) -
```

```
 sin(3*c)*1i))/(32*(b*1i + 3*d)) - (exp(a + b*x)*(cos(d*x) - sin(d*x)*1i)*(
cos(c) - sin(c)*1i))/(16*(b*1i + d)) - (exp(a + b*x)*(cos(3*d*x) + sin(3*d*
x)*1i)*(cos(3*c) + sin(3*c)*1i)*1i)/(32*(b + d*3i)) + (exp(a + b*x)*(cos(5*
d*x) + sin(5*d*x)*1i)*(cos(5*c) + sin(5*c)*1i)*1i)/(32*(b + d*5i))
```

3.44 $\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 129

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx=-\frac{de^{a+bx}\cos(2c+2dx)}{2(b^2+4d^2)}-\frac{de^{a+bx}\cos(4c+4dx)}{2(b^2+16d^2)}+\frac{be^{a+bx}\sin(2c+2dx)}{4(b^2+4d^2)}+\frac{be^{a+bx}\sin(4c+4dx)}{8(b^2+16d^2)}$$

```
[Out] -1/2*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)-1/2*d*exp(b*x+a)*cos(4*d*x+4*c
)/(b^2+16*d^2)+1/4*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)+1/8*b*exp(b*x+a)
*sin(4*d*x+4*c)/(b^2+16*d^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4557, 4517}

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx=\frac{be^{a+bx}\sin(2c+2dx)}{4(b^2+4d^2)}+\frac{be^{a+bx}\sin(4c+4dx)}{8(b^2+16d^2)}-\frac{de^{a+bx}\cos(2c+2dx)}{2(b^2+4d^2)}-\frac{de^{a+bx}\cos(4c+4dx)}{2(b^2+16d^2)}$$

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x],x]
```

```
[Out] -1/2*(d*E^(a + b*x)*Cos[2*c + 2*d*x])/(b^2 + 4*d^2) - (d*E^(a + b*x)*Cos[4*
c + 4*d*x])/(2*(b^2 + 16*d^2)) + (b*E^(a + b*x)*Sin[2*c + 2*d*x])/(4*(b^2 +
 4*d^2)) + (b*E^(a + b*x)*Sin[4*c + 4*d*x])/(8*(b^2 + 16*d^2))
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{4}e^{a+bx}\sin(2c+2dx) + \frac{1}{8}e^{a+bx}\sin(4c+4dx)\right) dx$$

$$= \frac{1}{8}\int e^{a+bx}\sin(4c+4dx)\,dx + \frac{1}{4}\int e^{a+bx}\sin(2c+2dx)\,dx$$

$$= -\frac{de^{a+bx}\cos(2c+2dx)}{2(b^2+4d^2)} - \frac{de^{a+bx}\cos(4c+4dx)}{2(b^2+16d^2)} + \frac{be^{a+bx}\sin(2c+2dx)}{4(b^2+4d^2)} + \frac{be^{a+bx}\sin(4c+4dx)}{8(b^2+16d^2)}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.63

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx = \frac{1}{8}e^{a+bx}\left(\frac{2(-2d\cos(2(c+dx))+b\sin(2(c+dx)))}{b^2+4d^2} + \frac{-4d\cos(4(c+dx))+b\sin(4(c+dx))}{b^2+16d^2}\right)$$

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x],x]
[Out] (E^(a + b*x)*((2*(-2*d*Cos[2*(c + d*x)] + b*Sin[2*(c + d*x)]))/(b^2 + 4*d^2
) + (-4*d*Cos[4*(c + d*x)] + b*Sin[4*(c + d*x)])/(b^2 + 16*d^2)))/8
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{\left(\left(b^{3}+4 b\,d^{2}\right) \sin \left(4 d x +4 c \right)+\left(-4 b^{2} d -16 d^{3}\right) \cos \left(4 d x +4 c \right)+2 \left(b^{2}+16 d^{2}\right) \left(b \sin \left(2 d x +2 c \right)-2 d \cos \left(2 d x +2 c \right)\right)\right) {\mathrm e}^{x b +a}}{8 b^{4}+160 b^{2} d^{2}+512 d^{4}}$
default	$-\frac{d\,{\mathrm e}^{x b +a} \cos \left(2 d x +2 c \right)}{2 \left(b^{2}+4 d^{2}\right)}-\frac{d\,{\mathrm e}^{x b +a} \cos \left(4 d x +4 c \right)}{2 \left(b^{2}+16 d^{2}\right)}+\frac{b\,{\mathrm e}^{x b +a} \sin \left(2 d x +2 c \right)}{4 b^{2}+16 d^{2}}+\frac{b\,{\mathrm e}^{x b +a} \sin \left(4 d x +4 c \right)}{8 b^{2}+128 d^{2}}$
risch	$-\frac{i {\mathrm e}^{x b +a} \left(-8 i d \left(b^{2}+4 d^{2}\right) \cos \left(4 d x +4 c \right)-i \left(-2 b^{3}-8 b\,d^{2}\right) \sin \left(4 d x +4 c \right)-8 i d \left(b^{2}+16 d^{2}\right) \cos \left(2 d x +2 c \right)-i \left(-4 b^{3}-64 b\,d^{2}\right) \sin \left(2 d x +2 c \right)\right.}{16 \left(4 i d +b \right) \left(2 i d +b \right) \left(2 i d -b \right) \left(4 i d -b \right)}$
norman	$-\frac{d \left(b^{2}+10 d^{2}\right) {\mathrm e}^{x b +a}}{b^{4}+20 b^{2} d^{2}+64 d^{4}}-\frac{6 b \left(b^{2}+2 d^{2}\right) {\mathrm e}^{x b +a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{b^{4}+20 b^{2} d^{2}+64 d^{4}}+\frac{6 b \left(b^{2}+2 d^{2}\right) {\mathrm e}^{x b +a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{b^{4}+20 b^{2} d^{2}+64 d^{4}}+\frac{2 b \left(b^{2}+10 d^{2}\right) {\mathrm e}^{x b +a} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{b^{4}+20 b^{2} d^{2}+64 d^{4}}-\frac{2 b \left(b^{2}+10 d\right.}{b^{4}}$

```
[In] int(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] ((b^3+4*b*d^2)*sin(4*d*x+4*c)+(-4*b^2*d-16*d^3)*cos(4*d*x+4*c)+2*(b^2+16*d^
2)*(b*sin(2*d*x+2*c)-2*d*cos(2*d*x+2*c)))*exp(b*x+a)/(8*b^4+160*b^2*d^2+512
*d^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx$$
$$= \frac{\left(6\,bd^2\cos\left(dx+c\right)+\left(b^3+4\,bd^2\right)\cos\left(dx+c\right)^3\right)e^{(bx+a)}\sin\left(dx+c\right)+\left(3\,b^2d\cos\left(dx+c\right)^2-4\left(b^2d+4\,d^3\right)\cos\right.}{b^4+20\,b^2d^2+64\,d^4}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] ((6*b*d^2*cos(d*x + c) + (b^3 + 4*b*d^2)*cos(d*x + c)^3)*e^(b*x + a)*sin(d*
x + c) + (3*b^2*d*cos(d*x + c)^2 - 4*(b^2*d + 4*d^3)*cos(d*x + c)^4 + 6*d^3
)*e^(b*x + a))/(b^4 + 20*b^2*d^2 + 64*d^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.38 (sec) , antiderivative size = 1357, normalized size of antiderivative = 10.52

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**3*sin(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-I*x*exp(a)*ex
p(-4*I*d*x)*sin(c + d*x)**4/16 - x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos
(c + d*x)/4 + 3*I*x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/8
+ x*exp(a)*exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - I*x*exp(a)*exp(-4
*I*d*x)*cos(c + d*x)**4/16 - exp(a)*exp(-4*I*d*x)*sin(c + d*x)**4/(24*d) +
5*I*exp(a)*exp(-4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) + 11*I*exp(a)*
exp(-4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) + exp(a)*exp(-4*I*d*x)*co
s(c + d*x)**4/(24*d), Eq(b, -4*I*d)), (I*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x
)**4/8 + x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + x*exp(a)*e
xp(-2*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 - I*x*exp(a)*exp(-2*I*d*x)*cos(
c + d*x)**4/8 + exp(a)*exp(-2*I*d*x)*sin(c + d*x)**4/(16*d) + exp(a)*exp(-2
*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**2/(4*d) - I*exp(a)*exp(-2*I*d*x)*sin(
c + d*x)*cos(c + d*x)**3/(6*d) - 7*exp(a)*exp(-2*I*d*x)*cos(c + d*x)**4/(48
*d), Eq(b, -2*I*d)), (-I*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**4/8 + x*exp(a)
*exp(2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/4 + x*exp(a)*exp(2*I*d*x)*sin(c
+ d*x)*cos(c + d*x)**3/4 + I*x*exp(a)*exp(2*I*d*x)*cos(c + d*x)**4/8 + exp(
a)*exp(2*I*d*x)*sin(c + d*x)**4/(16*d) + exp(a)*exp(2*I*d*x)*sin(c + d*x)**
2*cos(c + d*x)**2/(4*d) + I*exp(a)*exp(2*I*d*x)*sin(c + d*x)*cos(c + d*x)**
3/(6*d) - 7*exp(a)*exp(2*I*d*x)*cos(c + d*x)**4/(48*d), Eq(b, 2*I*d)), (I*x
*exp(a)*exp(4*I*d*x)*sin(c + d*x)**4/16 - x*exp(a)*exp(4*I*d*x)*sin(c + d*x
)**3*cos(c + d*x)/4 - 3*I*x*exp(a)*exp(4*I*d*x)*sin(c + d*x)**2*cos(c + d*x
)**2/8 + x*exp(a)*exp(4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/4 + I*x*exp(a)*
exp(4*I*d*x)*cos(c + d*x)**4/16 - exp(a)*exp(4*I*d*x)*sin(c + d*x)**4/(24*d
) - 5*I*exp(a)*exp(4*I*d*x)*sin(c + d*x)**3*cos(c + d*x)/(48*d) - 11*I*exp(
a)*exp(4*I*d*x)*sin(c + d*x)*cos(c + d*x)**3/(48*d) + exp(a)*exp(4*I*d*x)*c
os(c + d*x)**4/(24*d), Eq(b, 4*I*d)), (b**3*exp(a)*exp(b*x)*sin(c + d*x)*co
s(c + d*x)**3/(b**4 + 20*b**2*d**2 + 64*d**4) + 3*b**2*d*exp(a)*exp(b*x)*si
n(c + d*x)**2*cos(c + d*x)**2/(b**4 + 20*b**2*d**2 + 64*d**4) - b**2*d*exp(
a)*exp(b*x)*cos(c + d*x)**4/(b**4 + 20*b**2*d**2 + 64*d**4) + 6*b*d**2*exp(
a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)/(b**4 + 20*b**2*d**2 + 64*d**4) +
10*b*d**2*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**3/(b**4 + 20*b**2*d**2
 + 64*d**4) + 6*d**3*exp(a)*exp(b*x)*sin(c + d*x)**4/(b**4 + 20*b**2*d**2 +
 64*d**4) + 12*d**3*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**2/(b**4 +
 20*b**2*d**2 + 64*d**4) - 10*d**3*exp(a)*exp(b*x)*cos(c + d*x)**4/(b**4 +
20*b**2*d**2 + 64*d**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(117) = 234.

Time = 0.23 (sec) , antiderivative size = 550, normalized size of antiderivative = 4.26

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx = -\frac{(4\,b^2d\cos(4\,c)\,e^a + 16\,d^3\cos(4\,c)\,e^a - b^3e^a\sin(4\,c) - 4\,bd^2e^a\sin(4\,c))\cos(4\,dx)\,e^{(bx)} + (4\,b^2d\cos(4\,c)\,e^a}{}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*((4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 4*b
*d^2*e^a*sin(4*c))*cos(4*d*x)*e^(b*x) + (4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(
4*c)*e^a + b^3*e^a*sin(4*c) + 4*b*d^2*e^a*sin(4*c))*cos(4*d*x + 8*c)*e^(b*x
) + 2*(2*b^2*d*cos(4*c)*e^a + 32*d^3*cos(4*c)*e^a + b^3*e^a*sin(4*c) + 16*b
*d^2*e^a*sin(4*c))*cos(2*d*x + 6*c)*e^(b*x) + 2*(2*b^2*d*cos(4*c)*e^a + 32*
d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 16*b*d^2*e^a*sin(4*c))*cos(2*d*x - 2*
c)*e^(b*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*cos(4*c)*e^a + 4*b^2*d*e^a*sin(4*c
) + 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*c
os(4*c)*e^a - 4*b^2*d*e^a*sin(4*c) - 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x
 + 8*c) - 2*(b^3*cos(4*c)*e^a + 16*b*d^2*cos(4*c)*e^a - 2*b^2*d*e^a*sin(4*c
) - 32*d^3*e^a*sin(4*c))*e^(b*x)*sin(2*d*x + 6*c) - 2*(b^3*cos(4*c)*e^a + 1
6*b*d^2*cos(4*c)*e^a + 2*b^2*d*e^a*sin(4*c) + 32*d^3*e^a*sin(4*c))*e^(b*x)*
sin(2*d*x - 2*c))/(b^4*cos(4*c)^2 + b^4*sin(4*c)^2 + 64*(cos(4*c)^2 + sin(4
*c)^2)*d^4 + 20*(b^2*cos(4*c)^2 + b^2*sin(4*c)^2)*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx = -\frac{1}{8}\left(\frac{4\,d\cos(4\,dx+4\,c)}{b^2+16\,d^2} - \frac{b\sin(4\,dx+4\,c)}{b^2+16\,d^2}\right)e^{(bx+a)} - \frac{1}{4}\left(\frac{2\,d\cos(2\,dx+2\,c)}{b^2+4\,d^2} - \frac{b\sin(2\,dx+2\,c)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(4*d*cos(4*d*x + 4*c)/(b^2 + 16*d^2) - b*sin(4*d*x + 4*c)/(b^2 + 16*d^
2))*e^(b*x + a) - 1/4*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x + 2
*c)/(b^2 + 4*d^2))*e^(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.39

$$\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx = -\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(2\,d\,x\right)-\sin\left(2\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(2\,c\right)-\sin\left(2\,c\right)\,1\mathrm{i}\right)}{8\,\left(2\,d+b\,1\mathrm{i}\right)} - \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(4\,d\,x\right)-\sin\left(4\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(4\,c\right)-\sin\left(4\,c\right)\,1\mathrm{i}\right)}{16\,\left(4\,d+b\,1\mathrm{i}\right)} - \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(2\,d\,x\right)+\sin\left(2\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(2\,c\right)+\sin\left(2\,c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,\left(b+d\,2\mathrm{i}\right)} - \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(4\,d\,x\right)+\sin\left(4\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(4\,c\right)+\sin\left(4\,c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,\left(b+d\,4\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^3*exp(a + b*x)*sin(c + d*x),x)
```

```
[Out] - (exp(a + b*x)*(cos(2*d*x) - sin(2*d*x)*1i)*(cos(2*c) - sin(2*c)*1i))/(8*(
b*1i + 2*d)) - (exp(a + b*x)*(cos(4*d*x) - sin(4*d*x)*1i)*(cos(4*c) - sin(4
*c)*1i))/(16*(b*1i + 4*d)) - (exp(a + b*x)*(cos(2*d*x) + sin(2*d*x)*1i)*(co
s(2*c) + sin(2*c)*1i)*1i)/(8*(b + d*2i)) - (exp(a + b*x)*(cos(4*d*x) + sin(
4*d*x)*1i)*(cos(4*c) + sin(4*c)*1i)*1i)/(16*(b + d*4i))
```

3.45 $\int e^{a+bx} \cos^3(c+dx) \sin^2(c+dx) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 183

$$\int e^{a+bx} \cos^3(c+dx) \sin^2(c+dx) \, dx = \frac{be^{a+bx} \cos(c+dx)}{8(b^2+d^2)} - \frac{be^{a+bx} \cos(3c+3dx)}{16(b^2+9d^2)} - \frac{be^{a+bx} \cos(5c+5dx)}{16(b^2+25d^2)} + \frac{de^{a+bx} \sin(c+dx)}{8(b^2+d^2)} - \frac{3de^{a+bx} \sin(3c+3dx)}{16(b^2+9d^2)} - \frac{5de^{a+bx} \sin(5c+5dx)}{16(b^2+25d^2)}$$

```
[Out] 1/8*b*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-1/16*b*exp(b*x+a)*cos(3*d*x+3*c)/(b^2
+9*d^2)-1/16*b*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*d*exp(b*x+a)*sin(
d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-5/16*d*exp(b*
x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4557, 4518}

$$\int e^{a+bx} \cos^3(c+dx) \sin^2(c+dx) \, dx = \frac{de^{a+bx} \sin(c+dx)}{8(b^2+d^2)} - \frac{3de^{a+bx} \sin(3c+3dx)}{16(b^2+9d^2)} - \frac{5de^{a+bx} \sin(5c+5dx)}{16(b^2+25d^2)} + \frac{be^{a+bx} \cos(c+dx)}{8(b^2+d^2)} - \frac{be^{a+bx} \cos(3c+3dx)}{16(b^2+9d^2)} - \frac{be^{a+bx} \cos(5c+5dx)}{16(b^2+25d^2)}$$

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^2,x]
```

```
[Out] (b*E^(a + b*x)*Cos[c + d*x])/(8*(b^2 + d^2)) - (b*E^(a + b*x)*Cos[3*c + 3*d
*x])/(16*(b^2 + 9*d^2)) - (b*E^(a + b*x)*Cos[5*c + 5*d*x])/(16*(b^2 + 25*d^
2)) + (d*E^(a + b*x)*Sin[c + d*x])/(8*(b^2 + d^2)) - (3*d*E^(a + b*x)*Sin[3
*c + 3*d*x])/(16*(b^2 + 9*d^2)) - (5*d*E^(a + b*x)*Sin[5*c + 5*d*x])/(16*(b
^2 + 25*d^2))
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{8}e^{a+bx}\cos(c+dx)-\frac{1}{16}e^{a+bx}\cos(3c+3dx)-\frac{1}{16}e^{a+bx}\cos(5c+5dx)\right)\,dx \\
&= -\left(\frac{1}{16}\int e^{a+bx}\cos(3c+3dx)\,dx\right)-\frac{1}{16}\int e^{a+bx}\cos(5c+5dx)\,dx+\frac{1}{8}\int e^{a+bx}\cos(c+dx)\,dx \\
&= \frac{be^{a+bx}\cos(c+dx)}{8\,(b^2+d^2)}-\frac{be^{a+bx}\cos(3c+3dx)}{16\,(b^2+9d^2)}-\frac{be^{a+bx}\cos(5c+5dx)}{16\,(b^2+25d^2)} \\
&\quad +\frac{de^{a+bx}\sin(c+dx)}{8\,(b^2+d^2)}-\frac{3de^{a+bx}\sin(3c+3dx)}{16\,(b^2+9d^2)}-\frac{5de^{a+bx}\sin(5c+5dx)}{16\,(b^2+25d^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.60

$$
\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx=\frac{1}{16}e^{a+bx}\left(\frac{2(b\cos(c+dx)+d\sin(c+dx))}{b^2+d^2}-\frac{b\cos(3(c+dx))+3d\sin(3(c+dx))}{b^2+9d^2}-\frac{b\cos(5(c+dx))+5d\sin(5(c+dx))}{b^2+25d^2}\right)
$$

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^2,x]
```

```
[Out] (E^(a + b*x)*((2*(b*Cos[c + d*x] + d*Sin[c + d*x]))/(b^2 + d^2) - (b*Cos[3*
(c + d*x)] + 3*d*Sin[3*(c + d*x)])/(b^2 + 9*d^2) - (b*Cos[5*(c + d*x)] + 5*
d*Sin[5*(c + d*x)])/(b^2 + 25*d^2)))/16
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

method	result
default	$\frac{b\,\mathrm{e}^{xb+a}\cos(dx+c)}{8b^2+8d^2}-\frac{b\,\mathrm{e}^{xb+a}\cos(3dx+3c)}{16(b^2+9d^2)}-\frac{b\,\mathrm{e}^{xb+a}\cos(5dx+5c)}{16(b^2+25d^2)}+\frac{d\,\mathrm{e}^{xb+a}\sin(dx+c)}{8b^2+8d^2}-\frac{3d\,\mathrm{e}^{xb+a}\sin(3dx+3c)}{16(b^2+9d^2)}-\frac{5d\,\mathrm{e}^{x}}{1}$
parallelrisch	$\frac{\mathrm{e}^{xb+a}\left(\left(-\frac{1}{2}b^5-13d^2b^3-\frac{25}{2}d^4b\right)\cos(3dx+3c)+\left(-\frac{1}{2}b^5-5d^2b^3-\frac{9}{2}d^4b\right)\cos(5dx+5c)+3\left(-\frac{1}{2}b^4d-13d^3b^2-\frac{25}{2}d^5\right)\sin(3dx+3c)+(b\right.}{8b^6+280b^4d^2+2072b^2d^4+1800d^6}$
risch	$-\frac{\mathrm{e}^{xb+a}\left(4d\left(b^4+34b^2d^2+225d^4\right)\sin(dx+c)+\left(4b^5+136d^2b^3+900d^4b\right)\cos(dx+c)+\left(-2b^5-20d^2b^3-18d^4b\right)\cos(5dx+5c)-10d(b^4\right.}{32(5id+b)(3id+b)(id+b)(id-b)(3i}$

```
[In] int(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-1/16*b*exp(b*x+a)*cos(3*d*x+3*c)/(b^2
+9*d^2)-1/16*b*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*d*exp(b*x+a)*sin(
d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-5/16*d*exp(b*
x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.09

$$\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx = \frac{\left(6\,b^2d^3+30\,d^5-5\,(b^4d+10\,b^2d^3+9\,d^5)\cos{(dx+c)}^4+3\,(b^4d+6\,b^2d^3+5\,d^5)\cos{(dx+c)}^2\right)e^{(bx+a)}\sin{(dx}}{b^6+35\,b^4d}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((6*b^2*d^3 + 30*d^5 - 5*(b^4*d + 10*b^2*d^3 + 9*d^5)*cos(d*x + c)^4 + 3*(b
^4*d + 6*b^2*d^3 + 5*d^5)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c) - ((b^5
+ 10*b^3*d^2 + 9*b*d^4)*cos(d*x + c)^5 - (b^5 + 6*b^3*d^2 + 5*b*d^4)*cos(d*
x + c)^3 - 6*(b^3*d^2 + 5*b*d^4)*cos(d*x + c))*e^(b*x + a))/(b^6 + 35*b^4*d
^2 + 259*b^2*d^4 + 225*d^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.94 (sec) , antiderivative size = 2751, normalized size of antiderivative = 15.03

$$\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**3*sin(d*x+c)**2,x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)**2*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-I*x*exp(a)
*exp(-5*I*d*x)*sin(c + d*x)**5/32 - 5*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**
4*cos(c + d*x)/32 + 5*I*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)
**2/16 + 5*x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/16 - 5*I*
x*exp(a)*exp(-5*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/32 - x*exp(a)*exp(-5*I*
d*x)*cos(c + d*x)**5/32 - 31*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**5/(960*d) +
 25*I*exp(a)*exp(-5*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(192*d) + exp(a)*ex
p(-5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(6*d) - 3*exp(a)*exp(-5*I*d*x)*
sin(c + d*x)*cos(c + d*x)**4/(64*d) + I*exp(a)*exp(-5*I*d*x)*cos(c + d*x)**
5/(64*d), Eq(b, -5*I*d)), (I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**5/32 + 3*
x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/32 - I*x*exp(a)*exp(-3*
I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/16 + x*exp(a)*exp(-3*I*d*x)*sin(c +
d*x)**2*cos(c + d*x)**3/16 - 3*I*x*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c
+ d*x)**4/32 - x*exp(a)*exp(-3*I*d*x)*cos(c + d*x)**5/32 - 7*exp(a)*exp(-3*
I*d*x)*sin(c + d*x)**5/(192*d) + 9*I*exp(a)*exp(-3*I*d*x)*sin(c + d*x)**4*c
os(c + d*x)/(64*d) + exp(a)*exp(-3*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(
6*d) - 7*exp(a)*exp(-3*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(64*d) + 3*I*exp
(a)*exp(-3*I*d*x)*cos(c + d*x)**5/(64*d), Eq(b, -3*I*d)), (I*x*exp(a)*exp(-
I*d*x)*sin(c + d*x)**5/16 + x*exp(a)*exp(-I*d*x)*sin(c + d*x)**4*cos(c + d*
x)/16 + I*x*exp(a)*exp(-I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/8 + x*exp(a)
*exp(-I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/8 + I*x*exp(a)*exp(-I*d*x)*sin
(c + d*x)*cos(c + d*x)**4/16 + x*exp(a)*exp(-I*d*x)*cos(c + d*x)**5/16 + 5*
exp(a)*exp(-I*d*x)*sin(c + d*x)**5/(96*d) + I*exp(a)*exp(-I*d*x)*sin(c + d*
x)**4*cos(c + d*x)/(96*d) + exp(a)*exp(-I*d*x)*sin(c + d*x)**3*cos(c + d*x)
**2/(6*d) - 3*exp(a)*exp(-I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(32*d) + I*ex
p(a)*exp(-I*d*x)*cos(c + d*x)**5/(32*d), Eq(b, -I*d)), (-I*x*exp(a)*exp(I*d
*x)*sin(c + d*x)**5/16 + x*exp(a)*exp(I*d*x)*sin(c + d*x)**4*cos(c + d*x)/1
6 - I*x*exp(a)*exp(I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/8 + x*exp(a)*exp(
I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/8 - I*x*exp(a)*exp(I*d*x)*sin(c + d*
x)*cos(c + d*x)**4/16 + x*exp(a)*exp(I*d*x)*cos(c + d*x)**5/16 + 5*exp(a)*e
xp(I*d*x)*sin(c + d*x)**5/(96*d) - I*exp(a)*exp(I*d*x)*sin(c + d*x)**4*cos(
c + d*x)/(96*d) + exp(a)*exp(I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/(6*d) -
 3*exp(a)*exp(I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(32*d) - I*exp(a)*exp(I*d
*x)*cos(c + d*x)**5/(32*d), Eq(b, I*d)), (-I*x*exp(a)*exp(3*I*d*x)*sin(c +
d*x)**5/32 + 3*x*exp(a)*exp(3*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/32 + I*x*
```

```
exp(a)*exp(3*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/16 + x*exp(a)*exp(3*I*d
*x)*sin(c + d*x)**2*cos(c + d*x)**3/16 + 3*I*x*exp(a)*exp(3*I*d*x)*sin(c +
d*x)*cos(c + d*x)**4/32 - x*exp(a)*exp(3*I*d*x)*cos(c + d*x)**5/32 - 7*exp(
a)*exp(3*I*d*x)*sin(c + d*x)**5/(192*d) - 9*I*exp(a)*exp(3*I*d*x)*sin(c + d
*x)**4*cos(c + d*x)/(64*d) + exp(a)*exp(3*I*d*x)*sin(c + d*x)**3*cos(c + d*
x)**2/(6*d) - 7*exp(a)*exp(3*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/(64*d) - 3
*I*exp(a)*exp(3*I*d*x)*cos(c + d*x)**5/(64*d), Eq(b, 3*I*d)), (I*x*exp(a)*e
xp(5*I*d*x)*sin(c + d*x)**5/32 - 5*x*exp(a)*exp(5*I*d*x)*sin(c + d*x)**4*co
s(c + d*x)/32 - 5*I*x*exp(a)*exp(5*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**2/1
6 + 5*x*exp(a)*exp(5*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**3/16 + 5*I*x*exp(
a)*exp(5*I*d*x)*sin(c + d*x)*cos(c + d*x)**4/32 - x*exp(a)*exp(5*I*d*x)*cos
(c + d*x)**5/32 - 31*exp(a)*exp(5*I*d*x)*sin(c + d*x)**5/(960*d) - 25*I*exp
(a)*exp(5*I*d*x)*sin(c + d*x)**4*cos(c + d*x)/(192*d) + exp(a)*exp(5*I*d*x)
*sin(c + d*x)**3*cos(c + d*x)**2/(6*d) - 3*exp(a)*exp(5*I*d*x)*sin(c + d*x)
*cos(c + d*x)**4/(64*d) - I*exp(a)*exp(5*I*d*x)*cos(c + d*x)**5/(64*d), Eq(
b, 5*I*d)), (b**5*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**3/(b**6 + 3
5*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 3*b**4*d*exp(a)*exp(b*x)*sin(c +
d*x)**3*cos(c + d*x)**2/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) -
2*b**4*d*exp(a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)**4/(b**6 + 35*b**4*d**2
+ 259*b**2*d**4 + 225*d**6) + 6*b**3*d**2*exp(a)*exp(b*x)*sin(c + d*x)**4*c
os(c + d*x)/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 18*b**3*d**2
*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**3/(b**6 + 35*b**4*d**2 + 259
*b**2*d**4 + 225*d**6) + 2*b**3*d**2*exp(a)*exp(b*x)*cos(c + d*x)**5/(b**6
+ 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 6*b**2*d**3*exp(a)*exp(b*x)*si
n(c + d*x)**5/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 30*b**2*d*
*3*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)**2/(b**6 + 35*b**4*d**2 + 2
59*b**2*d**4 + 225*d**6) - 26*b**2*d**3*exp(a)*exp(b*x)*sin(c + d*x)*cos(c
+ d*x)**4/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 30*b*d**4*exp(
a)*exp(b*x)*sin(c + d*x)**4*cos(c + d*x)/(b**6 + 35*b**4*d**2 + 259*b**2*d*
*4 + 225*d**6) + 65*b*d**4*exp(a)*exp(b*x)*sin(c + d*x)**2*cos(c + d*x)**3/
(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 26*b*d**4*exp(a)*exp(b*x
)*cos(c + d*x)**5/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 225*d**6) + 30*d**
5*exp(a)*exp(b*x)*sin(c + d*x)**5/(b**6 + 35*b**4*d**2 + 259*b**2*d**4 + 22
5*d**6) + 75*d**5*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)**2/(b**6 + 3
5*b**4*d**2 + 259*b**2*d**4 + 225*d**6), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1144 vs. $2(165) = 330$.

Time = 0.25 (sec) , antiderivative size = 1144, normalized size of antiderivative = 6.25

$$\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/32*((b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)*e^a + 9*b*d^4*cos(5*c)*e^a +
 5*b^4*d*e^a*sin(5*c) + 50*b^2*d^3*e^a*sin(5*c) + 45*d^5*e^a*sin(5*c))*cos(
5*d*x)*e^(b*x) + (b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)*e^a + 9*b*d^4*cos(
5*c)*e^a - 5*b^4*d*e^a*sin(5*c) - 50*b^2*d^3*e^a*sin(5*c) - 45*d^5*e^a*sin(
5*c))*cos(5*d*x + 10*c)*e^(b*x) + (b^5*cos(5*c)*e^a + 26*b^3*d^2*cos(5*c)*e
^a + 25*b*d^4*cos(5*c)*e^a - 3*b^4*d*e^a*sin(5*c) - 78*b^2*d^3*e^a*sin(5*c)
 - 75*d^5*e^a*sin(5*c))*cos(3*d*x + 8*c)*e^(b*x) + (b^5*cos(5*c)*e^a + 26*b
^3*d^2*cos(5*c)*e^a + 25*b*d^4*cos(5*c)*e^a + 3*b^4*d*e^a*sin(5*c) + 78*b^2
*d^3*e^a*sin(5*c) + 75*d^5*e^a*sin(5*c))*cos(3*d*x - 2*c)*e^(b*x) - 2*(b^5*
cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a + 225*b*d^4*cos(5*c)*e^a - b^4*d*e^a
*sin(5*c) - 34*b^2*d^3*e^a*sin(5*c) - 225*d^5*e^a*sin(5*c))*cos(d*x + 6*c)*
e^(b*x) - 2*(b^5*cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a + 225*b*d^4*cos(5*c
)*e^a + b^4*d*e^a*sin(5*c) + 34*b^2*d^3*e^a*sin(5*c) + 225*d^5*e^a*sin(5*c)
)*cos(d*x - 4*c)*e^(b*x) + (5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos(5*c)*e^a
+ 45*d^5*cos(5*c)*e^a - b^5*e^a*sin(5*c) - 10*b^3*d^2*e^a*sin(5*c) - 9*b*d^
4*e^a*sin(5*c))*e^(b*x)*sin(5*d*x) + (5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos
(5*c)*e^a + 45*d^5*cos(5*c)*e^a + b^5*e^a*sin(5*c) + 10*b^3*d^2*e^a*sin(5*c
) + 9*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(5*d*x + 10*c) + (3*b^4*d*cos(5*c)*e^a
 + 78*b^2*d^3*cos(5*c)*e^a + 75*d^5*cos(5*c)*e^a + b^5*e^a*sin(5*c) + 26*b^
3*d^2*e^a*sin(5*c) + 25*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(3*d*x + 8*c) + (3*b
^4*d*cos(5*c)*e^a + 78*b^2*d^3*cos(5*c)*e^a + 75*d^5*cos(5*c)*e^a - b^5*e^a
*sin(5*c) - 26*b^3*d^2*e^a*sin(5*c) - 25*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(3*
d*x - 2*c) - 2*(b^4*d*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e^a + 225*d^5*cos(
5*c)*e^a + b^5*e^a*sin(5*c) + 34*b^3*d^2*e^a*sin(5*c) + 225*b*d^4*e^a*sin(5
*c))*e^(b*x)*sin(d*x + 6*c) - 2*(b^4*d*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e
^a + 225*d^5*cos(5*c)*e^a - b^5*e^a*sin(5*c) - 34*b^3*d^2*e^a*sin(5*c) - 22
5*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(d*x - 4*c))/(b^6*cos(5*c)^2 + b^6*sin(5*c
)^2 + 225*(cos(5*c)^2 + sin(5*c)^2)*d^6 + 259*(b^2*cos(5*c)^2 + b^2*sin(5*c
)^2)*d^4 + 35*(b^4*cos(5*c)^2 + b^4*sin(5*c)^2)*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.83

$$\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx = -\frac{1}{16}\left(\frac{b\cos(5\,dx+5\,c)}{b^2+25\,d^2}+\frac{5\,d\sin(5\,dx+5\,c)}{b^2+25\,d^2}\right)e^{(bx+a)} - \frac{1}{16}\left(\frac{b\cos(3\,dx+3\,c)}{b^2+9\,d^2}+\frac{3\,d\sin(3\,dx+3\,c)}{b^2+9\,d^2}\right)e^{(bx+a)} + \frac{1}{8}\left(\frac{b\cos(dx+c)}{b^2+d^2}+\frac{d\sin(dx+c)}{b^2+d^2}\right)e^{(bx+a)}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/16*(b*cos(5*d*x + 5*c)/(b^2 + 25*d^2) + 5*d*sin(5*d*x + 5*c)/(b^2 + 25*d
^2))*e^(b*x + a) - 1/16*(b*cos(3*d*x + 3*c)/(b^2 + 9*d^2) + 3*d*sin(3*d*x +
 3*c)/(b^2 + 9*d^2))*e^(b*x + a) + 1/8*(b*cos(d*x + c)/(b^2 + d^2) + d*sin(
d*x + c)/(b^2 + d^2))*e^(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 28.85 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.39

$$\begin{aligned}\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx &= \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1\mathrm{i}\right)}{16\,\left(b-d\,1\mathrm{i}\right)}\\ &\quad-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)+\sin\left(3\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,c\right)+\sin\left(3\,c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{32\,\left(-3\,d+b\,1\mathrm{i}\right)}\\ &\quad-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(5\,d\,x\right)+\sin\left(5\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(5\,c\right)+\sin\left(5\,c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{32\,\left(-5\,d+b\,1\mathrm{i}\right)}\\ &\quad+\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(d\,x\right)+\sin\left(d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(c\right)+\sin\left(c\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,\left(-d+b\,1\mathrm{i}\right)}\\ &\quad-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(3\,d\,x\right)-\sin\left(3\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(3\,c\right)-\sin\left(3\,c\right)\,1\mathrm{i}\right)}{32\,\left(b-d\,3\mathrm{i}\right)}\\ &\quad-\frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(5\,d\,x\right)-\sin\left(5\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(5\,c\right)-\sin\left(5\,c\right)\,1\mathrm{i}\right)}{32\,\left(b-d\,5\mathrm{i}\right)}\end{aligned}$$

```
[In] int(cos(c + d*x)^3*exp(a + b*x)*sin(c + d*x)^2,x)
```

```
[Out] (exp(a + b*x)*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i))/(16*(b - d*1i)
) - (exp(a + b*x)*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*1i)
/(32*(b*1i - 3*d)) - (exp(a + b*x)*(cos(5*d*x) + sin(5*d*x)*1i)*(cos(5*c) +
 sin(5*c)*1i)*1i)/(32*(b*1i - 5*d)) + (exp(a + b*x)*(cos(d*x) + sin(d*x)*1i
)*(cos(c) + sin(c)*1i)*1i)/(16*(b*1i - d)) - (exp(a + b*x)*(cos(3*d*x) - si
n(3*d*x)*1i)*(cos(3*c) - sin(3*c)*1i))/(32*(b - d*3i)) - (exp(a + b*x)*(cos
(5*d*x) - sin(5*d*x)*1i)*(cos(5*c) - sin(5*c)*1i))/(32*(b - d*5i))
```

3.46 $\int e^{a+bx} \cos^3(c+dx) \sin^3(c+dx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 129

$$\int e^{a+bx} \cos^3(c+dx) \sin^3(c+dx)\,dx = -\frac{3de^{a+bx}\cos(2c+2dx)}{16(b^2+4d^2)} + \frac{3de^{a+bx}\cos(6c+6dx)}{16(b^2+36d^2)} + \frac{3be^{a+bx}\sin(2c+2dx)}{32(b^2+4d^2)} - \frac{be^{a+bx}\sin(6c+6dx)}{32(b^2+36d^2)}$$

```
[Out] -3/16*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+3/16*d*exp(b*x+a)*cos(6*d*x+6
*c)/(b^2+36*d^2)+3/32*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)-1/32*b*exp(b*
x+a)*sin(6*d*x+6*c)/(b^2+36*d^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4557, 4517}

$$\int e^{a+bx} \cos^3(c+dx) \sin^3(c+dx)\,dx = \frac{3be^{a+bx}\sin(2c+2dx)}{32(b^2+4d^2)} - \frac{be^{a+bx}\sin(6c+6dx)}{32(b^2+36d^2)} - \frac{3de^{a+bx}\cos(2c+2dx)}{16(b^2+4d^2)} + \frac{3de^{a+bx}\cos(6c+6dx)}{16(b^2+36d^2)}$$

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^3,x]
```

```
[Out] (-3*d*E^(a + b*x)*Cos[2*c + 2*d*x])/(16*(b^2 + 4*d^2)) + (3*d*E^(a + b*x)*C
os[6*c + 6*d*x])/(16*(b^2 + 36*d^2)) + (3*b*E^(a + b*x)*Sin[2*c + 2*d*x])/(
32*(b^2 + 4*d^2)) - (b*E^(a + b*x)*Sin[6*c + 6*d*x])/(32*(b^2 + 36*d^2))
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{32}e^{a+bx}\sin(2c+2dx) - \frac{1}{32}e^{a+bx}\sin(6c+6dx)\right)dx \\
&= -\left(\frac{1}{32}\int e^{a+bx}\sin(6c+6dx)\,dx\right) + \frac{3}{32}\int e^{a+bx}\sin(2c+2dx)\,dx \\
&= -\frac{3de^{a+bx}\cos(2c+2dx)}{16\,(b^2+4d^2)} + \frac{3de^{a+bx}\cos(6c+6dx)}{16\,(b^2+36d^2)} \\
&\quad + \frac{3be^{a+bx}\sin(2c+2dx)}{32\,(b^2+4d^2)} - \frac{be^{a+bx}\sin(6c+6dx)}{32\,(b^2+36d^2)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx$$
$$= \frac{e^{a+bx}(-6d(b^2+36d^2)\cos(2(c+dx)) + 6d(b^2+4d^2)\cos(6(c+dx)) - 2b(-b^2-52d^2+(b^2+4d^2)\cos(4(c+}{32\,(b^4+40b^2d^2+144d^4)}$$

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*(-6*d*(b^2 + 36*d^2)*Cos[2*(c + d*x)] + 6*d*(b^2 + 4*d^2)*Cos[
6*(c + d*x)] - 2*b*(-b^2 - 52*d^2 + (b^2 + 4*d^2)*Cos[4*(c + d*x)])*Sin[2*(
c + d*x)]))/(32*(b^4 + 40*b^2*d^2 + 144*d^4))
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

method	result
parallelrisch	$-\frac{\left(\left(b^{3}+4 b \,d^{2}\right) \sin \left(6 d x +6 c \right)+\left(-6 b^{2} d -24 d^{3}\right) \cos \left(6 d x +6 c \right)-3 \left(b^{2}+36 d^{2}\right) \left(b \sin \left(2 d x +2 c \right)-2 d \cos \left(2 d x +2 c \right)\right)\right) {\mathrm e}^{x b +a}}{32 b^{4}+1280 b^{2} d^{2}+4608 d^{4}}$
default	$-\frac{3 d \,{\mathrm e}^{x b +a} \cos \left(2 d x +2 c \right)}{16 \left(b^{2}+4 d^{2}\right)}+\frac{3 d \,{\mathrm e}^{x b +a} \cos \left(6 d x +6 c \right)}{16 \left(b^{2}+36 d^{2}\right)}+\frac{3 b \,{\mathrm e}^{x b +a} \sin \left(2 d x +2 c \right)}{32 \left(b^{2}+4 d^{2}\right)}-\frac{b \,{\mathrm e}^{x b +a} \sin \left(6 d x +6 c \right)}{32 \left(b^{2}+36 d^{2}\right)}$
risch	$\frac{i {\mathrm e}^{x b +a} \left(-12 i d \left(b^{2}+4 d^{2}\right) \cos \left(6 d x +6 c \right)-i \left(-2 b^{3}-8 b \,d^{2}\right) \sin \left(6 d x +6 c \right)+12 i d \left(b^{2}+36 d^{2}\right) \cos \left(2 d x +2 c \right)-i \left(6 b^{3}+216 b \,d^{2}\right) \sin \left(2 d x +2 c\right.}{64 \left(6 i d +b \right) \left(2 i d +b \right) \left(2 i d -b \right) \left(6 i d -b \right)}$

```
[In] int(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -((b^3+4*b*d^2)*sin(6*d*x+6*c)+(-6*b^2*d-24*d^3)*cos(6*d*x+6*c)-3*(b^2+36*d
^2)*(b*sin(2*d*x+2*c)-2*d*cos(2*d*x+2*c)))*exp(b*x+a)/(32*b^4+1280*b^2*d^2+
4608*d^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.21

$$\int e^{a+bx} \cos^3(c+dx) \sin^3(c+dx)\,dx = -\frac{\left((b^3+4\,bd^2)\cos\left(dx+c\right)^5-6\,bd^2\cos\left(dx+c\right)-(b^3+4\,bd^2)\cos\left(dx+c\right)^3\right)e^{(bx+a)}\sin\left(dx+c\right)-3\,\left(2\,(b^2\right.}{b^4+40\,b^2d^2+144\,\ldots}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(((b^3 + 4*b*d^2)*cos(d*x + c)^5 - 6*b*d^2*cos(d*x + c) - (b^3 + 4*b*d^2)*
cos(d*x + c)^3)*e^(b*x + a)*sin(d*x + c) - 3*(2*(b^2*d + 4*d^3)*cos(d*x + c
)^6 + b^2*d*cos(d*x + c)^2 - 3*(b^2*d + 4*d^3)*cos(d*x + c)^4 + 2*d^3)*e^(b
*x + a))/(b^4 + 40*b^2*d^2 + 144*d^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 71.62 (sec) , antiderivative size = 1991, normalized size of antiderivative = 15.43

$$\int e^{a+bx} \cos^3(c+dx) \sin^3(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**3*sin(d*x+c)**3,x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)**3*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-I*x*exp(a)
*exp(-6*I*d*x)*sin(c + d*x)**6/64 - 3*x*exp(a)*exp(-6*I*d*x)*sin(c + d*x)**
5*cos(c + d*x)/32 + 15*I*x*exp(a)*exp(-6*I*d*x)*sin(c + d*x)**4*cos(c + d*x
)**2/64 + 5*x*exp(a)*exp(-6*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**3/16 - 15*
I*x*exp(a)*exp(-6*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**4/64 - 3*x*exp(a)*ex
p(-6*I*d*x)*sin(c + d*x)*cos(c + d*x)**5/32 + I*x*exp(a)*exp(-6*I*d*x)*cos(
c + d*x)**6/64 - exp(a)*exp(-6*I*d*x)*sin(c + d*x)**6/(160*d) + 7*I*exp(a)*
exp(-6*I*d*x)*sin(c + d*x)**5*cos(c + d*x)/(320*d) + 11*I*exp(a)*exp(-6*I*d
*x)*sin(c + d*x)**3*cos(c + d*x)**3/(96*d) + 7*I*exp(a)*exp(-6*I*d*x)*sin(c
 + d*x)*cos(c + d*x)**5/(320*d) + exp(a)*exp(-6*I*d*x)*cos(c + d*x)**6/(160
*d), Eq(b, -6*I*d)), (3*I*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**6/64 + 3*x*e
xp(a)*exp(-2*I*d*x)*sin(c + d*x)**5*cos(c + d*x)/32 + 3*I*x*exp(a)*exp(-2*I
*d*x)*sin(c + d*x)**4*cos(c + d*x)**2/64 + 3*x*exp(a)*exp(-2*I*d*x)*sin(c +
 d*x)**3*cos(c + d*x)**3/16 - 3*I*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**2*co
s(c + d*x)**4/64 + 3*x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)*cos(c + d*x)**5/32
 - 3*I*x*exp(a)*exp(-2*I*d*x)*cos(c + d*x)**6/64 - 3*exp(a)*exp(-2*I*d*x)*s
in(c + d*x)**6/(32*d) + 15*I*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**5*cos(c + d
*x)/(64*d) + 13*I*exp(a)*exp(-2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**3/(32*
d) + 15*I*exp(a)*exp(-2*I*d*x)*sin(c + d*x)*cos(c + d*x)**5/(64*d) + 3*exp(
a)*exp(-2*I*d*x)*cos(c + d*x)**6/(32*d), Eq(b, -2*I*d)), (-3*I*x*exp(a)*exp
(2*I*d*x)*sin(c + d*x)**6/64 + 3*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**5*cos(
c + d*x)/32 - 3*I*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**4*cos(c + d*x)**2/64
+ 3*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**3/16 + 3*I*x*exp(a)
*exp(2*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**4/64 + 3*x*exp(a)*exp(2*I*d*x)*
sin(c + d*x)*cos(c + d*x)**5/32 + 3*I*x*exp(a)*exp(2*I*d*x)*cos(c + d*x)**6
/64 - 3*exp(a)*exp(2*I*d*x)*sin(c + d*x)**6/(32*d) - 15*I*exp(a)*exp(2*I*d*
x)*sin(c + d*x)**5*cos(c + d*x)/(64*d) - 13*I*exp(a)*exp(2*I*d*x)*sin(c + d
*x)**3*cos(c + d*x)**3/(32*d) - 15*I*exp(a)*exp(2*I*d*x)*sin(c + d*x)*cos(c
 + d*x)**5/(64*d) + 3*exp(a)*exp(2*I*d*x)*cos(c + d*x)**6/(32*d), Eq(b, 2*I
*d)), (I*x*exp(a)*exp(6*I*d*x)*sin(c + d*x)**6/64 - 3*x*exp(a)*exp(6*I*d*x)
*sin(c + d*x)**5*cos(c + d*x)/32 - 15*I*x*exp(a)*exp(6*I*d*x)*sin(c + d*x)*
*4*cos(c + d*x)**2/64 + 5*x*exp(a)*exp(6*I*d*x)*sin(c + d*x)**3*cos(c + d*x
)**3/16 + 15*I*x*exp(a)*exp(6*I*d*x)*sin(c + d*x)**2*cos(c + d*x)**4/64 - 3
*x*exp(a)*exp(6*I*d*x)*sin(c + d*x)*cos(c + d*x)**5/32 - I*x*exp(a)*exp(6*I
*d*x)*cos(c + d*x)**6/64 - exp(a)*exp(6*I*d*x)*sin(c + d*x)**6/(160*d) - 7*
I*exp(a)*exp(6*I*d*x)*sin(c + d*x)**5*cos(c + d*x)/(320*d) - 11*I*exp(a)*ex
p(6*I*d*x)*sin(c + d*x)**3*cos(c + d*x)**3/(96*d) - 7*I*exp(a)*exp(6*I*d*x)
*sin(c + d*x)*cos(c + d*x)**5/(320*d) + exp(a)*exp(6*I*d*x)*cos(c + d*x)**6
/(160*d), Eq(b, 6*I*d)), (b**3*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)
**3/(b**4 + 40*b**2*d**2 + 144*d**4) + 3*b**2*d*exp(a)*exp(b*x)*sin(c + d*x
)**4*cos(c + d*x)**2/(b**4 + 40*b**2*d**2 + 144*d**4) - 3*b**2*d*exp(a)*exp
(b*x)*sin(c + d*x)**2*cos(c + d*x)**4/(b**4 + 40*b**2*d**2 + 144*d**4) + 6*
b*d**2*exp(a)*exp(b*x)*sin(c + d*x)**5*cos(c + d*x)/(b**4 + 40*b**2*d**2 +
144*d**4) + 16*b*d**2*exp(a)*exp(b*x)*sin(c + d*x)**3*cos(c + d*x)**3/(b**4
 + 40*b**2*d**2 + 144*d**4) + 6*b*d**2*exp(a)*exp(b*x)*sin(c + d*x)*cos(c +
```

```
 d*x)**5/(b**4 + 40*b**2*d**2 + 144*d**4) + 6*d**3*exp(a)*exp(b*x)*sin(c +
d*x)**6/(b**4 + 40*b**2*d**2 + 144*d**4) + 18*d**3*exp(a)*exp(b*x)*sin(c +
d*x)**4*cos(c + d*x)**2/(b**4 + 40*b**2*d**2 + 144*d**4) - 18*d**3*exp(a)*e
xp(b*x)*sin(c + d*x)**2*cos(c + d*x)**4/(b**4 + 40*b**2*d**2 + 144*d**4) -
6*d**3*exp(a)*exp(b*x)*cos(c + d*x)**6/(b**4 + 40*b**2*d**2 + 144*d**4), Tr
ue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(117) = 234$.

Time = 0.22 (sec) , antiderivative size = 550, normalized size of antiderivative = 4.26

$$\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx = \frac{(6\,b^2d\cos(6\,c)\,e^a + 24\,d^3\cos(6\,c)\,e^a - b^3e^a\sin(6\,c) - 4\,bd^2e^a\sin(6\,c))\cos(6\,dx)\,e^{(bx)} + (6\,b^2d\cos(6\,c)\,e^a +$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/64*((6*b^2*d*cos(6*c)*e^a + 24*d^3*cos(6*c)*e^a - b^3*e^a*sin(6*c) - 4*b*
d^2*e^a*sin(6*c))*cos(6*d*x)*e^(b*x) + (6*b^2*d*cos(6*c)*e^a + 24*d^3*cos(6
*c)*e^a + b^3*e^a*sin(6*c) + 4*b*d^2*e^a*sin(6*c))*cos(6*d*x + 12*c)*e^(b*x
) - 3*(2*b^2*d*cos(6*c)*e^a + 72*d^3*cos(6*c)*e^a + b^3*e^a*sin(6*c) + 36*b
*d^2*e^a*sin(6*c))*cos(2*d*x + 8*c)*e^(b*x) - 3*(2*b^2*d*cos(6*c)*e^a + 72*
d^3*cos(6*c)*e^a - b^3*e^a*sin(6*c) - 36*b*d^2*e^a*sin(6*c))*cos(2*d*x - 4*
c)*e^(b*x) - (b^3*cos(6*c)*e^a + 4*b*d^2*cos(6*c)*e^a + 6*b^2*d*e^a*sin(6*c
) + 24*d^3*e^a*sin(6*c))*e^(b*x)*sin(6*d*x) - (b^3*cos(6*c)*e^a + 4*b*d^2*c
os(6*c)*e^a - 6*b^2*d*e^a*sin(6*c) - 24*d^3*e^a*sin(6*c))*e^(b*x)*sin(6*d*x
 + 12*c) + 3*(b^3*cos(6*c)*e^a + 36*b*d^2*cos(6*c)*e^a - 2*b^2*d*e^a*sin(6*
c) - 72*d^3*e^a*sin(6*c))*e^(b*x)*sin(2*d*x + 8*c) + 3*(b^3*cos(6*c)*e^a +
36*b*d^2*cos(6*c)*e^a + 2*b^2*d*e^a*sin(6*c) + 72*d^3*e^a*sin(6*c))*e^(b*x)
*sin(2*d*x - 4*c))/(b^4*cos(6*c)^2 + b^4*sin(6*c)^2 + 144*(cos(6*c)^2 + sin
(6*c)^2)*d^4 + 40*(b^2*cos(6*c)^2 + b^2*sin(6*c)^2)*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

$$\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx = \frac{1}{32}\left(\frac{6\,d\cos(6\,dx+6\,c)}{b^2+36\,d^2} - \frac{b\sin(6\,dx+6\,c)}{b^2+36\,d^2}\right)e^{(bx+a)} - \frac{3}{32}\left(\frac{2\,d\cos(2\,dx+2\,c)}{b^2+4\,d^2} - \frac{b\sin(2\,dx+2\,c)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*(6*d*cos(6*d*x + 6*c)/(b^2 + 36*d^2) - b*sin(6*d*x + 6*c)/(b^2 + 36*d^
2))*e^(b*x + a) - 3/32*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x +
2*c)/(b^2 + 4*d^2))*e^(b*x + a)
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.38

$$\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx = -\frac{3\,\mathrm{e}^{a+b\,x}\,\left(\cos\left(2\,d\,x\right)-\sin\left(2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(2\,c\right)-\sin\left(2\,c\right)\,1{}\mathrm{i}\right)}{64\,\left(2\,d+b\,1{}\mathrm{i}\right)} + \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(6\,d\,x\right)-\sin\left(6\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(6\,c\right)-\sin\left(6\,c\right)\,1{}\mathrm{i}\right)}{64\,\left(6\,d+b\,1{}\mathrm{i}\right)} - \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(2\,d\,x\right)+\sin\left(2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(2\,c\right)+\sin\left(2\,c\right)\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{64\,\left(b+d\,2{}\mathrm{i}\right)} + \frac{\mathrm{e}^{a+b\,x}\,\left(\cos\left(6\,d\,x\right)+\sin\left(6\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(6\,c\right)+\sin\left(6\,c\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{64\,\left(b+d\,6{}\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^3*exp(a + b*x)*sin(c + d*x)^3,x)
```

```
[Out] (exp(a + b*x)*(cos(6*d*x) - sin(6*d*x)*1i)*(cos(6*c) - sin(6*c)*1i))/(64*(b
*1i + 6*d)) - (3*exp(a + b*x)*(cos(2*d*x) - sin(2*d*x)*1i)*(cos(2*c) - sin(
2*c)*1i))/(64*(b*1i + 2*d)) - (exp(a + b*x)*(cos(2*d*x) + sin(2*d*x)*1i)*(c
os(2*c) + sin(2*c)*1i)*3i)/(64*(b + d*2i)) + (exp(a + b*x)*(cos(6*d*x) + si
n(6*d*x)*1i)*(cos(6*c) + sin(6*c)*1i)*1i)/(64*(b + d*6i))
```

3.47 $\int e^x x \sin(x) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 30

$$\int e^x x \sin(x) \, dx = \frac{1}{2} e^x \cos(x) - \frac{1}{2} e^x x \cos(x) + \frac{1}{2} e^x x \sin(x)$$

```
[Out] 1/2*exp(x)*cos(x)-1/2*exp(x)*x*cos(x)+1/2*exp(x)*x*sin(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4517, 4553, 4518}

$$\int e^x x \sin(x) \, dx = \frac{1}{2} e^x x \sin(x) + \frac{1}{2} e^x \cos(x) - \frac{1}{2} e^x x \cos(x)$$

```
[In] Int[E^x*x*Sin[x],x]
```

```
[Out] (E^x*Cos[x])/2 - (E^x*x*Cos[x])/2 + (E^x*x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
```

```
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)-\int\left(-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)dx\\ &= -\frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)+\frac{1}{2}\int e^x\cos(x)\,dx-\frac{1}{2}\int e^x\sin(x)\,dx\\ &= \frac{1}{2}e^x\cos(x)-\frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int e^x x\sin(x)\,dx=\frac{1}{2}e^x(\cos(x)-x\cos(x)+x\sin(x))$$

```
[In] Integrate[E^x*x*Sin[x],x]
[Out] (E^x*(Cos[x] - x*Cos[x] + x*Sin[x]))/2
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

method	result	size
parallelrisch	$-\frac{((x-1)\cos(x)-x\sin(x))\mathrm{e}^x}{2}$	17
default	$\left(-\frac{x}{2}+\frac{1}{2}\right)\mathrm{e}^x\cos(x)+\frac{\mathrm{e}^x x\sin(x)}{2}$	19
risch	$\left(-\frac{1}{8}-\frac{i}{8}\right)(-1+i+2x)\,\mathrm{e}^{(1+i)x}+\left(-\frac{1}{8}+\frac{i}{8}\right)(-1-i+2x)\,\mathrm{e}^{(1-i)x}$	36
norman	$\frac{\mathrm{e}^x x\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^x x}{2}-\frac{\mathrm{e}^x\tan\left(\frac{x}{2}\right)^2}{2}+\frac{\mathrm{e}^x x\tan\left(\frac{x}{2}\right)^2}{2}+\frac{\mathrm{e}^x}{2}}{1+\tan\left(\frac{x}{2}\right)^2}$	51

```
[In] int(exp(x)*x*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((x-1)*cos(x)-x*sin(x))*exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \sin(x) \, dx = -\frac{1}{2} (x - 1) \cos(x) e^x + \frac{1}{2} x e^x \sin(x)$$

```
[In] integrate(exp(x)*x*sin(x),x, algorithm="fricas")
[Out] -1/2*(x - 1)*cos(x)*e^x + 1/2*x*e^x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^x x \sin(x) \, dx = \frac{x e^x \sin(x)}{2} - \frac{x e^x \cos(x)}{2} + \frac{e^x \cos(x)}{2}$$

```
[In] integrate(exp(x)*x*sin(x),x)
[Out] x*exp(x)*sin(x)/2 - x*exp(x)*cos(x)/2 + exp(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \sin(x) \, dx = -\frac{1}{2} (x - 1) \cos(x) e^x + \frac{1}{2} x e^x \sin(x)$$

```
[In] integrate(exp(x)*x*sin(x),x, algorithm="maxima")
[Out] -1/2*(x - 1)*cos(x)*e^x + 1/2*x*e^x*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

$$\int e^x x \sin(x) \, dx = -\frac{1}{2} \left((x-1) \cos(x) - x \sin(x) \right) e^x$$

```
[In] integrate(exp(x)*x*sin(x),x, algorithm="giac")
[Out] -1/2*((x - 1)*cos(x) - x*sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 26.39 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

$$\int e^x x \sin(x) \, dx = \frac{e^x \left(\cos(x) - x \cos(x) + x \sin(x)\right)}{2}$$

```
[In] int(x*exp(x)*sin(x),x)
[Out] (exp(x)*(cos(x) - x*cos(x) + x*sin(x)))/2
```

3.48 $\int e^x x^2 \sin(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 50

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}e^x\cos(x) + e^x x\cos(x) - \frac{1}{2}e^x x^2\cos(x) - \frac{1}{2}e^x\sin(x) + \frac{1}{2}e^x x^2\sin(x)$$

```
[Out] -1/2*exp(x)*cos(x)+exp(x)*x*cos(x)-1/2*exp(x)*x^2*cos(x)-1/2*exp(x)*sin(x)+
1/2*exp(x)*x^2*sin(x)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4517, 4553, 14, 4518, 4554}

$$\int e^x x^2 \sin(x)\,dx = \frac{1}{2}e^x x^2\sin(x) - \frac{1}{2}e^x x^2\cos(x) - \frac{1}{2}e^x\sin(x) + e^x x\cos(x) - \frac{1}{2}e^x\cos(x)$$

```
[In] Int[E^x*x^2*Sin[x],x]
```

```
[Out] -1/2*(E^x*Cos[x]) + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x])/2 + (E
^x*x^2*Sin[x])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
```

```
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) - 2\int x\left(-\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)\right)dx \\
&= -\frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) - 2\int\left(-\frac{1}{2}e^x x\cos(x) + \frac{1}{2}e^x x\sin(x)\right)dx \\
&= -\frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) + \int e^x x\cos(x)\,dx - \int e^x x\sin(x)\,dx \\
&= e^x x\cos(x) - \frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) \\
&\quad + \int\left(-\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)\right)dx - \int\left(\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)\right)dx \\
&= e^x x\cos(x) - \frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) - 2\left(\frac{1}{2}\int e^x\cos(x)\,dx\right) \\
&= e^x x\cos(x) - \frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) - 2\left(\frac{1}{4}e^x\cos(x) + \frac{1}{4}e^x\sin(x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.50

$$\int e^x x^2 \sin(x)\, dx = \frac{1}{2} e^x \left(-(-1+x)^2 \cos(x) + \left(-1+x^2\right) \sin(x)\right)$$

```
[In] Integrate[E^x*x^2*Sin[x],x]
[Out] (E^x*(-((-1 + x)^2*Cos[x]) + (-1 + x^2)*Sin[x]))/2
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.46

method	result	size
parallelrisch	$-\frac{(x-1)\mathrm{e}^x((-x-1)\sin(x)+(x-1)\cos(x))}{2}$	23
default	$\left(-\frac{1}{2}x^2+x-\frac{1}{2}\right)\mathrm{e}^x\cos\left(x\right)+\left(\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^x\sin\left(x\right)$	27
risch	$\left(-\frac{1}{4}-\frac{i}{4}\right)\left(x^2+ix-x-i\right)\mathrm{e}^{(1+i)x}+\left(-\frac{1}{4}+\frac{i}{4}\right)\left(x^2-ix-x+i\right)\mathrm{e}^{(1-i)x}$	48
norman	$\frac{\mathrm{e}^x x+\mathrm{e}^x x^2\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^x x^2}{2}-\mathrm{e}^x\tan\left(\frac{x}{2}\right)+\frac{\mathrm{e}^x\tan\left(\frac{x}{2}\right)^2}{2}-\mathrm{e}^x x\tan\left(\frac{x}{2}\right)^2+\frac{\mathrm{e}^x x^2\tan\left(\frac{x}{2}\right)^2}{2}-\frac{\mathrm{e}^x}{2}}{1+\tan\left(\frac{x}{2}\right)^2}$	80

```
[In] int(exp(x)*x^2*sin(x),x,method=_RETURNVERBOSE)
[Out] -1/2*(x-1)*exp(x)*((-x-1)*sin(x)+(x-1)*cos(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int e^x x^2 \sin(x)\, dx = -\frac{1}{2}\left(x^2-2x+1\right)\cos\left(x\right)e^x + \frac{1}{2}\left(x^2-1\right)e^x\sin\left(x\right)$$

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="fricas")
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int e^x x^2 \sin(x)\,dx = \frac{x^2e^x\sin(x)}{2} - \frac{x^2e^x\cos(x)}{2} + xe^x\cos(x) - \frac{e^x\sin(x)}{2} - \frac{e^x\cos(x)}{2}$$

```
[In] integrate(exp(x)*x**2*sin(x),x)
```

```
[Out] x**2*exp(x)*sin(x)/2 - x**2*exp(x)*cos(x)/2 + x*exp(x)*cos(x) - exp(x)*sin(
x)/2 - exp(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}\left(x^2-2x+1\right)\cos(x)e^x + \frac{1}{2}\left(x^2-1\right)e^x\sin(x)$$

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.50

$$\int e^x x^2 \sin(x)\,dx = -\frac{1}{2}\left(\left(x^2-2x+1\right)\cos(x) - \left(x^2-1\right)\sin(x)\right)e^x$$

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*((x^2 - 2*x + 1)*cos(x) - (x^2 - 1)*sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 27.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.42

$$\int e^x x^2 \sin(x)\,dx = \frac{e^x\,(x-1)\,\left(\cos(x)+\sin(x)-x\,\cos(x)+x\,\sin(x)\right)}{2}$$

```
[In] int(x^2*exp(x)*sin(x),x)
```

```
[Out] (exp(x)*(x - 1)*(cos(x) + sin(x) - x*cos(x) + x*sin(x)))/2
```

3.49 $\int e^x x \cos(x) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 30

$$\int e^x x \cos(x) \, dx = \frac{1}{2} e^x x \cos(x) - \frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x x \sin(x)$$

```
[Out] 1/2*exp(x)*x*cos(x)-1/2*exp(x)*sin(x)+1/2*exp(x)*x*sin(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4518, 4554, 4517}

$$\int e^x x \cos(x) \, dx = -\frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x x \sin(x) + \frac{1}{2} e^x x \cos(x)$$

```
[In] Int[E^x*x*Cos[x],x]
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
```

```
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)-\int\left(\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)dx\\
&= \frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)-\frac{1}{2}\int e^x\cos(x)\,dx-\frac{1}{2}\int e^x\sin(x)\,dx\\
&= \frac{1}{2}e^x x\cos(x)-\frac{1}{2}e^x\sin(x)+\frac{1}{2}e^x x\sin(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int e^x x\cos(x)\,dx=\frac{1}{2}e^x(x\cos(x)+(-1+x)\sin(x))$$

```
[In] Integrate[E^x*x*Cos[x],x]
[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

method	result	size
parallelrisch	$\frac{e^x((x-1)\sin(x)+x\cos(x))}{2}$	16
default	$\frac{e^x x\cos(x)}{2}-\left(-\frac{x}{2}+\frac{1}{2}\right)e^x\sin(x)$	20
risch	$\left(\frac{1}{8}-\frac{i}{8}\right)(-1+i+2x)\,e^{(1+i)x}+\left(\frac{1}{8}+\frac{i}{8}\right)(-1-i+2x)\,e^{(1-i)x}$	36
norman	$\frac{e^x x\tan\left(\frac{x}{2}\right)+\frac{e^x x}{2}-e^x\tan\left(\frac{x}{2}\right)-\frac{e^x x\tan\left(\frac{x}{2}\right)^2}{2}}{1+\tan\left(\frac{x}{2}\right)^2}$	45

```
[In] int(exp(x)*x*cos(x),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)*((x-1)*sin(x)+x*cos(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x)\, dx = \frac{1}{2}\, x \cos\left(x\right) e^{x} + \frac{1}{2}\, (x-1)e^{x} \sin\left(x\right)$$

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="fricas")
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^x x \cos(x)\, dx = \frac{xe^{x}\sin\left(x\right)}{2} + \frac{xe^{x}\cos\left(x\right)}{2} - \frac{e^{x}\sin\left(x\right)}{2}$$

```
[In] integrate(exp(x)*x*cos(x),x)
[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x)\, dx = \frac{1}{2}\, x \cos\left(x\right) e^{x} + \frac{1}{2}\, (x-1)e^{x} \sin\left(x\right)$$

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="maxima")
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.50

$$\int e^x x \cos(x) \, dx = \frac{1}{2} (x \cos(x) + (x - 1) \sin(x)) e^x$$

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="giac")
[Out] 1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 27.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x) \, dx = \frac{e^x (x \cos(x) - \sin(x) + x \sin(x))}{2}$$

```
[In] int(x*exp(x)*cos(x),x)
[Out] (exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2
```

3.50 $\int e^x x^2 \cos(x)\, dx$

Optimal result

Integrand size = 9, antiderivative size = 51

$$\int e^x x^2 \cos(x)\, dx = -\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x \sin(x) - e^x x \sin(x) + \frac{1}{2}e^x x^2 \sin(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*x^2*cos(x)+1/2*exp(x)*sin(x)-exp(x)*x*sin(x)+
1/2*exp(x)*x^2*sin(x)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4518, 4554, 14, 4517, 4553}

$$\int e^x x^2 \cos(x)\, dx = \frac{1}{2}e^x x^2 \sin(x) + \frac{1}{2}e^x x^2 \cos(x) - e^x x \sin(x) + \frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

```
[In] Int[E^x*x^2*Cos[x],x]
```

```
[Out] -1/2*(E^x*Cos[x]) + (E^x*x^2*Cos[x])/2 + (E^x*Sin[x])/2 - E^x*x*Sin[x] + (E
^x*x^2*Sin[x])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
```

```
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - 2\int x\left(\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right)\,dx \\
&= \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - 2\int \left(\frac{1}{2}e^x x \cos(x) + \frac{1}{2}e^x x \sin(x)\right)\,dx \\
&= \frac{1}{2}e^x x^2 \cos(x) + \frac{1}{2}e^x x^2 \sin(x) - \int e^x x \cos(x)\,dx - \int e^x x \sin(x)\,dx \\
&= \frac{1}{2}e^x x^2 \cos(x) - e^x x \sin(x) + \frac{1}{2}e^x x^2 \sin(x) \\
&\quad + \int \left(-\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right)\,dx + \int \left(\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right)\,dx \\
&= \frac{1}{2}e^x x^2 \cos(x) - e^x x \sin(x) + \frac{1}{2}e^x x^2 \sin(x) + 2\left(\frac{1}{2}\int e^x \sin(x)\,dx\right) \\
&= \frac{1}{2}e^x x^2 \cos(x) - e^x x \sin(x) + \frac{1}{2}e^x x^2 \sin(x) + 2\left(-\frac{1}{4}e^x \cos(x) + \frac{1}{4}e^x \sin(x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.45

$$\int e^x x^2 \cos(x)\,dx = \frac{1}{2}e^x(-1+x)((1+x)\cos(x)+(-1+x)\sin(x))$$

```
[In] Integrate[E^x*x^2*Cos[x],x]
[Out] (E^x*(-1 + x)*((1 + x)*Cos[x] + (-1 + x)*Sin[x]))/2
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.41

method	result	size
parallelrisch	$\frac{(x-1)((x+1)\cos(x)+(x-1)\sin(x))\mathrm{e}^{x}}{2}$	21
default	$\left(\frac{x^{2}}{2}-\frac{1}{2}\right)\mathrm{e}^{x}\cos\left(x\right)-\left(-\frac{1}{2}x^{2}+x-\frac{1}{2}\right)\mathrm{e}^{x}\sin\left(x\right)$	28
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)\left(x^{2}+ix-x-i\right)\mathrm{e}^{(1+i)x}+\left(\frac{1}{4}+\frac{i}{4}\right)\left(x^{2}-ix-x+i\right)\mathrm{e}^{(1-i)x}$	48
norman	$\frac{\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)+\mathrm{e}^{x}x^{2}\tan\left(\frac{x}{2}\right)+\frac{\mathrm{e}^{x}x^{2}}{2}+\frac{\mathrm{e}^{x}\tan\left(\frac{x}{2}\right)^{2}}{2}-2\,\mathrm{e}^{x}x\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^{x}x^{2}\tan\left(\frac{x}{2}\right)^{2}}{2}-\frac{\mathrm{e}^{x}}{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}$	73

```
[In] int(exp(x)*x^2*cos(x),x,method=_RETURNVERBOSE)
[Out] 1/2*(x-1)*((x+1)*cos(x)+(x-1)*sin(x))*exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

$$\int e^x x^2 \cos(x)\,dx = \frac{1}{2}\left(x^2-1\right)\cos\left(x\right)e^x + \frac{1}{2}\left(x^2-2\,x+1\right)e^x\sin\left(x\right)$$

```
[In] integrate(exp(x)*x^2*cos(x),x, algorithm="fricas")
[Out] 1/2*(x^2 - 1)*cos(x)*e^x + 1/2*(x^2 - 2*x + 1)*e^x*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int e^x x^2 \cos(x)\, dx = \frac{x^2 e^x \sin(x)}{2} + \frac{x^2 e^x \cos(x)}{2} - x e^x \sin(x) + \frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

```
[In] integrate(exp(x)*x**2*cos(x),x)
[Out] x**2*exp(x)*sin(x)/2 + x**2*exp(x)*cos(x)/2 - x*exp(x)*sin(x) + exp(x)*sin(
x)/2 - exp(x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

$$\int e^x x^2 \cos(x)\, dx = \frac{1}{2}\left(x^2 - 1\right)\cos(x)\, e^x + \frac{1}{2}\left(x^2 - 2x + 1\right) e^x \sin(x)$$

```
[In] integrate(exp(x)*x^2*cos(x),x, algorithm="maxima")
[Out] 1/2*(x^2 - 1)*cos(x)*e^x + 1/2*(x^2 - 2*x + 1)*e^x*sin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.47

$$\int e^x x^2 \cos(x)\, dx = \frac{1}{2}\left(\left(x^2 - 1\right)\cos(x) + \left(x^2 - 2x + 1\right)\sin(x)\right) e^x$$

```
[In] integrate(exp(x)*x^2*cos(x),x, algorithm="giac")
[Out] 1/2*((x^2 - 1)*cos(x) + (x^2 - 2*x + 1)*sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.43

$$\int e^x x^2 \cos(x)\, dx = \frac{e^x\,(x-1)\,\left(\cos(x) - \sin(x) + x\,\cos(x) + x\,\sin(x)\right)}{2}$$

```
[In] int(x^2*exp(x)*cos(x),x)
[Out] (exp(x)*(x - 1)*(cos(x) - sin(x) + x*cos(x) + x*sin(x)))/2
```

3.51 $\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 27

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx=-\frac{23}{25}e^{3x}\cos(4x)-\frac{14}{25}e^{3x}\sin(4x)$$

```
[Out] -23/25*exp(3*x)*cos(4*x)-14/25*exp(3*x)*sin(4*x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6874, 4518, 4517}

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx=-\frac{14}{25}e^{3x}\sin(4x)-\frac{23}{25}e^{3x}\cos(4x)$$

```
[In] Int[E^(3*x)*(-5*Cos[4*x] + 2*Sin[4*x]),x]
```

```
[Out] (-23*E^(3*x)*Cos[4*x])/25 - (14*E^(3*x)*Sin[4*x])/25
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
```

```
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-5e^{3x}\cos(4x)+2e^{3x}\sin(4x)\right)\,dx\\ &= 2\int e^{3x}\sin(4x)\,dx-5\int e^{3x}\cos(4x)\,dx\\ &= -\frac{23}{25}e^{3x}\cos(4x)-\frac{14}{25}e^{3x}\sin(4x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx=-\frac{1}{25}e^{3x}(23\cos(4x)+14\sin(4x))$$

```
[In] Integrate[E^(3*x)*(-5*Cos[4*x] + 2*Sin[4*x]),x]
[Out] -1/25*(E^(3*x)*(23*Cos[4*x] + 14*Sin[4*x]))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result
parallelrisch	$-\frac{e^{3x}(23\cos(4x)+14\sin(4x))}{25}$
parts	$-\frac{23\,e^{3x}\cos(4x)}{25}-\frac{14\,e^{3x}\sin(4x)}{25}$
risch	$-\frac{23\,e^{(3+4i)x}}{50}+\frac{7i e^{(3+4i)x}}{25}-\frac{23\,e^{(3-4i)x}}{50}-\frac{7i e^{(3-4i)x}}{25}$
norman	$\frac{-\frac{28\,e^{3x}\tan(2x)}{25}+\frac{23\,e^{3x}\tan(2x)^2}{25}-\frac{23\,e^{3x}}{25}}{1+\tan(2x)^2}$
default	$-\frac{8(3\cos(x)+4\sin(x))e^{3x}\cos(x)^3}{5}+\frac{8(3\cos(x)+2\sin(x))e^{3x}\cos(x)}{5}-\frac{3\,e^{3x}}{5}-\frac{8\,e^{3x}\cos(4x)}{25}+\frac{6\,e^{3x}\sin(4x)}{25}-\frac{8\,e^{3x}\cos(}{13}$

```
[In] int(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x,method=_RETURNVERBOSE)
[Out] -1/25*exp(3*x)*(23*cos(4*x)+14*sin(4*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx=-\frac{23}{25}\cos\left(4\,x\right)e^{\left(3\,x\right)}-\frac{14}{25}\,e^{\left(3\,x\right)}\sin\left(4\,x\right)$$

```
[In] integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x, algorithm="fricas")
[Out] -23/25*cos(4*x)*e^(3*x) - 14/25*e^(3*x)*sin(4*x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx=-\frac{14e^{3x}\sin\left(4x\right)}{25}-\frac{23e^{3x}\cos\left(4x\right)}{25}$$

```
[In] integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x)
[Out] -14*exp(3*x)*sin(4*x)/25 - 23*exp(3*x)*cos(4*x)/25
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\begin{aligned}\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx=&-\frac{2}{25}\left(4\,\cos\left(4\,x\right)-3\,\sin\left(4\,x\right)\right)e^{\left(3\,x\right)}\\&-\frac{1}{5}\left(3\,\cos\left(4\,x\right)+4\,\sin\left(4\,x\right)\right)e^{\left(3\,x\right)}\end{aligned}$$

```
[In] integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x, algorithm="maxima")
[Out] -2/25*(4*cos(4*x) - 3*sin(4*x))*e^(3*x) - 1/5*(3*cos(4*x) + 4*sin(4*x))*e^(
3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx = -\frac{2}{25}\,(4\,\cos\left(4\,x\right)-3\,\sin\left(4\,x\right))e^{(3\,x)} - \frac{1}{5}\,(3\,\cos\left(4\,x\right)+4\,\sin\left(4\,x\right))e^{(3\,x)}$$

```
[In] integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x, algorithm="giac")
[Out] -2/25*(4*cos(4*x) - 3*sin(4*x))*e^(3*x) - 1/5*(3*cos(4*x) + 4*sin(4*x))*e^(
3*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx = -\frac{\mathrm{e}^{3\,x}\,\left(23\,\cos\left(4\,x\right)+14\,\sin\left(4\,x\right)\right)}{25}$$

```
[In] int(-exp(3*x)*(5*cos(4*x) - 2*sin(4*x)),x)
[Out] -(exp(3*x)*(23*cos(4*x) + 14*sin(4*x)))/25
```

3.52 $\int (e^{-x} \sin(x) + e^x \sin(x)) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \left(e^{-x} \sin(x) + e^x \sin(x)\right) \, dx = -\frac{1}{2} e^{-x} \cos(x) - \frac{1}{2} e^x \cos(x) - \frac{1}{2} e^{-x} \sin(x) + \frac{1}{2} e^x \sin(x)$$

```
[Out] -1/2*cos(x)/exp(x)-1/2*exp(x)*cos(x)-1/2*sin(x)/exp(x)+1/2*exp(x)*sin(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4517}

$$\int \left(e^{-x} \sin(x) + e^x \sin(x)\right) \, dx = -\frac{1}{2} e^{-x} \sin(x) + \frac{1}{2} e^x \sin(x) - \frac{1}{2} e^{-x} \cos(x) - \frac{1}{2} e^x \cos(x)$$

```
[In] Int[Sin[x]/E^x + E^x*Sin[x],x]
[Out] -1/2*Cos[x]/E^x - (E^x*Cos[x])/2 - Sin[x]/(2*E^x) + (E^x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int e^{-x} \sin(x) \, dx + \int e^x \sin(x) \, dx \\ &= -\frac{1}{2} e^{-x} \cos(x) - \frac{1}{2} e^x \cos(x) - \frac{1}{2} e^{-x} \sin(x) + \frac{1}{2} e^x \sin(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \left(e^{-x}\sin(x)+e^x\sin(x)\right)\,dx=-\frac{1}{2}e^x\left(1+e^{-2x}\right)\cos(x)-\frac{1}{2}e^x\left(-1+e^{-2x}\right)\sin(x)$$

```
[In] Integrate[Sin[x]/E^x + E^x*Sin[x],x]
[Out] -1/2*(E^x*(1 + E^(-2*x))*Cos[x]) - (E^x*(-1 + E^(-2*x))*Sin[x])/2
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{(-\cos(x)-\sin(x))\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}(\cos(x)-\sin(x))}{2}$	28
default	$-\frac{\mathrm{e}^{x}\cos(x)}{2}+\frac{\mathrm{e}^{x}\sin(x)}{2}-\frac{\mathrm{e}^{-x}\cos(x)}{2}-\frac{\mathrm{e}^{-x}\sin(x)}{2}$	30
parts	$-\frac{\mathrm{e}^{x}\cos(x)}{2}+\frac{\mathrm{e}^{x}\sin(x)}{2}-\frac{\mathrm{e}^{-x}\cos(x)}{2}-\frac{\mathrm{e}^{-x}\sin(x)}{2}$	30
norman	$\frac{\left(-\frac{1}{2}+\mathrm{e}^{2x}\tan\left(\frac{x}{2}\right)-\frac{\mathrm{e}^{2x}}{2}+\frac{\tan\left(\frac{x}{2}\right)^{2}}{2}+\frac{\mathrm{e}^{2x}\tan\left(\frac{x}{2}\right)^{2}}{2}-\tan\left(\frac{x}{2}\right)\right)\mathrm{e}^{-x}}{1+\tan\left(\frac{x}{2}\right)^{2}}$	59
risch	$-\frac{\mathrm{e}^{(-1+i)x}}{4}+\frac{i\mathrm{e}^{(-1+i)x}}{4}-\frac{\mathrm{e}^{(-1-i)x}}{4}-\frac{i\mathrm{e}^{(-1-i)x}}{4}-\frac{\mathrm{e}^{(1+i)x}}{4}-\frac{i\mathrm{e}^{(1+i)x}}{4}-\frac{\mathrm{e}^{(1-i)x}}{4}+\frac{i\mathrm{e}^{(1-i)x}}{4}$	70

```
[In] int(sin(x)/exp(x)+exp(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 1/2*(-cos(x)-sin(x))*exp(-x)-1/2*exp(x)*(cos(x)-sin(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \left(e^{-x}\sin(x)+e^x\sin(x)\right)\,dx=-\frac{1}{2}\left(\cos\left(x\right)e^{(2x)}-\left(e^{(2x)}-1\right)\sin\left(x\right)+\cos\left(x\right)\right)e^{(-x)}$$

```
[In] integrate(sin(x)/exp(x)+exp(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*(cos(x)*e^(2*x) - (e^(2*x) - 1)*sin(x) + cos(x))*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \left(e^{-x}\sin(x)+e^{x}\sin(x)\right)\,dx=\frac{e^{x}\sin{(x)}}{2}-\frac{e^{x}\cos{(x)}}{2}-\frac{e^{-x}\sin{(x)}}{2}-\frac{e^{-x}\cos{(x)}}{2}$$

```
[In] integrate(sin(x)/exp(x)+exp(x)*sin(x),x)
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2 - exp(-x)*sin(x)/2 - exp(-x)*cos(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \left(e^{-x}\sin(x)+e^{x}\sin(x)\right)\,dx=-\frac{1}{2}\left(\cos{(x)}+\sin{(x)}\right)e^{(-x)}-\frac{1}{2}\left(\cos{(x)}-\sin{(x)}\right)e^{x}$$

```
[In] integrate(sin(x)/exp(x)+exp(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) + sin(x))*e^(-x) - 1/2*(cos(x) - sin(x))*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

$$\int \left(e^{-x}\sin(x)+e^{x}\sin(x)\right)\,dx=-\frac{1}{2}\left(\cos{(x)}+\sin{(x)}\right)e^{(-x)}-\frac{1}{2}\left(\cos{(x)}-\sin{(x)}\right)e^{x}$$

```
[In] integrate(sin(x)/exp(x)+exp(x)*sin(x),x, algorithm="giac")
[Out] -1/2*(cos(x) + sin(x))*e^(-x) - 1/2*(cos(x) - sin(x))*e^x
```

Mupad [B] (verification not implemented)

Time = 27.97 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \left(e^{-x}\sin(x)+e^{x}\sin(x)\right)\,dx=-\mathrm{e}^{-x}\left(\frac{\cos{(x)}}{2}+\frac{\sin{(x)}}{2}+\frac{\mathrm{e}^{2\,x}\,\cos{(x)}}{2}-\frac{\mathrm{e}^{2\,x}\,\sin{(x)}}{2}\right)$$

```
[In] int(exp(x)*sin(x) + exp(-x)*sin(x),x)
[Out] -exp(-x)*(cos(x)/2 + sin(x)/2 + (exp(2*x)*cos(x))/2 - (exp(2*x)*sin(x))/2)
```

3.53 $\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \frac{iF^{a+bx}}{be\log(F)} - \frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)}{be\log(F)}$$

```
[Out] I*F^(b*x+a)/b/e/ln(F)-2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(F
)/d],I*exp(I*(d*x+c)))/b/e/ln(F)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4547, 4527, 2225, 2283}

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \frac{iF^{a+bx}}{be\log(F)} - \frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)}{be\log(F)}$$

```
[In] Int[(F^(a + b*x)*Cos[c + d*x])/(e + e*Sin[c + d*x]),x]
```

```
[Out] (I*F^(a + b*x))/(b*e*Log[F]) - ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, ((-I
)*b*Log[F])/d, 1 - (I*b*Log[F])/d, I*E^(I*(c + d*x))])/(b*e*Log[F])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4547

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a +
 b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d,
 e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int F^{a+bx}\tan\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{e}\\
&= -\frac{i\int\left(-F^{a+bx}+\frac{2F^{a+bx}}{1+e^{2i\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)}}\right)dx}{e}\\
&= \frac{i\int F^{a+bx}\,dx}{e}-\frac{(2i)\int\frac{F^{a+bx}}{1+e^{2i\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)}}\,dx}{e}\\
&= \frac{iF^{a+bx}}{be\log(F)}-\frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)}{be\log(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = -\frac{iF^{a+bx}\left(-1+2\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)\right)}{be\log(F)}$$

```
[In] Integrate[(F^(a + b*x)*Cos[c + d*x])/(e + e*Sin[c + d*x]),x]
[Out] ((-I)*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*
Log[F])/d, I*E^(I*(c + d*x))]))/(b*e*Log[F])
```

Maple [F]

$$\int \frac{F^{xb+a}\cos(dx+c)}{e+e\sin(dx+c)}dx$$

```
[In] int(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x)
[Out] int(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x)
```

Fricas [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \int \frac{F^{bx+a}\cos(dx+c)}{e\sin(dx+c)+e}\,dx$$

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x, algorithm="fricas")
[Out] integral(F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) + e), x)
```

Sympy [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \frac{\int \frac{F^{a+bx}\cos(c+dx)}{\sin(c+dx)+1}\,dx}{e}$$

```
[In] integrate(F**(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x)
[Out] Integral(F**(a + b*x)*cos(c + d*x)/(sin(c + d*x) + 1), x)/e
```

Maxima [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \int \frac{F^{bx+a}\cos\left(dx+c\right)}{e\sin\left(dx+c\right)+e}\,dx$$

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*F^(b*x)*F^a*b*d*cos(d*x + c)*log(F) + 2*F^(b*x)*F^a*d^2*sin(d*x + c) +
(F^a*b^2*log(F)^2 + F^a*d^2)*F^(b*x)*cos(d*x + c)^2 + (F^a*b^2*log(F)^2 + F
^a*d^2)*F^(b*x)*sin(d*x + c)^2 - (F^a*b^2*log(F)^2 - F^a*d^2)*F^(b*x) - 2*(
(F^a*b^3*d*log(F)^3 + F^a*b*d^3*log(F))*e*cos(d*x + c)^2 + (F^a*b^3*d*log(F
)^3 + F^a*b*d^3*log(F))*e*sin(d*x + c)^2 + 2*(F^a*b^3*d*log(F)^3 + F^a*b*d^
3*log(F))*e*sin(d*x + c) + (F^a*b^3*d*log(F)^3 + F^a*b*d^3*log(F))*e)*integ
rate((2*F^(b*x)*b*cos(d*x + c)*log(F) + F^(b*x)*b*log(F)*sin(2*d*x + 2*c) -
 F^(b*x)*d*cos(2*d*x + 2*c) + 2*F^(b*x)*d*sin(d*x + c) + F^(b*x)*d)/((b^2*l
og(F)^2 + d^2)*e*cos(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c)
^2 + 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c)*sin(2*d*x + 2*c) + (b^2*log(F)^2
 + d^2)*e*sin(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*sin(d*x + c)^2 + 4*
(b^2*log(F)^2 + d^2)*e*sin(d*x + c) + (b^2*log(F)^2 + d^2)*e - 2*(2*(b^2*lo
g(F)^2 + d^2)*e*sin(d*x + c) + (b^2*log(F)^2 + d^2)*e)*cos(2*d*x + 2*c)), x
))/((b^3*log(F)^3 + b*d^2*log(F))*e*cos(d*x + c)^2 + (b^3*log(F)^3 + b*d^2*
log(F))*e*sin(d*x + c)^2 + 2*(b^3*log(F)^3 + b*d^2*log(F))*e*sin(d*x + c) +
 (b^3*log(F)^3 + b*d^2*log(F))*e)
```

Giac [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \int \frac{F^{bx+a}\cos\left(dx+c\right)}{e\sin\left(dx+c\right)+e}\,dx$$

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx = \int \frac{F^{a+b\,x}\,\cos\left(c+d\,x\right)}{e+e\,\sin\left(c+d\,x\right)}\,dx$$

```
[In] int((F^(a + b*x)*cos(c + d*x))/(e + e*sin(c + d*x)),x)
```

```
[Out] int((F^(a + b*x)*cos(c + d*x))/(e + e*sin(c + d*x)), x)
```

3.54 $\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = -\frac{iF^{a+bx}}{be\log(F)} + \frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-ie^{i(c+dx)}\right)}{be\log(F)}$$

```
[Out] -I*F^(b*x+a)/b/e/ln(F)+2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(
F)/d],-I*exp(I*(d*x+c)))/b/e/ln(F)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4547, 4527, 2225, 2283}

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = \frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-ie^{i(c+dx)}\right)}{be\log(F)} - \frac{iF^{a+bx}}{be\log(F)}$$

```
[In] Int[(F^(a + b*x)*Cos[c + d*x])/(e - e*Sin[c + d*x]),x]
```

```
[Out] ((-I)*F^(a + b*x))/(b*e*Log[F]) + ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, (
(-I)*b*Log[F])/d, 1 - (I*b*Log[F])/d, (-I)*E^(I*(c + d*x))])/(b*e*Log[F])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4547

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a +
 b*x))*Tan[f*(Pi/(4*g)) - d/2 - e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d,
 e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int F^{a+bx}\tan\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)\,dx}{e} \\
&= \frac{i\int\left(-F^{a+bx}+\frac{2F^{a+bx}}{1+e^{2i\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}}\right)dx}{e} \\
&= -\frac{i\int F^{a+bx}\,dx}{e}+\frac{(2i)\int\frac{F^{a+bx}}{1+e^{2i\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}}\,dx}{e} \\
&= -\frac{iF^{a+bx}}{be\log(F)}+\frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-ie^{i(c+dx)}\right)}{be\log(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = \frac{iF^{a+bx}\left(-1+2\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-ie^{i(c+dx)}\right)\right)}{be\log(F)}$$

```
[In] Integrate[(F^(a + b*x)*Cos[c + d*x])/(e - e*Sin[c + d*x]),x]
[Out] (I*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*Log
[F])/d, (-I)*E^(I*(c + d*x))]))/(b*e*Log[F])
```

Maple [F]

$$\int \frac{F^{xb+a}\cos(dx+c)}{e-e\sin(dx+c)}dx$$

```
[In] int(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x)
[Out] int(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x)
```

Fricas [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = \int -\frac{F^{bx+a}\cos(dx+c)}{e\sin(dx+c)-e}\,dx$$

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x, algorithm="fricas")
[Out] integral(-F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) - e), x)
```

Sympy [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = -\frac{\int \frac{F^{a+bx}\cos(c+dx)}{\sin(c+dx)-1}\,dx}{e}$$

```
[In] integrate(F**(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x)
[Out] -Integral(F**(a + b*x)*cos(c + d*x)/(sin(c + d*x) - 1), x)/e
```

Maxima [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = \int -\frac{F^{bx+a}\cos(dx+c)}{e\sin(dx+c)-e}\,dx$$

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*F^(b*x)*F^a*b*d*cos(d*x + c)*log(F) + 2*F^(b*x)*F^a*d^2*sin(d*x + c) -
(F^a*b^2*log(F)^2 + F^a*d^2)*F^(b*x)*cos(d*x + c)^2 - (F^a*b^2*log(F)^2 + F
^a*d^2)*F^(b*x)*sin(d*x + c)^2 + (F^a*b^2*log(F)^2 - F^a*d^2)*F^(b*x) + 2*(
(F^a*b^3*d*log(F)^3 + F^a*b*d^3*log(F))*e*cos(d*x + c)^2 + (F^a*b^3*d*log(F
)^3 + F^a*b*d^3*log(F))*e*sin(d*x + c)^2 - 2*(F^a*b^3*d*log(F)^3 + F^a*b*d^
3*log(F))*e*sin(d*x + c) + (F^a*b^3*d*log(F)^3 + F^a*b*d^3*log(F))*e)*integ
rate(-(2*F^(b*x)*b*cos(d*x + c)*log(F) - F^(b*x)*b*log(F)*sin(2*d*x + 2*c)
+ F^(b*x)*d*cos(2*d*x + 2*c) + 2*F^(b*x)*d*sin(d*x + c) - F^(b*x)*d)/((b^2*
log(F)^2 + d^2)*e*cos(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c
)^2 - 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c)*sin(2*d*x + 2*c) + (b^2*log(F)^
2 + d^2)*e*sin(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*sin(d*x + c)^2 - 4
*(b^2*log(F)^2 + d^2)*e*sin(d*x + c) + (b^2*log(F)^2 + d^2)*e + 2*(2*(b^2*l
og(F)^2 + d^2)*e*sin(d*x + c) - (b^2*log(F)^2 + d^2)*e)*cos(2*d*x + 2*c)),
x))/((b^3*log(F)^3 + b*d^2*log(F))*e*cos(d*x + c)^2 + (b^3*log(F)^3 + b*d^2
*log(F))*e*sin(d*x + c)^2 - 2*(b^3*log(F)^3 + b*d^2*log(F))*e*sin(d*x + c)
+ (b^3*log(F)^3 + b*d^2*log(F))*e)
```

Giac [F]

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = \int -\frac{F^{bx+a}\cos(dx+c)}{e\sin(dx+c)-e}\,dx$$

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(-F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) - e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx = \int \frac{F^{a+b\,x}\,\cos(c+d\,x)}{e-e\,\sin(c+d\,x)}\,dx$$

```
[In] int((F^(a + b*x)*cos(c + d*x))/(e - e*sin(c + d*x)),x)
```

```
[Out] int((F^(a + b*x)*cos(c + d*x))/(e - e*sin(c + d*x)), x)
```

3.55 $\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = -\frac{iF^{a+bx}}{be\log(F)}+\frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-e^{i(c+dx)}\right)}{be\log(F)}$$

```
[Out] -I*F^(b*x+a)/b/e/ln(F)+2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(
F)/d],-exp(I*(d*x+c)))/b/e/ln(F)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4548, 4527, 2225, 2283}

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = \frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-e^{i(c+dx)}\right)}{be\log(F)} - \frac{iF^{a+bx}}{be\log(F)}$$

```
[In] Int[(F^(a + b*x)*Sin[c + d*x])/(e + e*Cos[c + d*x]),x]
```

```
[Out] ((-I)*F^(a + b*x))/(b*e*Log[F]) + ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, (
(-I)*b*Log[F])/d, 1 - (I*b*Log[F])/d, -E^(I*(c + d*x))])/(b*e*Log[F])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4548

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Dist[f^n, Int[F^(c*(a +
 b*x))*Tan[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] &
& EqQ[f - g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int F^{a+bx}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,dx}{e} \\
&= \frac{i\int\left(-F^{a+bx}+\frac{2F^{a+bx}}{1+e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}\right)dx}{e} \\
&= -\frac{i\int F^{a+bx}\,dx}{e}+\frac{(2i)\int\frac{F^{a+bx}}{1+e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}\,dx}{e} \\
&= -\frac{iF^{a+bx}}{be\log(F)}+\frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-e^{i(c+dx)}\right)}{be\log(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = \frac{iF^{a+bx}\left(-1+2\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-\cos(c+dx)-i\sin(c+dx)\right)\right)}{be\log(F)}$$

```
[In] Integrate[(F^(a + b*x)*Sin[c + d*x])/(e + e*Cos[c + d*x]),x]
[Out] (I*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*Log
[F])/d, -Cos[c + d*x] - I*Sin[c + d*x]]))/(b*e*Log[F])
```

Maple [F]

$$\int \frac{F^{xb+a}\sin(dx+c)}{e+e\cos(dx+c)}dx$$

```
[In] int(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x)
[Out] int(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x)
```

Fricas [F]

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = \int \frac{F^{bx+a}\sin(dx+c)}{e\cos(dx+c)+e}\,dx$$

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x, algorithm="fricas")
[Out] integral(F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) + e), x)
```

Sympy [F]

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = \frac{\int \frac{F^{a+bx}\sin(c+dx)}{\cos(c+dx)+1}\,dx}{e}$$

```
[In] integrate(F**(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x)
[Out] Integral(F**(a + b*x)*sin(c + d*x)/(cos(c + d*x) + 1), x)/e
```

Maxima [F]

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = \int \frac{F^{bx+a}\sin\left(dx+c\right)}{e\cos\left(dx+c\right)+e}\,dx$$

[In] integrate(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x, algorithm="maxima")

[Out] 2*(F^(b*x)*F^a*b*log(F)*sin(d*x + c) - F^(b*x)*F^a*d*cos(d*x + c) - F^(b*x)*F^a*d + ((F^a*b^2*d*log(F)^2 + F^a*d^3)*e*cos(d*x + c)^2 + (F^a*b^2*d*log(F)^2 + F^a*d^3)*e*sin(d*x + c)^2 + 2*(F^a*b^2*d*log(F)^2 + F^a*d^3)*e*cos(d*x + c) + (F^a*b^2*d*log(F)^2 + F^a*d^3)*e)*integrate((F^(b*x)*b*cos(2*d*x + 2*c)*log(F) + 2*F^(b*x)*b*cos(d*x + c)*log(F) + F^(b*x)*b*log(F) + F^(b*x)*d*sin(2*d*x + 2*c) + 2*F^(b*x)*d*sin(d*x + c))/((b^2*log(F)^2 + d^2)*e*cos(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c)^2 + (b^2*log(F)^2 + d^2)*e*sin(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*sin(2*d*x + 2*c)*sin(d*x + c) + 4*(b^2*log(F)^2 + d^2)*e*sin(d*x + c)^2 + 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c) + (b^2*log(F)^2 + d^2)*e + 2*(2*(b^2*log(F)^2 + d^2)*e*cos(d*x + c) + (b^2*log(F)^2 + d^2)*e)*cos(2*d*x + 2*c)), x))/((b^2*log(F)^2 + d^2)*e*cos(d*x + c)^2 + (b^2*log(F)^2 + d^2)*e*sin(d*x + c)^2 + 2*(b^2*log(F)^2 + d^2)*e*cos(d*x + c) + (b^2*log(F)^2 + d^2)*e)

Giac [F]

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = \int \frac{F^{bx+a}\sin\left(dx+c\right)}{e\cos\left(dx+c\right)+e}\,dx$$

[In] integrate(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) + e), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx = \int \frac{F^{a+b\,x}\,\sin\left(c+d\,x\right)}{e+e\,\cos\left(c+d\,x\right)}\,dx$$

[In] int((F^(a + b*x)*sin(c + d*x))/(e + e*cos(c + d*x)),x)

[Out] int((F^(a + b*x)*sin(c + d*x))/(e + e*cos(c + d*x)), x)

3.56 $\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = \frac{iF^{a+bx}}{be\log(F)} - \frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},e^{i(c+dx)}\right)}{be\log(F)}$$

```
[Out] I*F^(b*x+a)/b/e/ln(F)-2*I*F^(b*x+a)*hypergeom([1, -I*b*ln(F)/d],[1-I*b*ln(F
)/d],exp(I*(d*x+c)))/b/e/ln(F)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4549, 4528, 2225, 2283}

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = \frac{iF^{a+bx}}{be\log(F)} - \frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},e^{i(c+dx)}\right)}{be\log(F)}$$

```
[In] Int[(F^(a + b*x)*Sin[c + d*x])/(e - e*Cos[c + d*x]),x]
```

```
[Out] (I*F^(a + b*x))/(b*e*Log[F]) - ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, ((-I
)*b*Log[F])/d, 1 - (I*b*Log[F])/d, E^(I*(c + d*x))])/(b*e*Log[F])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4528

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*I*(d + e
*x)))^n/(1 - E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rule 4549

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Dist[f^n, Int[F^(c*(a +
 b*x))*Cot[d/2 + e*(x/2)]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] &
& EqQ[f + g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int F^{a+bx}\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\,dx}{e}\\
&= -\frac{i\int\left(-F^{a+bx}-\frac{2F^{a+bx}}{-1+e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}\right)\,dx}{e}\\
&= \frac{i\int F^{a+bx}\,dx}{e}+\frac{(2i)\int\frac{F^{a+bx}}{-1+e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}\,dx}{e}\\
&= \frac{iF^{a+bx}}{be\log(F)}-\frac{2iF^{a+bx}\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},e^{i(c+dx)}\right)}{be\log(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = -\frac{iF^{a+bx}\left(-1+2\,\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},\cos(c+dx)+i\sin(c+dx)\right)\right)}{be\log(F)}$$

```
[In] Integrate[(F^(a + b*x)*Sin[c + d*x])/(e - e*Cos[c + d*x]),x]
[Out] ((-I)*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*
Log[F])/d, Cos[c + d*x] + I*Sin[c + d*x]]))/(b*e*Log[F])
```

Maple [F]

$$\int \frac{F^{xb+a}\sin\left(dx+c\right)}{e-e\cos\left(dx+c\right)}dx$$

```
[In] int(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x)
[Out] int(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x)
```

Fricas [F]

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = \int -\frac{F^{bx+a}\sin\left(dx+c\right)}{e\cos\left(dx+c\right)-e}\,dx$$

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x, algorithm="fricas")
[Out] integral(-F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) - e), x)
```

Sympy [F]

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = -\frac{\int \frac{F^{a+bx}\sin\left(c+dx\right)}{\cos\left(c+dx\right)-1}\,dx}{e}$$

```
[In] integrate(F**(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x)
[Out] -Integral(F**(a + b*x)*sin(c + d*x)/(cos(c + d*x) - 1), x)/e
```

Maxima [F]

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = \int -\frac{F^{bx+a}\sin(dx+c)}{e\cos(dx+c)-e}\,dx$$

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(F^(b*x)*F^a*b*log(F)*sin(d*x + c) - F^(b*x)*F^a*d*cos(d*x + c) + F^(b*x)
*F^a*d - ((F^a*b^2*d*log(F)^2 + F^a*d^3)*e*cos(d*x + c)^2 + (F^a*b^2*d*log(
F)^2 + F^a*d^3)*e*sin(d*x + c)^2 - 2*(F^a*b^2*d*log(F)^2 + F^a*d^3)*e*cos(d
*x + c) + (F^a*b^2*d*log(F)^2 + F^a*d^3)*e)*integrate((F^(b*x)*b*cos(2*d*x
+ 2*c)*log(F) - 2*F^(b*x)*b*cos(d*x + c)*log(F) + F^(b*x)*b*log(F) + F^(b*x
)*d*sin(2*d*x + 2*c) - 2*F^(b*x)*d*sin(d*x + c))/((b^2*log(F)^2 + d^2)*e*co
s(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c)^2 + (b^2*log(F)^2
+ d^2)*e*sin(2*d*x + 2*c)^2 - 4*(b^2*log(F)^2 + d^2)*e*sin(2*d*x + 2*c)*sin
(d*x + c) + 4*(b^2*log(F)^2 + d^2)*e*sin(d*x + c)^2 - 4*(b^2*log(F)^2 + d^2
)*e*cos(d*x + c) + (b^2*log(F)^2 + d^2)*e - 2*(2*(b^2*log(F)^2 + d^2)*e*cos
(d*x + c) - (b^2*log(F)^2 + d^2)*e)*cos(2*d*x + 2*c)), x))/((b^2*log(F)^2 +
 d^2)*e*cos(d*x + c)^2 + (b^2*log(F)^2 + d^2)*e*sin(d*x + c)^2 - 2*(b^2*log
(F)^2 + d^2)*e*cos(d*x + c) + (b^2*log(F)^2 + d^2)*e)
```

Giac [F]

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = \int -\frac{F^{bx+a}\sin(dx+c)}{e\cos(dx+c)-e}\,dx$$

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(-F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) - e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx = \int \frac{F^{a+b\,x}\,\sin(c+d\,x)}{e-e\,\cos(c+d\,x)}\,dx$$

```
[In] int((F^(a + b*x)*sin(c + d*x))/(e - e*cos(c + d*x)),x)
```

```
[Out] int((F^(a + b*x)*sin(c + d*x))/(e - e*cos(c + d*x)), x)
```

3.57 $\int e^{x^2} \sin(bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 69

$$\int e^{x^2} \sin(bx)\,dx = \frac{1}{4}ie^{\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-ib+2x)\right) - \frac{1}{4}ie^{\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(ib+2x)\right)$$

```
[Out] -1/4*I*exp(1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)-1/4*I*exp(1/4*b^2)*erfi(1/2*I*
b+x)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4560, 2266, 2235}

$$\int e^{x^2} \sin(bx)\,dx = \frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}}\text{erfi}\left(\frac{1}{2}(2x-ib)\right) - \frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}}\text{erfi}\left(\frac{1}{2}(2x+ib)\right)$$

```
[In] Int[E^x^2*Sin[b*x],x]
```

```
[Out] (I/4)*E^(b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(b^2/4)*Sqrt[Pi]*
Erfi[(I*b + 2*x)/2]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2} i e^{-ibx+x^2} - \frac{1}{2} i e^{ibx+x^2}\right) dx \\ &= \frac{1}{2} i \int e^{-ibx+x^2}\, dx - \frac{1}{2} i \int e^{ibx+x^2}\, dx \\ &= \frac{1}{2}\left(i e^{\frac{b^2}{4}}\right) \int e^{\frac{1}{4}(-ib+2x)^2}\, dx - \frac{1}{2}\left(i e^{\frac{b^2}{4}}\right) \int e^{\frac{1}{4}(ib+2x)^2}\, dx \\ &= \frac{1}{4} i e^{\frac{b^2}{4}} \sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-ib+2x)\right) - \frac{1}{4} i e^{\frac{b^2}{4}} \sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(ib+2x)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int e^{x^2} \sin(bx)\, dx = \frac{1}{4} e^{\frac{b^2}{4}} \sqrt{\pi} \left(\text{erf}\left(\frac{b}{2} - ix\right) + \text{erf}\left(\frac{b}{2} + ix\right)\right)$$

```
[In] Integrate[E^x^2*Sin[b*x],x]
[Out] (E^(b^2/4)*Sqrt[Pi]*(Erf[b/2 - I*x] + Erf[b/2 + I*x]))/4
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{\sqrt{\pi}\, e^{\frac{b^2}{4}}\, \text{erf}\left(-ix+\frac{b}{2}\right)}{4} + \frac{\sqrt{\pi}\, e^{\frac{b^2}{4}}\, \text{erf}\left(ix+\frac{b}{2}\right)}{4}$	42

```
[In] int(exp(x^2)*sin(x*b),x,method=_RETURNVERBOSE)
[Out] 1/4*Pi^(1/2)*exp(1/4*b^2)*erf(-I*x+1/2*b)+1/4*Pi^(1/2)*exp(1/4*b^2)*erf(I*x
+1/2*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.43

$$\int e^{x^2}\sin(bx)\,dx = \frac{1}{4}\sqrt{\pi}\left(\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)-\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)\right)e^{\left(\frac{1}{4}\,b^2\right)}$$

```
[In] integrate(exp(x^2)*sin(b*x),x, algorithm="fricas")
[Out] 1/4*sqrt(pi)*(erf(1/2*b + I*x) - erf(-1/2*b + I*x))*e^(1/4*b^2)
```

Sympy [F]

$$\int e^{x^2}\sin(bx)\,dx = \int e^{x^2}\sin(bx)\,dx$$

```
[In] integrate(exp(x**2)*sin(b*x),x)
[Out] Integral(exp(x**2)*sin(b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int e^{x^2}\sin(bx)\,dx = \frac{1}{4}\sqrt{\pi}\left(\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}-\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}\right)$$

```
[In] integrate(exp(x^2)*sin(b*x),x, algorithm="maxima")
[Out] 1/4*sqrt(pi)*(erf(1/2*b + I*x)*e^(1/4*b^2) - erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Giac [F]

$$\int e^{x^2}\sin(bx)\,dx = \int e^{\left(x^2\right)}\sin(bx)\,dx$$

```
[In] integrate(exp(x^2)*sin(b*x),x, algorithm="giac")
[Out] integrate(e^(x^2)*sin(b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \sin(bx)\,dx = \int \mathrm{e}^{x^2}\, \sin\left(b\,x\right)\,dx$$

```
[In] int(exp(x^2)*sin(b*x),x)
[Out] int(exp(x^2)*sin(b*x), x)
```

3.58 $\int e^{x^2} \cos(bx) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 65

$$\int e^{x^2} \cos(bx) \, dx = \frac{1}{4} e^{\frac{b^2}{4}} \sqrt{\pi} \operatorname{erfi}\left(\frac{1}{2}(-ib + 2x)\right) + \frac{1}{4} e^{\frac{b^2}{4}} \sqrt{\pi} \operatorname{erfi}\left(\frac{1}{2}(ib + 2x)\right)$$

```
[Out] -1/4*exp(1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)+1/4*exp(1/4*b^2)*erfi(1/2*I*b+x)
*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4561, 2266, 2235}

$$\int e^{x^2} \cos(bx) \, dx = \frac{1}{4} \sqrt{\pi} e^{\frac{b^2}{4}} \operatorname{erfi}\left(\frac{1}{2}(2x - ib)\right) + \frac{1}{4} \sqrt{\pi} e^{\frac{b^2}{4}} \operatorname{erfi}\left(\frac{1}{2}(2x + ib)\right)$$

```
[In] Int[E^x^2*Cos[b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(b^2/4)*Sqrt[Pi]*Erfi[(I
*b + 2*x)/2])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2}e^{-ibx+x^2}+\frac{1}{2}e^{ibx+x^2}\right)\,dx\\ &= \frac{1}{2}\int e^{-ibx+x^2}\,dx+\frac{1}{2}\int e^{ibx+x^2}\,dx\\ &= \frac{1}{2}e^{\frac{b^2}{4}}\int e^{\frac{1}{4}(-ib+2x)^2}\,dx+\frac{1}{2}e^{\frac{b^2}{4}}\int e^{\frac{1}{4}(ib+2x)^2}\,dx\\ &= \frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(ib+2x)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int e^{x^2}\cos(bx)\,dx=\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\text{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\text{erfi}\left(\frac{1}{2}(ib+2x)\right)\right)$$

```
[In] Integrate[E^x^2*Cos[b*x],x]
[Out] (E^(b^2/4)*Sqrt[Pi]*(Erfi[((-I)*b + 2*x)/2] + Erfi[(I*b + 2*x)/2]))/4
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}+\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}$	44

```
[In] int(exp(x^2)*cos(x*b),x,method=_RETURNVERBOSE)
[Out] -1/4*I*Pi^(1/2)*exp(1/4*b^2)*erf(I*x+1/2*b)+1/4*I*Pi^(1/2)*exp(1/4*b^2)*erf
(-I*x+1/2*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.49

$$\int e^{x^2}\cos(bx)\,dx = \frac{1}{4}\sqrt{\pi}\left(-i\,\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)-i\,\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)\right)e^{\left(\frac{1}{4}\,b^2\right)}$$

```
[In] integrate(exp(x^2)*cos(b*x),x, algorithm="fricas")
[Out] 1/4*sqrt(pi)*(-I*erf(1/2*b + I*x) - I*erf(-1/2*b + I*x))*e^(1/4*b^2)
```

Sympy [F]

$$\int e^{x^2}\cos(bx)\,dx = \int e^{x^2}\cos\left(bx\right)dx$$

```
[In] integrate(exp(x**2)*cos(b*x),x)
[Out] Integral(exp(x**2)*cos(b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int e^{x^2}\cos(bx)\,dx = -\frac{1}{4}\sqrt{\pi}\left(i\,\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}+i\,\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}\right)$$

```
[In] integrate(exp(x^2)*cos(b*x),x, algorithm="maxima")
[Out] -1/4*sqrt(pi)*(I*erf(1/2*b + I*x)*e^(1/4*b^2) + I*erf(-1/2*b + I*x)*e^(1/4*
b^2))
```

Giac [F]

$$\int e^{x^2}\cos(bx)\,dx = \int \cos\left(bx\right)e^{\left(x^2\right)}\,dx$$

```
[In] integrate(exp(x^2)*cos(b*x),x, algorithm="giac")
[Out] integrate(cos(b*x)*e^(x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\cos(bx)\,dx = \int \mathrm{e}^{x^2}\,\cos(b\,x)\,dx$$

```
[In] int(exp(x^2)*cos(b*x),x)
[Out] int(exp(x^2)*cos(b*x), x)
```

3.59 $\int e^{x^2} \sin(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 81

$$\int e^{x^2}\sin(a+bx)\,dx=\frac{1}{4}ie^{-ia+\frac{b^2}{4}}\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-ib+2x)\right)-\frac{1}{4}ie^{ia+\frac{b^2}{4}}\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(ib+2x)\right)$$

```
[Out] -1/4*I*exp(-I*a+1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)-1/4*I*exp(I*a+1/4*b^2)*er
fi(1/2*I*b+x)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4560, 2266, 2235}

$$\int e^{x^2}\sin(a+bx)\,dx=\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}-ia}\operatorname{erfi}\left(\frac{1}{2}(2x-ib)\right)-\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}+ia}\operatorname{erfi}\left(\frac{1}{2}(2x+ib)\right)$$

```
[In] Int[E^x^2*Sin[a + b*x],x]
```

```
[Out] (I/4)*E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(I*a + b
^2/4)*Sqrt[Pi]*Erfi[(I*b + 2*x)/2]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}ie^{-ia-ibx+x^2}-\frac{1}{2}ie^{ia+ibx+x^2}\right)dx \\
&= \frac{1}{2}i\int e^{-ia-ibx+x^2}\,dx-\frac{1}{2}i\int e^{ia+ibx+x^2}\,dx \\
&= \frac{1}{2}\left(ie^{-ia+\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(-ib+2x)^2}\,dx-\frac{1}{2}\left(ie^{ia+\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(ib+2x)^2}\,dx \\
&= \frac{1}{4}ie^{-ia+\frac{b^2}{4}}\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(-ib+2x)\right)-\frac{1}{4}ie^{ia+\frac{b^2}{4}}\sqrt{\pi}\operatorname{erfi}\left(\frac{1}{2}(ib+2x)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$
\int e^{x^2}\sin(a+bx)\,dx=\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\cos(a)\operatorname{erf}\left(\frac{b}{2}-ix\right)+\cos(a)\operatorname{erf}\left(\frac{b}{2}+ix\right)+\left(\operatorname{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\operatorname{erfi}\left(\frac{1}{2}(ib+2x)\right)\right)\sin(a)\right)
$$

```
[In] Integrate[E^x^2*Sin[a + b*x],x]
[Out] (E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erf[b/2 - I*x] + Cos[a]*Erf[b/2 + I*x] + (Erfi[
((-I)*b + 2*x)/2] + Erfi[(I*b + 2*x)/2])*Sin[a]))/4
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{ia}\operatorname{erf}\left(-ix+\frac{b}{2}\right)}{4}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{-ia}\operatorname{erf}\left(ix+\frac{b}{2}\right)}{4}$	52

```
[In] int(exp(x^2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(1/4*b^2)*exp(I*a)*erf(-I*x+1/2*b)+1/4*Pi^(1/2)*exp(1/4*b^2
)*exp(-I*a)*erf(I*x+1/2*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.56

$$\int e^{x^2}\sin(a+bx)\,dx = -\frac{1}{4}\,\sqrt{\pi}\left(\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2+i\,a\right)} - \operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2-i\,a\right)}\right)$$

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(pi)*(erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - erf(1/2*b + I*x)*e^(1/
4*b^2 - I*a))
```

Sympy [F]

$$\int e^{x^2}\sin(a+bx)\,dx = \int e^{x^2}\sin\left(a+bx\right)dx$$

```
[In] integrate(exp(x**2)*sin(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sin(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

$$\int e^{x^2}\sin(a+bx)\,dx$$
$$= \frac{1}{4}\,\sqrt{\pi}\left(\left(\cos\left(a\right) - i\,\sin\left(a\right)\right)\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)} - \left(\cos\left(a\right) + i\,\sin\left(a\right)\right)\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}\right)$$

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*((cos(a) - I*sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) - (cos(a) +
I*sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Giac [F]

$$\int e^{x^2} \sin(a+bx)\,dx = \int e^{(x^2)} \sin(bx+a)\,dx$$

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="giac")
[Out] integrate(e^(x^2)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \sin(a+bx)\,dx = \int e^{x^2} \sin(a+b\,x)\,dx$$

```
[In] int(exp(x^2)*sin(a + b*x),x)
[Out] int(exp(x^2)*sin(a + b*x), x)
```

3.60 $\int e^{x^2} \cos(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int e^{x^2} \cos(a+bx)\,dx = \frac{1}{4}e^{-ia+\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{ia+\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(ib+2x)\right)$$

```
[Out] -1/4*exp(-I*a+1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)+1/4*exp(I*a+1/4*b^2)*erfi(1
/2*I*b+x)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4561, 2266, 2235}

$$\int e^{x^2} \cos(a+bx)\,dx = \frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}-ia}\text{erfi}\left(\frac{1}{2}(2x-ib)\right)+\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}+ia}\text{erfi}\left(\frac{1}{2}(2x+ib)\right)$$

```
[In] Int[E^x^2*Cos[a + b*x],x]
```

```
[Out] (E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(I*a + b^2/4)*S
qrt[Pi]*Erfi[(I*b + 2*x)/2])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-ia-ibx+x^2}+\frac{1}{2}e^{ia+ibx+x^2}\right) dx \\
&= \frac{1}{2}\int e^{-ia-ibx+x^2}\,dx+\frac{1}{2}\int e^{ia+ibx+x^2}\,dx \\
&= \frac{1}{2}e^{-ia+\frac{b^2}{4}}\int e^{\frac{1}{4}(-ib+2x)^2}\,dx+\frac{1}{2}e^{ia+\frac{b^2}{4}}\int e^{\frac{1}{4}(ib+2x)^2}\,dx \\
&= \frac{1}{4}e^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(ib+2x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int e^{x^2}\cos(a+bx)\,dx=\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\cos(a)\text{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\cos(a)\text{erfi}\left(\frac{1}{2}(ib+2x)\right)-\left(\text{erf}\left(\frac{b}{2}-ix\right)+\text{erf}\left(\frac{b}{2}+ix\right)\right)\sin(a)\right)$$

```
[In] Integrate[E^x^2*Cos[a + b*x],x]
[Out] (E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erfi[((-I)*b + 2*x)/2] + Cos[a]*Erfi[(I*b + 2*x
)/2] - (Erf[b/2 - I*x] + Erf[b/2 + I*x])*Sin[a]))/4
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{i\sqrt{\pi}\,e^{\frac{b^2}{4}}\,e^{-ia}\,\text{erf}\left(ix+\frac{b}{2}\right)}{4}+\frac{i\sqrt{\pi}\,e^{\frac{b^2}{4}}\,e^{ia}\,\text{erf}\left(-ix+\frac{b}{2}\right)}{4}$	54

```
[In] int(exp(x^2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(1/4*b^2)*exp(-I*a)*erf(I*x+1/2*b)+1/4*I*Pi^(1/2)*exp(1/
4*b^2)*exp(I*a)*erf(-I*x+1/2*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60

$$\int e^{x^2}\cos(a+bx)\,dx=\frac{1}{4}\,\sqrt{\pi}\left(-i\,\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2+i\,a\right)}-i\,\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2-i\,a\right)}\right)$$

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(-I*erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - I*erf(1/2*b + I*x)*e
^(1/4*b^2 - I*a))
```

Sympy [F]

$$\int e^{x^2}\cos(a+bx)\,dx=\int e^{x^2}\cos\left(a+bx\right)dx$$

```
[In] integrate(exp(x**2)*cos(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cos(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int e^{x^2}\cos(a+bx)\,dx=$$
$$-\frac{1}{4}\,\sqrt{\pi}\left(\left(i\,\cos\left(a\right)+\sin\left(a\right)\right)\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}+\left(i\,\cos\left(a\right)-\sin\left(a\right)\right)\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}\right)$$

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*((I*cos(a) + sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) + (I*cos(a)
 - sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Giac [F]

$$\int e^{x^2}\cos(a+bx)\,dx = \int \cos\left(bx+a\right)e^{\left(x^2\right)}\,dx$$

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*e^(x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\cos(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\mathrm{e}^{x^2}\,dx$$

```
[In] int(cos(a + b*x)*exp(x^2),x)
[Out] int(cos(a + b*x)*exp(x^2), x)
```

3.61 $\int e^{2x^2} x \cos(2x^2)\, dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int e^{2x^2} x \cos(2x^2)\, dx = \frac{1}{8}e^{2x^2}\cos(2x^2) + \frac{1}{8}e^{2x^2}\sin(2x^2)$$

```
[Out] 1/8*exp(2*x^2)*cos(2*x^2)+1/8*exp(2*x^2)*sin(2*x^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6847, 4518}

$$\int e^{2x^2} x \cos(2x^2)\, dx = \frac{1}{8}e^{2x^2}\sin(2x^2) + \frac{1}{8}e^{2x^2}\cos(2x^2)$$

```
[In] Int[E^(2*x^2)*x*Cos[2*x^2],x]
```

```
[Out] (E^(2*x^2)*Cos[2*x^2])/8 + (E^(2*x^2)*Sin[2*x^2])/8
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
```

```
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int e^{2x}\cos(2x)\,dx, x, x^2\right) \\ &= \frac{1}{8}e^{2x^2}\cos\left(2x^2\right)+\frac{1}{8}e^{2x^2}\sin\left(2x^2\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int e^{2x^2}x\cos\left(2x^2\right)\,dx=\frac{1}{8}e^{2x^2}\left(\cos\left(2x^2\right)+\sin\left(2x^2\right)\right)$$

```
[In] Integrate[E^(2*x^2)*x*Cos[2*x^2],x]
[Out] (E^(2*x^2)*(Cos[2*x^2] + Sin[2*x^2]))/8
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{e^{2x^2}\left(\cos\left(2x^2\right)+\sin\left(2x^2\right)\right)}{8}$	22
derivativedivides	$\frac{e^{2x^2}\cos\left(2x^2\right)}{8}+\frac{e^{2x^2}\sin\left(2x^2\right)}{8}$	30
default	$\frac{e^{2x^2}\cos\left(2x^2\right)}{8}+\frac{e^{2x^2}\sin\left(2x^2\right)}{8}$	30
risch	$\frac{e^{(2+2i)x^2}}{16}-\frac{ie^{(2+2i)x^2}}{16}+\frac{e^{(2-2i)x^2}}{16}+\frac{ie^{(2-2i)x^2}}{16}$	44
norman	$\frac{\frac{e^{2x^2}\tan\left(x^2\right)}{4}-\frac{e^{2x^2}\tan\left(x^2\right)^2}{8}+\frac{e^{2x^2}}{8}}{1+\tan\left(x^2\right)^2}$	47

```
[In] int(exp(2*x^2)*x*cos(2*x^2),x,method=_RETURNVERBOSE)
[Out] 1/8*exp(2*x^2)*(cos(2*x^2)+sin(2*x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int e^{2x^2} x \cos\left(2x^2\right)\, dx = \frac{1}{8}\cos\left(2\,x^2\right)e^{\left(2\,x^2\right)} + \frac{1}{8}e^{\left(2\,x^2\right)}\sin\left(2\,x^2\right)$$

```
[In] integrate(exp(2*x^2)*x*cos(2*x^2),x, algorithm="fricas")
[Out] 1/8*cos(2*x^2)*e^(2*x^2) + 1/8*e^(2*x^2)*sin(2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int e^{2x^2} x \cos\left(2x^2\right)\, dx = \frac{e^{2x^2}\sin\left(2x^2\right)}{8} + \frac{e^{2x^2}\cos\left(2x^2\right)}{8}$$

```
[In] integrate(exp(2*x**2)*x*cos(2*x**2),x)
[Out] exp(2*x**2)*sin(2*x**2)/8 + exp(2*x**2)*cos(2*x**2)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int e^{2x^2} x \cos\left(2x^2\right)\, dx = \frac{1}{8}\cos\left(2\,x^2\right)e^{\left(2\,x^2\right)} + \frac{1}{8}e^{\left(2\,x^2\right)}\sin\left(2\,x^2\right)$$

```
[In] integrate(exp(2*x^2)*x*cos(2*x^2),x, algorithm="maxima")
[Out] 1/8*cos(2*x^2)*e^(2*x^2) + 1/8*e^(2*x^2)*sin(2*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int e^{2x^2} x \cos\left(2x^2\right)\, dx = \frac{1}{8}\left(\cos\left(2\,x^2\right) + \sin\left(2\,x^2\right)\right)e^{\left(2\,x^2\right)}$$

```
[In] integrate(exp(2*x^2)*x*cos(2*x^2),x, algorithm="giac")
[Out] 1/8*(cos(2*x^2) + sin(2*x^2))*e^(2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int e^{2x^2} x \cos\left(2x^2\right)\, dx = \frac{\mathrm{e}^{2\,x^2}\,\left(\cos\left(2\,x^2\right)+\sin\left(2\,x^2\right)\right)}{8}$$

```
[In] int(x*exp(2*x^2)*cos(2*x^2),x)
[Out] (exp(2*x^2)*(cos(2*x^2) + sin(2*x^2)))/8
```

3.62 $\int e^x \sin(e^x)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 6

$$\int e^x \sin(e^x)\, dx = -\cos(e^x)$$

```
[Out] -cos(exp(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 2718}

$$\int e^x \sin(e^x)\, dx = -\cos(e^x)$$

```
[In] Int[E^x*Sin[E^x],x]
```

```
[Out] -Cos[E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \sin(x)\,dx, x, e^x\right) \\ &= -\cos\left(e^x\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int e^x \sin\left(e^x\right)\,dx = -\cos\left(e^x\right)$$

```
[In] Integrate[E^x*Sin[E^x],x]
[Out] -Cos[E^x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\cos\left(\mathrm{e}^{x}\right)$	6
default	$-\cos\left(\mathrm{e}^{x}\right)$	6
risch	$-\cos\left(\mathrm{e}^{x}\right)$	6
parallelrisch	$-\cos\left(\mathrm{e}^{x}\right)-1$	8
norman	$-\frac{2}{1+\tan\left(\frac{\mathrm{e}^{x}}{2}\right)^{2}}$	14

```
[In] int(exp(x)*sin(exp(x)),x,method=_RETURNVERBOSE)
[Out] -cos(exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \sin\left(e^x\right)\,dx = -\cos\left(e^x\right)$$

```
[In] integrate(exp(x)*sin(exp(x)),x, algorithm="fricas")
[Out] -cos(e^x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \sin(e^x) \, dx = -\cos(e^x)$$

```
[In] integrate(exp(x)*sin(exp(x)),x)
[Out] -cos(exp(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \sin(e^x) \, dx = -\cos(e^x)$$

```
[In] integrate(exp(x)*sin(exp(x)),x, algorithm="maxima")
[Out] -cos(e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \sin(e^x) \, dx = -\cos(e^x)$$

```
[In] integrate(exp(x)*sin(exp(x)),x, algorithm="giac")
[Out] -cos(e^x)
```

Mupad [B] (verification not implemented)

Time = 27.65 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \sin(e^x) \, dx = -\cos(\mathrm{e}^x)$$

```
[In] int(sin(exp(x))*exp(x),x)
[Out] -cos(exp(x))
```

3.63 $\int e^x \csc(e^x) \sec(e^x) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 5

$$\int e^x \csc(e^x) \sec(e^x) \, dx = \log(\tan(e^x))$$

```
[Out] ln(tan(exp(x)))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 2700, 29}

$$\int e^x \csc(e^x) \sec(e^x) \, dx = \log(\tan(e^x))$$

```
[In] Int[E^x*Csc[E^x]*Sec[E^x],x]
[Out] Log[Tan[E^x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \csc(x)\sec(x)\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan\left(e^x\right)\right)\\ &= \log\left(\tan\left(e^x\right)\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. $2(5) = 10$.

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 4.20

$$\int e^x \csc\left(e^x\right)\sec\left(e^x\right)\,dx = 2\left(-\frac{1}{2}\log\left(\cos\left(e^x\right)\right) + \frac{1}{2}\log\left(\sin\left(e^x\right)\right)\right)$$

```
[In] Integrate[E^x*Csc[E^x]*Sec[E^x],x]
[Out] 2*(-1/2*Log[Cos[E^x]] + Log[Sin[E^x]]/2)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\ln\left(\tan\left(e^x\right)\right)$	5
default	$\ln\left(\tan\left(e^x\right)\right)$	5
risch	$-\ln\left(e^{2ie^x}+1\right)+\ln\left(e^{2ie^x}-1\right)$	22
norman	$-\ln\left(\tan\left(\frac{e^x}{2}\right)-1\right)-\ln\left(\tan\left(\frac{e^x}{2}\right)+1\right)+\ln\left(\tan\left(\frac{e^x}{2}\right)\right)$	28
parallelrisch	$-\ln\left(\tan\left(\frac{e^x}{2}\right)-1\right)-\ln\left(\tan\left(\frac{e^x}{2}\right)+1\right)+\ln\left(\tan\left(\frac{e^x}{2}\right)\right)$	28

```
[In] int(exp(x)*csc(exp(x))*sec(exp(x)),x,method=_RETURNVERBOSE)
[Out] ln(tan(exp(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(4) = 8$.

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 4.20

$$\int e^x \csc(e^x)\sec(e^x)\,dx = -\frac{1}{2}\log\left(\cos(e^x)^2\right) + \frac{1}{2}\log\left(-\frac{1}{4}\cos(e^x)^2 + \frac{1}{4}\right)$$

```
[In] integrate(exp(x)*csc(exp(x))*sec(exp(x)),x, algorithm="fricas")
[Out] -1/2*log(cos(e^x)^2) + 1/2*log(-1/4*cos(e^x)^2 + 1/4)
```

Sympy [F]

$$\int e^x \csc(e^x)\sec(e^x)\,dx = \int e^x \csc(e^x)\sec(e^x)\,dx$$

```
[In] integrate(exp(x)*csc(exp(x))*sec(exp(x)),x)
[Out] Integral(exp(x)*csc(exp(x))*sec(exp(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(4) = 8$.

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.80

$$\int e^x \csc(e^x)\sec(e^x)\,dx = -\frac{1}{2}\log\left(\sin(e^x)^2 - 1\right) + \frac{1}{2}\log\left(\sin(e^x)^2\right)$$

```
[In] integrate(exp(x)*csc(exp(x))*sec(exp(x)),x, algorithm="maxima")
[Out] -1/2*log(sin(e^x)^2 - 1) + 1/2*log(sin(e^x)^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(4) = 8$.

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int e^x \csc(e^x)\sec(e^x)\,dx = -\frac{1}{2}\log\left(\left|\sin(e^x)^2 - 1\right|\right) + \log\left(\left|\sin(e^x)\right|\right)$$

```
[In] integrate(exp(x)*csc(exp(x))*sec(exp(x)),x, algorithm="giac")
[Out] -1/2*log(abs(sin(e^x)^2 - 1)) + log(abs(sin(e^x)))
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 43, normalized size of antiderivative = 8.60

$$\int e^x \csc(e^x) \sec(e^x)\, dx = -\ln\left(-16\,\mathrm{e}^{2\,x} - 16\,\mathrm{e}^{2\,x}\,\mathrm{e}^{\mathrm{e}^x\,2\mathrm{i}}\right) + \ln\left(16\,\mathrm{e}^{2\,x} - 16\,\mathrm{e}^{2\,x}\,\mathrm{e}^{\mathrm{e}^x\,2\mathrm{i}}\right)$$

```
[In] int(exp(x)/(cos(exp(x))*sin(exp(x))),x)
```

```
[Out] log(16*exp(2*x) - 16*exp(2*x)*exp(exp(x)*2i)) - log(- 16*exp(2*x) - 16*exp(
2*x)*exp(exp(x)*2i))
```

3.64 $\int e^x \cos(e^x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 4

$$\int e^x \cos(e^x) \, dx = \sin(e^x)$$

```
[Out] sin(exp(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 2717}

$$\int e^x \cos(e^x) \, dx = \sin(e^x)$$

```
[In] Int[E^x*Cos[E^x],x]
```

```
[Out] Sin[E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \cos(x)\,dx, x, e^x\right) \\ &= \sin\left(e^x\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int e^x \cos\left(e^x\right)\,dx = \sin\left(e^x\right)$$

```
[In] Integrate[E^x*Cos[E^x],x]
[Out] Sin[E^x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\sin\left(\mathrm{e}^x\right)$	4
default	$\sin\left(\mathrm{e}^x\right)$	4
risch	$\sin\left(\mathrm{e}^x\right)$	4
parallelrisch	$\sin\left(\mathrm{e}^x\right)$	4
norman	$\frac{2\tan\left(\frac{\mathrm{e}^x}{2}\right)}{1+\tan\left(\frac{\mathrm{e}^x}{2}\right)^2}$	19

```
[In] int(exp(x)*cos(exp(x)),x,method=_RETURNVERBOSE)
[Out] sin(exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^x \cos(e^x) \, dx = \sin(e^x)$$

```
[In] integrate(exp(x)*cos(exp(x)),x, algorithm="fricas")
[Out] sin(e^x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^x \cos(e^x) \, dx = \sin(e^x)$$

```
[In] integrate(exp(x)*cos(exp(x)),x)
[Out] sin(exp(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^x \cos(e^x) \, dx = \sin(e^x)$$

```
[In] integrate(exp(x)*cos(exp(x)),x, algorithm="maxima")
[Out] sin(e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^x \cos(e^x) \, dx = \sin(e^x)$$

```
[In] integrate(exp(x)*cos(exp(x)),x, algorithm="giac")
[Out] sin(e^x)
```

Mupad [B] (verification not implemented)

Time = 27.70 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^x \cos\left(e^x\right) \, dx = \sin\left(\mathrm{e}^x\right)$$

```
[In] int(cos(exp(x))*exp(x),x)
[Out] sin(exp(x))
```

3.65 $\int e^{2x} \cos(e^{2x}) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 10

$$\int e^{2x} \cos(e^{2x}) \, dx = \frac{1}{2} \sin(e^{2x})$$

```
[Out] 1/2*sin(exp(2*x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2320, 2717}

$$\int e^{2x} \cos(e^{2x}) \, dx = \frac{1}{2} \sin(e^{2x})$$

```
[In] Int[E^(2*x)*Cos[E^(2*x)],x]
```

```
[Out] Sin[E^(2*x)]/2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \cos(x)\,dx, x, e^{2x}\right)\\
&= \frac{1}{2}\sin\left(e^{2x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int e^{2x}\cos\left(e^{2x}\right)\,dx = \frac{1}{2}\sin\left(e^{2x}\right)$$

```
[In] Integrate[E^(2*x)*Cos[E^(2*x)],x]
[Out] Sin[E^(2*x)]/2
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\sin\left(e^{2x}\right)}{2}$	8
default	$\frac{\sin\left(e^{2x}\right)}{2}$	8
risch	$\frac{\sin\left(e^{2x}\right)}{2}$	8
parallelrisch	$\frac{\sin\left(e^{2x}\right)}{2}$	8
norman	$\frac{\tan\left(\frac{e^{2x}}{2}\right)}{1+\tan\left(\frac{e^{2x}}{2}\right)^{2}}$	22

```
[In] int(exp(2*x)*cos(exp(2*x)),x,method=_RETURNVERBOSE)
[Out] 1/2*sin(exp(2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2x}\cos\left(e^{2x}\right)\,dx=\frac{1}{2}\,\sin\left(e^{(2\,x)}\right)$$

```
[In] integrate(exp(2*x)*cos(exp(2*x)),x, algorithm="fricas")
[Out] 1/2*sin(e^(2*x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2x}\cos\left(e^{2x}\right)\,dx=\frac{\sin\left(e^{2x}\right)}{2}$$

```
[In] integrate(exp(2*x)*cos(exp(2*x)),x)
[Out] sin(exp(2*x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2x}\cos\left(e^{2x}\right)\,dx=\frac{1}{2}\,\sin\left(e^{(2\,x)}\right)$$

```
[In] integrate(exp(2*x)*cos(exp(2*x)),x, algorithm="maxima")
[Out] 1/2*sin(e^(2*x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2x}\cos\left(e^{2x}\right)\,dx=\frac{1}{2}\,\sin\left(e^{(2\,x)}\right)$$

```
[In] integrate(exp(2*x)*cos(exp(2*x)),x, algorithm="giac")
[Out] 1/2*sin(e^(2*x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{2x} \cos\left(e^{2x}\right) \, dx = \frac{\sin\left(\mathrm{e}^{2\,x}\right)}{2}$$

```
[In] int(exp(2*x)*cos(exp(2*x)),x)
[Out] sin(exp(2*x))/2
```

3.66 $\int e^{-2x} \cos\left(e^{-2x}\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 10

$$\int e^{-2x} \cos\left(e^{-2x}\right) \, dx = -\frac{1}{2} \sin\left(e^{-2x}\right)$$

```
[Out] -1/2*sin(exp(-2*x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2320, 2717}

$$\int e^{-2x} \cos\left(e^{-2x}\right) \, dx = -\frac{1}{2} \sin\left(e^{-2x}\right)$$

```
[In] Int[Cos[E^(-2*x)]/E^(2*x),x]
```

```
[Out] -1/2*Sin[E^(-2*x)]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \cos(x)\,dx, x, e^{-2x}\right)\right)\\ &= -\frac{1}{2}\sin\left(e^{-2x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int e^{-2x}\cos\left(e^{-2x}\right)\,dx = -\frac{1}{2}\sin\left(e^{-2x}\right)$$

```
[In] Integrate[Cos[E^(-2*x)]/E^(2*x),x]
[Out] -1/2*Sin[E^(-2*x)]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\sin\left(\mathrm{e}^{-2x}\right)}{2}$	8
risch	$-\frac{\sin\left(\mathrm{e}^{-2x}\right)}{2}$	8
norman	$-\frac{\tan\left(\frac{\mathrm{e}^{-2x}}{2}\right)}{1+\tan\left(\frac{\mathrm{e}^{-2x}}{2}\right)^{2}}$	23

```
[In] int(cos(exp(-2*x))/exp(2*x),x,method=_RETURNVERBOSE)
[Out] -1/2*sin(exp(-2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{-2x}\cos\left(e^{-2x}\right)\,dx = -\frac{1}{2}\sin\left(e^{(-2x)}\right)$$

```
[In] integrate(cos(exp(-2*x))/exp(2*x),x, algorithm="fricas")
[Out] -1/2*sin(e^(-2*x))
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int e^{-2x}\cos\left(e^{-2x}\right)\,dx = -\frac{\sin\left(e^{-2x}\right)}{2}$$

```
[In] integrate(cos(exp(-2*x))/exp(2*x),x)
[Out] -sin(exp(-2*x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{-2x}\cos\left(e^{-2x}\right)\,dx = -\frac{1}{2}\sin\left(e^{(-2\,x)}\right)$$

```
[In] integrate(cos(exp(-2*x))/exp(2*x),x, algorithm="maxima")
[Out] -1/2*sin(e^(-2*x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{-2x}\cos\left(e^{-2x}\right)\,dx = -\frac{1}{2}\sin\left(e^{(-2\,x)}\right)$$

```
[In] integrate(cos(exp(-2*x))/exp(2*x),x, algorithm="giac")
[Out] -1/2*sin(e^(-2*x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{-2x}\cos\left(e^{-2x}\right)\,dx = -\frac{\sin\left(\mathrm{e}^{-2\,x}\right)}{2}$$

```
[In] int(exp(-2*x)*cos(exp(-2*x)),x)
[Out] -sin(exp(-2*x))/2
```

3.67 $\int e^{2x}\cos(e^x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int e^{2x}\cos(e^x)\,dx=\cos(e^x)+e^x\sin(e^x)$$

```
[Out] cos(exp(x))+exp(x)*sin(exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2320, 3377, 2718}

$$\int e^{2x}\cos(e^x)\,dx=e^x\sin(e^x)+\cos(e^x)$$

```
[In] Int[E^(2*x)*Cos[E^x],x]
```

```
[Out] Cos[E^x] + E^x*Sin[E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x\cos(x)\,dx, x, e^x\right)\\ &= e^x\sin\left(e^x\right) - \text{Subst}\left(\int \sin(x)\,dx, x, e^x\right)\\ &= \cos\left(e^x\right) + e^x\sin\left(e^x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{2x}\cos\left(e^x\right)\,dx = \cos\left(e^x\right) + e^x\sin\left(e^x\right)$$

```
[In] Integrate[E^(2*x)*Cos[E^x],x]
[Out] Cos[E^x] + E^x*Sin[E^x]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
risch	$\cos\left(\mathrm{e}^{x}\right)+\mathrm{e}^{x}\sin\left(\mathrm{e}^{x}\right)$	11
norman	$\frac{2\,\mathrm{e}^{x}\tan\left(\frac{\mathrm{e}^{x}}{2}\right)+2}{1+\tan\left(\frac{\mathrm{e}^{x}}{2}\right)^{2}}$	24

```
[In] int(exp(2*x)*cos(exp(x)),x,method=_RETURNVERBOSE)
[Out] cos(exp(x))+exp(x)*sin(exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{2x}\cos\left(e^{x}\right)\,dx=e^{x}\sin\left(e^{x}\right)+\cos\left(e^{x}\right)$$

```
[In] integrate(exp(2*x)*cos(exp(x)),x, algorithm="fricas")
[Out] e^x*sin(e^x) + cos(e^x)
```

Sympy [A] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int e^{2x}\cos\left(e^{x}\right)\,dx=e^{x}\sin\left(e^{x}\right)+\cos\left(e^{x}\right)$$

```
[In] integrate(exp(2*x)*cos(exp(x)),x)
[Out] exp(x)*sin(exp(x)) + cos(exp(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{2x}\cos\left(e^{x}\right)\,dx=e^{x}\sin\left(e^{x}\right)+\cos\left(e^{x}\right)$$

```
[In] integrate(exp(2*x)*cos(exp(x)),x, algorithm="maxima")
[Out] e^x*sin(e^x) + cos(e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{2x}\cos\left(e^{x}\right)\,dx=e^{x}\sin\left(e^{x}\right)+\cos\left(e^{x}\right)$$

```
[In] integrate(exp(2*x)*cos(exp(x)),x, algorithm="giac")
[Out] e^x*sin(e^x) + cos(e^x)
```

Mupad [B] (verification not implemented)

Time = 27.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{2x} \cos(e^x) \, dx = \cos(e^x) + \sin(e^x) \, e^x$$

```
[In] int(cos(exp(x))*exp(2*x),x)
[Out] cos(exp(x)) + sin(exp(x))*exp(x)
```

3.68 $\int e^{-1+3x} \cos \left(e^{-1+3x}\right) \sin \left(1+e^{-1+3x}\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 30

$$\int e^{-1+3x} \cos \left(e^{-1+3x}\right) \sin \left(1+e^{-1+3x}\right) \, dx = -\frac{1}{12} \cos \left(1+2e^{-1+3x}\right)+\frac{1}{6}e^{-1+3x} \sin(1)$$

```
[Out] -1/12*cos(1+2*exp(-1+3*x))+1/6*exp(-1+3*x)*sin(1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2320, 4670, 2718}

$$\int e^{-1+3x} \cos \left(e^{-1+3x}\right) \sin \left(1+e^{-1+3x}\right) \, dx = \frac{1}{6}e^{3x-1} \sin(1)-\frac{1}{12} \cos \left(2e^{3x-1}+1\right)$$

```
[In] Int[E^(-1 + 3*x)*Cos[E^(-1 + 3*x)]*Sin[1 + E^(-1 + 3*x)],x]
```

```
[Out] -1/12*Cos[1 + 2*E^(-1 + 3*x)] + (E^(-1 + 3*x)*Sin[1])/6
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \cos(x)\sin(1+x)\,dx, x, e^{-1+3x}\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{\sin(1)}{2}+\frac{1}{2}\sin(1+2x)\right)\,dx, x, e^{-1+3x}\right)\\
&= \frac{1}{6}e^{-1+3x}\sin(1)+\frac{1}{6}\text{Subst}\left(\int \sin(1+2x)\,dx, x, e^{-1+3x}\right)\\
&= -\frac{1}{12}\cos\left(1+2e^{-1+3x}\right)+\frac{1}{6}e^{-1+3x}\sin(1)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)\,dx = -\frac{1}{12}\cos\left(1+2e^{-1+3x}\right)+\frac{1}{6}e^{-1+3x}\sin(1)$$

```
[In] Integrate[E^(-1 + 3*x)*Cos[E^(-1 + 3*x)]*Sin[1 + E^(-1 + 3*x)],x]
[Out] -1/12*Cos[1 + 2*E^(-1 + 3*x)] + (E^(-1 + 3*x)*Sin[1])/6
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{\cos(1+2\,\mathrm{e}^{-1+3x})}{12}+\frac{\mathrm{e}^{-1+3x}\sin(1)}{6}$
default	$-\frac{\cos(1+2\,\mathrm{e}^{-1+3x})}{12}+\frac{\mathrm{e}^{-1+3x}\sin(1)}{6}$
risch	$-\frac{\cos(1+2\,\mathrm{e}^{-1+3x})}{12}+\frac{\mathrm{e}^{-1+3x}\sin(1)}{6}$
parallelrisch	$\frac{\mathrm{e}^{-1+3x}\sin(1)}{6}-\frac{\cos(1+2\,\mathrm{e}^{-1+3x})}{12}-\frac{\cos(1)}{12}+\frac{1}{6}$
norman	$\frac{\frac{2\tan\left(\frac{\mathrm{e}^{-1+3x}}{2}\right)\tan\left(\frac{1}{2}+\frac{\mathrm{e}^{-1+3x}}{2}\right)}{3}-\frac{\mathrm{e}^{-1+3x}\tan\left(\frac{\mathrm{e}^{-1+3x}}{2}\right)}{3}+\frac{\mathrm{e}^{-1+3x}\tan\left(\frac{1}{2}+\frac{\mathrm{e}^{-1+3x}}{2}\right)}{3}+\frac{\mathrm{e}^{-1+3x}\tan\left(\frac{\mathrm{e}^{-1+3x}}{2}\right)\tan\left(\frac{1}{2}+\frac{\mathrm{e}^{-1+}}{2}\right.}{3}}{\left(1+\tan\left(\frac{\mathrm{e}^{-1+3x}}{2}\right)^2\right)\left(1+\tan\left(\frac{1}{2}+\frac{\mathrm{e}^{-1+3x}}{2}\right)^2\right)}$

```
[In] int(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/12*cos(1+2*exp(-1+3*x))+1/6*exp(-1+3*x)*sin(1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)\,dx = -\frac{1}{6}\cos(1)\cos\left(e^{(3x-1)}\right)^2+\frac{1}{6}\cos\left(e^{(3x-1)}\right)\sin(1)\sin\left(e^{(3x-1)}\right)+\frac{1}{6}e^{(3x-1)}\sin(1)$$

```
[In] integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x, algorithm="fri
cas")
```

```
[Out] -1/6*cos(1)*cos(e^(3*x - 1))^2 + 1/6*cos(e^(3*x - 1))*sin(1)*sin(e^(3*x - 1
)) + 1/6*e^(3*x - 1)*sin(1)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)\,dx = \text{Timed out}$$

```
[In] integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)\,dx=\frac{1}{6}\,e^{(3\,x-1)}\sin\left(1\right)-\frac{1}{12}\,\cos\left(2\,e^{(3\,x-1)}+1\right)$$

```
[In] integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x, algorithm="max
ima")
```

```
[Out] 1/6*e^(3*x - 1)*sin(1) - 1/12*cos(2*e^(3*x - 1) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)\,dx=\frac{1}{6}\,e^{(3\,x-1)}\sin\left(1\right)-\frac{1}{12}\,\cos\left(2\,e^{(3\,x-1)}+1\right)$$

```
[In] integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x, algorithm="gia
c")
```

```
[Out] 1/6*e^(3*x - 1)*sin(1) - 1/12*cos(2*e^(3*x - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)\,dx=\frac{\mathrm{e}^{3\,x-1}\,\sin\left(1\right)}{6}-\frac{\cos\left(2\,\mathrm{e}^{3\,x-1}+1\right)}{12}$$

```
[In] int(exp(3*x - 1)*sin(exp(3*x - 1) + 1)*cos(exp(3*x - 1)),x)
```

```
[Out] (exp(3*x - 1)*sin(1))/6 - cos(2*exp(3*x - 1) + 1)/12
```

3.69 $\int e^x \tan(e^x)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 7

$$\int e^x \tan(e^x)\, dx = -\log(\cos(e^x))$$

```
[Out] -ln(cos(exp(x)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 3556}

$$\int e^x \tan(e^x)\, dx = -\log(\cos(e^x))$$

```
[In] Int[E^x*Tan[E^x],x]
```

```
[Out] -Log[Cos[E^x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \tan(x)\,dx, x, e^x\right)\\ &= -\log\left(\cos\left(e^x\right)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int e^x \tan\left(e^x\right)\,dx = -\log\left(\cos\left(e^x\right)\right)$$

```
[In] Integrate[E^x*Tan[E^x],x]
[Out] -Log[Cos[E^x]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\ln\left(\cos\left(\mathrm{e}^{x}\right)\right)$	7
default	$-\ln\left(\cos\left(\mathrm{e}^{x}\right)\right)$	7
norman	$\frac{\ln\left(1+\tan\left(\mathrm{e}^{x}\right)^{2}\right)}{2}$	11
parallelrisch	$\frac{\ln\left(1+\tan\left(\mathrm{e}^{x}\right)^{2}\right)}{2}$	11
risch	$i\mathrm{e}^{x}-\ln\left(\mathrm{e}^{2i\mathrm{e}^{x}}+1\right)$	18

```
[In] int(exp(x)*tan(exp(x)),x,method=_RETURNVERBOSE)
[Out] -ln(cos(exp(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int e^x \tan\left(e^x\right)\, dx = -\frac{1}{2}\,\log\left(\frac{1}{\tan\left(e^x\right)^2+1}\right)$$

```
[In] integrate(exp(x)*tan(exp(x)),x, algorithm="fricas")
[Out] -1/2*log(1/(tan(e^x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

$$\int e^x \tan\left(e^x\right)\, dx = \frac{\log\left(\tan^2\left(e^x\right)+1\right)}{2}$$

```
[In] integrate(exp(x)*tan(exp(x)),x)
[Out] log(tan(exp(x))**2 + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.57

$$\int e^x \tan\left(e^x\right)\, dx = \log\left(\sec\left(e^x\right)\right)$$

```
[In] integrate(exp(x)*tan(exp(x)),x, algorithm="maxima")
[Out] log(sec(e^x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int e^x \tan\left(e^x\right)\, dx = -\log\left(\left|\cos\left(e^x\right)\right|\right)$$

```
[In] integrate(exp(x)*tan(exp(x)),x, algorithm="giac")
[Out] -log(abs(cos(e^x)))
```

Mupad [B] (verification not implemented)

Time = 27.55 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

$$\int e^x \tan(e^x)\, dx = \frac{\ln\left(\tan(\mathrm{e}^x)^2 + 1\right)}{2}$$

```
[In] int(tan(exp(x))*exp(x),x)
[Out] log(tan(exp(x))^2 + 1)/2
```

3.70 $\int e^x \sec(e^x)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 5

$$\int e^x \sec(e^x)\, dx = \operatorname{arctanh}(\sin(e^x))$$

```
[Out] arctanh(sin(exp(x)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 3855}

$$\int e^x \sec(e^x)\, dx = \operatorname{arctanh}(\sin(e^x))$$

```
[In] Int[E^x*Sec[E^x],x]
```

```
[Out] ArcTanh[Sin[E^x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \sec(x)\,dx, x, e^x\right)$$
$$= \operatorname{arctanh}(\sin(e^x))$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int e^x \sec(e^x)\ dx = \operatorname{arctanh}(\sin(e^x))$$

```
[In] Integrate[E^x*Sec[E^x],x]
[Out] ArcTanh[Sin[E^x]]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

method	result	size
derivativedivides	$\ln(\sec(e^x) + \tan(e^x))$	9
default	$\ln(\sec(e^x) + \tan(e^x))$	9
norman	$-\ln\left(\tan\left(\frac{e^x}{2}\right) - 1\right) + \ln\left(\tan\left(\frac{e^x}{2}\right) + 1\right)$	20
parallelrisch	$-\ln\left(\tan\left(\frac{e^x}{2}\right) - 1\right) + \ln\left(\tan\left(\frac{e^x}{2}\right) + 1\right)$	20
risch	$-\ln\left(e^{ie^x} - i\right) + \ln\left(e^{ie^x} + i\right)$	24

```
[In] int(exp(x)*sec(exp(x)),x,method=_RETURNVERBOSE)
[Out] ln(sec(exp(x))+tan(exp(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(4) = 8$.

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.80

$$\int e^x \sec(e^x)\ dx = \frac{1}{2}\log(\sin(e^x) + 1) - \frac{1}{2}\log(-\sin(e^x) + 1)$$

```
[In] integrate(exp(x)*sec(exp(x)),x, algorithm="fricas")
[Out] 1/2*log(sin(e^x) + 1) - 1/2*log(-sin(e^x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

$$\int e^x \sec(e^x) \, dx = \log(\tan(e^x) + \sec(e^x))$$

```
[In] integrate(exp(x)*sec(exp(x)),x)
[Out] log(tan(exp(x)) + sec(exp(x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

$$\int e^x \sec(e^x) \, dx = \log(\sec(e^x) + \tan(e^x))$$

```
[In] integrate(exp(x)*sec(exp(x)),x, algorithm="maxima")
[Out] log(sec(e^x) + tan(e^x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 5.80

$$\int e^x \sec(e^x) \, dx = \frac{1}{4} \log\left(\left|\frac{1}{\sin(e^x)} + \sin(e^x) + 2\right|\right) - \frac{1}{4} \log\left(\left|\frac{1}{\sin(e^x)} + \sin(e^x) - 2\right|\right)$$

```
[In] integrate(exp(x)*sec(exp(x)),x, algorithm="giac")
[Out] 1/4*log(abs(1/sin(e^x) + sin(e^x) + 2)) - 1/4*log(abs(1/sin(e^x) + sin(e^x)
 - 2))
```

Mupad [B] (verification not implemented)

Time = 27.97 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

$$\int e^x \sec(e^x) \, dx = -\mathrm{atan}\left(\mathrm{e}^{\mathrm{e}^x\,1\mathrm{i}}\right)\,2\mathrm{i}$$

```
[In] int(exp(x)/cos(exp(x)),x)
[Out] -atan(exp(exp(x)*1i))*2i
```

3.71 $\int e^x \sec(e^x) \tan(e^x)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 4

$$\int e^x \sec(e^x) \tan(e^x)\, dx = \sec(e^x)$$

```
[Out] sec(exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 2686, 8}

$$\int e^x \sec(e^x) \tan(e^x)\, dx = \sec(e^x)$$

```
[In] Int[E^x*Sec[E^x]*Tan[E^x],x]
```

```
[Out] Sec[E^x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \sec(x)\tan(x)\,dx, x, e^x\right)$$
$$= \text{Subst}\left(\int 1\,dx, x, \sec\left(e^x\right)\right)$$
$$= \sec\left(e^x\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int e^x \sec\left(e^x\right)\tan\left(e^x\right)\,dx = \sec\left(e^x\right)$$

```
[In] Integrate[E^x*Sec[E^x]*Tan[E^x],x]
[Out] Sec[E^x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\sec\left(\mathrm{e}^{x}\right)$	4
default	$\sec\left(\mathrm{e}^{x}\right)$	4
risch	$\frac{2\,\mathrm{e}^{i\mathrm{e}^{x}}}{\mathrm{e}^{2i\mathrm{e}^{x}}+1}$	19

```
[In] int(exp(x)*sec(exp(x))*tan(exp(x)),x,method=_RETURNVERBOSE)
[Out] sec(exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int e^x \sec(e^x) \tan(e^x) \, dx = \frac{1}{\cos(e^x)}$$

```
[In] integrate(exp(x)*sec(exp(x))*tan(exp(x)),x, algorithm="fricas")
[Out] 1/cos(e^x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^x \sec(e^x) \tan(e^x) \, dx = \sec(e^x)$$

```
[In] integrate(exp(x)*sec(exp(x))*tan(exp(x)),x)
[Out] sec(exp(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int e^x \sec(e^x) \tan(e^x) \, dx = \frac{1}{\cos(e^x)}$$

```
[In] integrate(exp(x)*sec(exp(x))*tan(exp(x)),x, algorithm="maxima")
[Out] 1/cos(e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int e^x \sec(e^x) \tan(e^x) \, dx = \frac{1}{\cos(e^x)}$$

```
[In] integrate(exp(x)*sec(exp(x))*tan(exp(x)),x, algorithm="giac")
[Out] 1/cos(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int e^x \sec(e^x) \tan(e^x) \, dx = \frac{1}{\cos(e^x)}$$

```
[In] int((tan(exp(x))*exp(x))/cos(exp(x)),x)
[Out] 1/cos(exp(x))
```

3.72 $\int e^x \csc^2(e^x)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 6

$$\int e^x \csc^2(e^x)\, dx = -\cot(e^x)$$

```
[Out] -cot(exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 3852, 8}

$$\int e^x \csc^2(e^x)\, dx = -\cot(e^x)$$

```
[In] Int[E^x*Csc[E^x]^2,x]
```

```
[Out] -Cot[E^x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \csc^2(x)\,dx, x, e^x\right)\\ &= -\text{Subst}\left(\int 1\,dx, x, \cot\left(e^x\right)\right)\\ &= -\cot\left(e^x\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int e^x \csc^2\left(e^x\right)\,dx = -\cot\left(e^x\right)$$

```
[In] Integrate[E^x*Csc[E^x]^2,x]
[Out] -Cot[E^x]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\cot\left(e^x\right)$	6
default	$-\cot\left(e^x\right)$	6
risch	$-\frac{2i}{e^{2ie^x}-1}$	14
parallelrisch	$-\frac{\cot\left(\frac{e^x}{2}\right)}{2}+\frac{\tan\left(\frac{e^x}{2}\right)}{2}$	16
norman	$\frac{-\frac{1}{2}+\frac{\tan\left(\frac{e^x}{2}\right)^2}{2}}{\tan\left(\frac{e^x}{2}\right)}$	20

```
[In] int(exp(x)*csc(exp(x))^2,x,method=_RETURNVERBOSE)
[Out] -cot(exp(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.67

$$\int e^x \csc^2(e^x) \, dx = -\frac{\cos(e^x)}{\sin(e^x)}$$

```
[In] integrate(exp(x)*csc(exp(x))^2,x, algorithm="fricas")
[Out] -cos(e^x)/sin(e^x)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^x \csc^2(e^x) \, dx = -\cot(e^x)$$

```
[In] integrate(exp(x)*csc(exp(x))**2,x)
[Out] -cot(exp(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int e^x \csc^2(e^x) \, dx = -\frac{1}{\tan(e^x)}$$

```
[In] integrate(exp(x)*csc(exp(x))^2,x, algorithm="maxima")
[Out] -1/tan(e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int e^x \csc^2(e^x) \, dx = -\frac{1}{\tan(e^x)}$$

```
[In] integrate(exp(x)*csc(exp(x))^2,x, algorithm="giac")
[Out] -1/tan(e^x)
```

Mupad [B] (verification not implemented)

Time = 28.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.17

$$\int e^x \csc^2(e^x)\,dx = -\frac{2\mathrm{i}}{\mathrm{e}^{\mathrm{e}^x\,2\mathrm{i}} - 1}$$

```
[In] int(exp(x)/sin(exp(x))^2,x)
[Out] -2i/(exp(exp(x)*2i) - 1)
```

3.73 $\int e^x \sin(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int e^x \sin(a+bx)\,dx = -\frac{be^x\cos(a+bx)}{1+b^2}+\frac{e^x\sin(a+bx)}{1+b^2}$$

```
[Out] -b*exp(x)*cos(b*x+a)/(b^2+1)+exp(x)*sin(b*x+a)/(b^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4517}

$$\int e^x \sin(a+bx)\,dx = \frac{e^x\sin(a+bx)}{b^2+1}-\frac{be^x\cos(a+bx)}{b^2+1}$$

```
[In] Int[E^x*Sin[a + b*x],x]
```

```
[Out] -((b*E^x*Cos[a + b*x])/(1 + b^2)) + (E^x*Sin[a + b*x])/(1 + b^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{be^x\cos(a+bx)}{1+b^2}+\frac{e^x\sin(a+bx)}{1+b^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int e^x \sin(a+bx)\,dx = \frac{e^x(-b\cos(a+bx)+\sin(a+bx))}{1+b^2}$$

```
[In] Integrate[E^x*Sin[a + b*x],x]
[Out] (E^x*(-(b*Cos[a + b*x]) + Sin[a + b*x]))/(1 + b^2)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

method	result	size
parallelrisch	$\frac{e^{x}\left(-b\cos\left(x b+a\right)+\sin\left(x b+a\right)\right)}{b^{2}+1}$	27
default	$-\frac{b\,e^{x}\cos\left(x b+a\right)}{b^{2}+1}+\frac{e^{x}\sin\left(x b+a\right)}{b^{2}+1}$	36
risch	$\frac{e^{x}\left(2b\cos\left(x b+a\right)-2\sin\left(x b+a\right)\right)}{2\left(-b+i\right)\left(i+b\right)}$	37
norman	$\frac{\frac{b\,e^{x}\tan\left(\frac{a}{2}+\frac{x b}{2}\right)^{2}}{b^{2}+1}-\frac{b\,e^{x}}{b^{2}+1}+\frac{2\,e^{x}\tan\left(\frac{a}{2}+\frac{x b}{2}\right)}{b^{2}+1}}{1+\tan\left(\frac{a}{2}+\frac{x b}{2}\right)^{2}}$	72

```
[In] int(exp(x)*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] exp(x)/(b^2+1)*(-b*cos(b*x+a)+sin(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int e^x \sin(a+bx)\,dx = -\frac{b\cos\left(bx+a\right)e^{x}-e^{x}\sin\left(bx+a\right)}{b^{2}+1}$$

```
[In] integrate(exp(x)*sin(b*x+a),x, algorithm="fricas")
[Out] -(b*cos(b*x + a)*e^x - e^x*sin(b*x + a))/(b^2 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.08

$$\int e^x \sin(a+bx)\,dx = \begin{cases} \frac{xe^x \sin{(a-ix)}}{2} + \frac{ixe^x \cos{(a-ix)}}{2} + \frac{e^x \sin{(a-ix)}}{2} & \text{for } b=-i \\ \frac{xe^x \sin{(a+ix)}}{2} - \frac{ixe^x \cos{(a+ix)}}{2} + \frac{ie^x \cos{(a+ix)}}{2} & \text{for } b=i \\ -\frac{be^x \cos{(a+bx)}}{b^2+1} + \frac{e^x \sin{(a+bx)}}{b^2+1} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(x)*sin(b*x+a),x)
```

```
[Out] Piecewise((x*exp(x)*sin(a - I*x)/2 + I*x*exp(x)*cos(a - I*x)/2 + exp(x)*sin
(a - I*x)/2, Eq(b, -I)), (x*exp(x)*sin(a + I*x)/2 - I*x*exp(x)*cos(a + I*x)
/2 + I*exp(x)*cos(a + I*x)/2, Eq(b, I)), (-b*exp(x)*cos(a + b*x)/(b**2 + 1)
 + exp(x)*sin(a + b*x)/(b**2 + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int e^x \sin(a+bx)\,dx = -\frac{(b\cos{(bx+a)} - \sin{(bx+a)})e^x}{b^2+1}$$

```
[In] integrate(exp(x)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -(b*cos(b*x + a) - sin(b*x + a))*e^x/(b^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int e^x \sin(a+bx)\,dx = -\left(\frac{b\cos{(bx+a)}}{b^2+1} - \frac{\sin{(bx+a)}}{b^2+1}\right)e^x$$

```
[In] integrate(exp(x)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*cos(b*x + a)/(b^2 + 1) - sin(b*x + a)/(b^2 + 1))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int e^x \sin(a+bx)\,dx = \frac{e^x\,(\sin(a+b\,x)-b\,\cos(a+b\,x))}{b^2+1}$$

```
[In] int(exp(x)*sin(a + b*x),x)
[Out] (exp(x)*(sin(a + b*x) - b*cos(a + b*x)))/(b^2 + 1)
```

3.74 $\int e^x \sin(a+cx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 115

$$\int e^x \sin(a+cx^2)\,dx = \frac{(-1)^{3/4} e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)} \sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{(-1)^{3/4} e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)} \sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] 1/4*(-1)^(3/4)*exp(1/4*I*(4*a+1/c))*erf(1/2*(-1)^(1/4)*(1+2*I*c*x)/c^(1/2))
*Pi^(1/2)/c^(1/2)+1/4*(-1)^(3/4)*erfi(1/2*(-1)^(1/4)*(1-2*I*c*x)/c^(1/2))*P
i^(1/2)/exp(1/4*I*(4*a+1/c))/c^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4560, 2266, 2235, 2236}

$$\int e^x \sin(a+cx^2)\,dx = \frac{(-1)^{3/4} \sqrt{\pi} e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)} \operatorname{erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{(-1)^{3/4} \sqrt{\pi} e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)} \operatorname{erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[In] Int[E^x*Sin[a + c*x^2],x]
```

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*a + c^(-1)))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + (2*I)*c*
x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(3/4)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 -
(2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]*E^((I/4)*(4*a + c^(-1))))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2} i e^{-ia+x-icx^2} - \frac{1}{2} i e^{ia+x+icx^2}\right) dx \\
&= \frac{1}{2} i \int e^{-ia+x-icx^2}\, dx - \frac{1}{2} i \int e^{ia+x+icx^2}\, dx \\
&= \frac{1}{2}\left(i e^{-\frac{1}{4} i\left(4a+\frac{1}{c}\right)}\right) \int e^{\frac{i(1-2icx)^2}{4c}}\, dx - \frac{1}{2}\left(i e^{\frac{1}{4} i\left(4a+\frac{1}{c}\right)}\right) \int e^{-\frac{i(1+2icx)^2}{4c}}\, dx \\
&= \frac{(-1)^{3/4} e^{\frac{1}{4} i\left(4a+\frac{1}{c}\right)} \sqrt{\pi}\, \text{erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{(-1)^{3/4} e^{-\frac{1}{4} i\left(4a+\frac{1}{c}\right)} \sqrt{\pi}\, \text{erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int e^x \sin\left(a+cx^2\right) dx = -\frac{\sqrt[4]{-1}e^{-\frac{i}{4}/c}\sqrt{\pi}\left(e^{\frac{i}{2}/c}\text{erfi}\left(\frac{\sqrt[4]{-1}(-i+2cx)}{2\sqrt{c}}\right)(\cos(a)+i\sin(a))+\text{erfi}\left(\frac{(-1)^{3/4}(i+2cx)}{2\sqrt{c}}\right)(i\cos(a)+\sin(a))\right)}{4\sqrt{c}}$$

```
[In] Integrate[E^x*Sin[a + c*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[Pi]*(E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + 2*c*x))/(2*Sq
rt[c])]*(Cos[a] + I*Sin[a]) + Erfi[((-1)^(3/4)*(I + 2*c*x))/(2*Sqrt[c])]*(I
*Cos[a] + Sin[a])))/(Sqrt[c]*E^((I/4)/c))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{i(4ac+1)}{4c}}\,\mathrm{erf}\left(\sqrt{-ic}\,x-\frac{1}{2\sqrt{-ic}}\right)}{4\sqrt{-ic}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{-\frac{i(4ac+1)}{4c}}\,\mathrm{erf}\left(\sqrt{ic}\,x-\frac{1}{2\sqrt{ic}}\right)}{4\sqrt{ic}}$	88

```
[In] int(exp(x)*sin(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(1/4*I*(4*a*c+1)/c)/(-I*c)^(1/2)*erf((-I*c)^(1/2)*x-1/2/
(-I*c)^(1/2))+1/4*I*Pi^(1/2)*exp(-1/4*I*(4*a*c+1)/c)/(I*c)^(1/2)*erf((I*c)^
(1/2)*x-1/2/(I*c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(73) = 146.

Time = 0.24 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.26

$$\int e^x \sin\left(a+cx^2\right) dx = \frac{\sqrt{2}\left(i\pi\cos\left(\frac{4ac+1}{4c}\right)+\pi\sin\left(\frac{4ac+1}{4c}\right)\right)\sqrt{\frac{c}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}(2cx+i)\sqrt{\frac{c}{\pi}}}{2c}\right)+\sqrt{2}\left(i\pi\cos\left(\frac{4ac+1}{4c}\right)-\pi\sin\left(\frac{4ac+1}{4c}\right)\right)\sqrt{\frac{c}{\pi}}\,\mathrm{C}\left(-\right.}{}$$

```
[In] integrate(exp(x)*sin(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(I*pi*cos(1/4*(4*a*c + 1)/c) + pi*sin(1/4*(4*a*c + 1)/c))*sqrt
(c/pi)*fresnel_cos(1/2*sqrt(2)*(2*c*x + I)*sqrt(c/pi)/c) + sqrt(2)*(I*pi*co
s(1/4*(4*a*c + 1)/c) - pi*sin(1/4*(4*a*c + 1)/c))*sqrt(c/pi)*fresnel_cos(-1
/2*sqrt(2)*(2*c*x - I)*sqrt(c/pi)/c) + sqrt(2)*(pi*cos(1/4*(4*a*c + 1)/c) -
 I*pi*sin(1/4*(4*a*c + 1)/c))*sqrt(c/pi)*fresnel_sin(1/2*sqrt(2)*(2*c*x + I
)*sqrt(c/pi)/c) - sqrt(2)*(pi*cos(1/4*(4*a*c + 1)/c) + I*pi*sin(1/4*(4*a*c
+ 1)/c))*sqrt(c/pi)*fresnel_sin(-1/2*sqrt(2)*(2*c*x - I)*sqrt(c/pi)/c))/c
```

Sympy [F]

$$\int e^x \sin\left(a + cx^2\right)\,dx = \int e^x \sin\left(a + cx^2\right)dx$$

```
[In] integrate(exp(x)*sin(c*x**2+a),x)
```

```
[Out] Integral(exp(x)*sin(a + c*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int e^x \sin\left(a + cx^2\right)\,dx =$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\cos\left(\frac{4\,ac+1}{4\,c}\right) + (i-1)\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2i\,cx-1}{2\,\sqrt{i\,c}}\right) + \left(-(i-1)\cos\left(\frac{4\,ac+1}{4\,c}\right) + (i+1)\sin\left(\right.\right.}{8\,\sqrt{c}}$$

```
[In] integrate(exp(x)*sin(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*cos(1/4*(4*a*c + 1)/c) + (I - 1)*sin(1/4*(
4*a*c + 1)/c))*erf(1/2*(2*I*c*x - 1)/sqrt(I*c)) + (-(I - 1)*cos(1/4*(4*a*c
+ 1)/c) + (I + 1)*sin(1/4*(4*a*c + 1)/c))*erf(1/2*(2*I*c*x + 1)/sqrt(-I*c))
)/sqrt(c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\int e^x \sin\left(a+cx^2\right)\,dx = \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{4}i\,\sqrt{2}\left(2\,x+\frac{i}{c}\right)\left(-\frac{i\,c}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{4i\,ac+i}{4\,c}\right)}}{4\left(-\frac{i\,c}{|c|}+1\right)\sqrt{|c|}} + \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}i\,\sqrt{2}\left(2\,x-\frac{i}{c}\right)\left(\frac{i\,c}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-4i\,ac-i}{4\,c}\right)}}{4\left(\frac{i\,c}{|c|}+1\right)\sqrt{|c|}}$$

```
[In] integrate(exp(x)*sin(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(pi)*erf(1/4*I*sqrt(2)*(2*x + I/c)*(-I*c/abs(c) + 1)*sqrt(a
bs(c)))*e^(-1/4*(4*I*a*c + I)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c))) + 1/4*sqr
t(2)*sqrt(pi)*erf(-1/4*I*sqrt(2)*(2*x - I/c)*(I*c/abs(c) + 1)*sqrt(abs(c)))
*e^(-1/4*(-4*I*a*c - I)/c)/((I*c/abs(c) + 1)*sqrt(abs(c)))
```

Mupad [F(-1)]

Timed out.

$$\int e^x \sin\left(a+cx^2\right)\,dx = \int \mathrm{e}^x\,\sin\left(c\,x^2+a\right)\,dx$$

```
[In] int(exp(x)*sin(a + c*x^2),x)
```

```
[Out] int(exp(x)*sin(a + c*x^2), x)
```

3.75 $\int e^x \sin(a+bx+cx^2)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 144

$$\int e^x \sin(a+bx+cx^2)\,dx = \frac{(-1)^{3/4} e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(1+ib+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{(-1)^{3/4} e^{-ia+\frac{i(i+b)^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(1-ib-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] 1/4*(-1)^(3/4)*exp(1/4*I*(4*a+(1+I*b)^2/c))*erf(1/2*(-1)^(1/4)*(1+I*b+2*I*c
*x)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/4*(-1)^(3/4)*exp(-I*a+1/4*I*(I+b)^2/c)*erfi
(1/2*(-1)^(1/4)*(1-I*b-2*I*c*x)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4560, 2266, 2235, 2236}

$$\int e^x \sin(a+bx+cx^2)\,dx = \frac{(-1)^{3/4}\sqrt{\pi} e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(ib+2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{(-1)^{3/4}\sqrt{\pi} e^{\frac{i(b+i)^2}{4c}-ia}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(-ib-2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[In] Int[E^x*Sin[a + b*x + c*x^2],x]
```

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*a + (1 + I*b)^2/c))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + I
*b + (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(3/4)*E^((-I)*a + ((I/4)
*(I + b)^2)/c)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 - I*b - (2*I)*c*x))/(2*Sqrt[c])
])/(4*Sqrt[c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}ie^{-ia+(1-ib)x-icx^2}-\frac{1}{2}ie^{ia+(1+ib)x+icx^2}\right)\,dx\\
&= \frac{1}{2}i\int e^{-ia+(1-ib)x-icx^2}\,dx-\frac{1}{2}i\int e^{ia+(1+ib)x+icx^2}\,dx\\
&= -\left(\frac{1}{2}\left(ie^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\right)\int e^{-\frac{i(1+ib+2icx)^2}{4c}}\,dx\right)+\frac{1}{2}\left(ie^{-\frac{i\left(1-2ib-b^2+4ac\right)}{4c}}\right)\int e^{\frac{i(1-ib-2icx)^2}{4c}}\,dx\\
&= \frac{(-1)^{3/4}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(1+ib+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}\\
&\quad+\frac{(-1)^{3/4}e^{-\frac{i\left(1-2ib-b^2+4ac\right)}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(1-ib-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.93

$$\int e^x \sin\left(a+bx+cx^2\right)\,dx = -\frac{\sqrt[4]{-1}e^{-\frac{i\left(1-2ib+b^2\right)}{4c}}\sqrt{\pi}\left(e^{\frac{i}{2}/c}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(-i+b+2cx)}{2\sqrt{c}}\right)(\cos(a)+i\sin(a))+e^{\frac{ib^2}{2c}}\operatorname{erfi}\left(\frac{(-1)^{3/4}(i+b+2cx)}{2\sqrt{c}}\right)(i\cos(a)+\right.}{4\sqrt{c}}$$

```
[In] Integrate[E^x*Sin[a + b*x + c*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[Pi]*(E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + b + 2*c*x))/(
2*Sqrt[c])]*(Cos[a] + I*Sin[a]) + E^(((I/2)*b^2)/c)*Erfi[((-1)^(3/4)*(I + b
 + 2*c*x))/(2*Sqrt[c])]*(I*Cos[a] + Sin[a])))/(Sqrt[c]*E^(((I/4)*(1 - (2*I)
*b + b^2))/c))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{i\left(4ac-b^2+2ib+1\right)}{4c}}\operatorname{erf}\left(-\sqrt{-ic}\,x+\frac{ib+1}{2\sqrt{-ic}}\right)}{4\sqrt{-ic}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{-\frac{i\left(4ac-b^2-2ib+1\right)}{4c}}\operatorname{erf}\left(\sqrt{ic}\,x-\frac{-ib+1}{2\sqrt{ic}}\right)}{4\sqrt{ic}}$	119

```
[In] int(exp(x)*sin(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*exp(1/4*I*(-b^2+2*I*b+4*a*c+1)/c)/(-I*c)^(1/2)*erf(-(-I*c)^(
1/2)*x+1/2*(1+I*b)/(-I*c)^(1/2))+1/4*I*Pi^(1/2)*exp(-1/4*I*(-b^2-2*I*b+4*a*
c+1)/c)/(I*c)^(1/2)*erf((I*c)^(1/2)*x-1/2*(-I*b+1)/(I*c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(91) = 182.

Time = 0.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.59

$$\int e^x \sin\left(a+bx+cx^2\right)\,dx = \frac{i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{ib^2-4iac-2b-i}{4c}\right)}\operatorname{C}\left(\frac{\sqrt{2}(2cx+b+i)\sqrt{\frac{c}{\pi}}}{2c}\right)+i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{-ib^2+4iac-2b+i}{4c}\right)}\operatorname{C}\left(-\frac{\sqrt{2}(2cx+b-i)\sqrt{\frac{c}{\pi}}}{2c}\right)+\sqrt{2}\pi\sqrt{\frac{c}{\pi}}}{4c}$$

```
[In] integrate(exp(x)*sin(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/c)*fresnel_
cos(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) + I*sqrt(2)*pi*sqrt(c/pi)*e^(
1/4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_cos(-1/2*sqrt(2)*(2*c*x + b - I
)*sqrt(c/pi)/c) + sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/
c)*fresnel_sin(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) - sqrt(2)*pi*sqrt(
c/pi)*e^(1/4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_sin(-1/2*sqrt(2)*(2*c*
x + b - I)*sqrt(c/pi)/c))/c
```

Sympy [F]

$$\int e^x \sin\left(a + bx + cx^2\right)\,dx = \int e^x \sin\left(a + bx + cx^2\right)dx$$

```
[In] integrate(exp(x)*sin(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x)*sin(a + b*x + c*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91

$$\int e^x \sin\left(a + bx + cx^2\right)\,dx = -\frac{\sqrt{2}\sqrt{\pi}\left(\left((i+1)\cos\left(-\frac{b^2-4\,ac-1}{4\,c}\right) - (i-1)\sin\left(-\frac{b^2-4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{i\,(2i\,cx+i\,b-1)\sqrt{i\,c}}{2\,c}\right) + \left(-(i-1)\cos\left(-\right.\right.\right.}{8\,\sqrt{c}}$$

```
[In] integrate(exp(x)*sin(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*(((I + 1)*cos(-1/4*(b^2 - 4*a*c - 1)/c) - (I - 1)*sin
(-1/4*(b^2 - 4*a*c - 1)/c))*erf(1/2*I*(2*I*c*x + I*b - 1)*sqrt(I*c)/c) + (-
(I - 1)*cos(-1/4*(b^2 - 4*a*c - 1)/c) + (I + 1)*sin(-1/4*(b^2 - 4*a*c - 1)/
c))*erf(1/2*I*(2*I*c*x + I*b + 1)*sqrt(-I*c)/c))*e^(-1/2*b/c)/sqrt(c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

$$\int e^x \sin\left(a+bx+cx^2\right)\,dx = \frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}i\sqrt{2}\left(2x+\frac{b-i}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2-4iac+2b-i}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}} + \frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{4}i\sqrt{2}\left(2x+\frac{b+i}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-ib^2+4iac+2b+i}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

```
[In] integrate(exp(x)*sin(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(pi)*erf(-1/4*I*sqrt(2)*(2*x + (b - I)/c)*(I*c/abs(c) + 1)*
sqrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c + 2*b - I)/c)/((I*c/abs(c) + 1)*sqrt
(abs(c))) + 1/4*sqrt(2)*sqrt(pi)*erf(1/4*I*sqrt(2)*(2*x + (b + I)/c)*(-I*c/
abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c + 2*b + I)/c)/((-I*c/ab
s(c) + 1)*sqrt(abs(c)))
```

Mupad [F(-1)]

Timed out.

$$\int e^x \sin\left(a+bx+cx^2\right)\,dx = \int \sin\left(c\,x^2+b\,x+a\right)\,\mathrm{e}^x\,dx$$

```
[In] int(sin(a + b*x + c*x^2)*exp(x),x)
```

```
[Out] int(sin(a + b*x + c*x^2)*exp(x), x)
```

3.76 $\int e^{x^2} \sin(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 81

$$\int e^{x^2} \sin(a+bx)\,dx = \frac{1}{4}ie^{-ia+\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-ib+2x)\right) - \frac{1}{4}ie^{ia+\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(ib+2x)\right)$$

```
[Out] -1/4*I*exp(-I*a+1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)-1/4*I*exp(I*a+1/4*b^2)*er
fi(1/2*I*b+x)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4560, 2266, 2235}

$$\int e^{x^2} \sin(a+bx)\,dx = \frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}-ia}\text{erfi}\left(\frac{1}{2}(2x-ib)\right) - \frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}+ia}\text{erfi}\left(\frac{1}{2}(2x+ib)\right)$$

```
[In] Int[E^x^2*Sin[a + b*x],x]
```

```
[Out] (I/4)*E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(I*a + b
^2/4)*Sqrt[Pi]*Erfi[(I*b + 2*x)/2]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}ie^{-ia-ibx+x^2}-\frac{1}{2}ie^{ia+ibx+x^2}\right)dx\\
&= \frac{1}{2}i\int e^{-ia-ibx+x^2}\,dx-\frac{1}{2}i\int e^{ia+ibx+x^2}\,dx\\
&= \frac{1}{2}\left(ie^{-ia+\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(-ib+2x)^2}\,dx-\frac{1}{2}\left(ie^{ia+\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(ib+2x)^2}\,dx\\
&= \frac{1}{4}ie^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-ib+2x)\right)-\frac{1}{4}ie^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(ib+2x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int e^{x^2}\sin(a+bx)\,dx=\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\cos(a)\text{erf}\left(\frac{b}{2}-ix\right)+\cos(a)\text{erf}\left(\frac{b}{2}+ix\right)+\left(\text{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\text{erfi}\left(\frac{1}{2}(ib+2x)\right)\right)\sin(a)\right)$$

```
[In] Integrate[E^x^2*Sin[a + b*x],x]
[Out] (E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erf[b/2 - I*x] + Cos[a]*Erf[b/2 + I*x] + (Erfi[
((-I)*b + 2*x)/2] + Erfi[(I*b + 2*x)/2])*Sin[a]))/4
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{\sqrt{\pi}\,e^{\frac{b^2}{4}}\,e^{ia}\,\text{erf}\left(-ix+\frac{b}{2}\right)}{4}+\frac{\sqrt{\pi}\,e^{\frac{b^2}{4}}\,e^{-ia}\,\text{erf}\left(ix+\frac{b}{2}\right)}{4}$	52

```
[In] int(exp(x^2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(1/4*b^2)*exp(I*a)*erf(-I*x+1/2*b)+1/4*Pi^(1/2)*exp(1/4*b^2
)*exp(-I*a)*erf(I*x+1/2*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.56

$$\int e^{x^2}\sin(a+bx)\,dx = -\frac{1}{4}\sqrt{\pi}\left(\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2+i\,a\right)} - \operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2-i\,a\right)}\right)$$

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(pi)*(erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - erf(1/2*b + I*x)*e^(1/
4*b^2 - I*a))
```

Sympy [F]

$$\int e^{x^2}\sin(a+bx)\,dx = \int e^{x^2}\sin\left(a+bx\right)dx$$

```
[In] integrate(exp(x**2)*sin(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sin(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

$$\int e^{x^2}\sin(a+bx)\,dx$$
$$= \frac{1}{4}\sqrt{\pi}\left(\left(\cos\left(a\right)-i\,\sin\left(a\right)\right)\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)} - \left(\cos\left(a\right)+i\,\sin\left(a\right)\right)\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}\right)$$

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*((cos(a) - I*sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) - (cos(a) +
I*sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Giac [F]

$$\int e^{x^2} \sin(a+bx)\,dx = \int e^{(x^2)} \sin(bx+a)\,dx$$

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="giac")
[Out] integrate(e^(x^2)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \sin(a+bx)\,dx = \int \mathrm{e}^{x^2} \sin(a+b\,x)\,dx$$

```
[In] int(exp(x^2)*sin(a + b*x),x)
[Out] int(exp(x^2)*sin(a + b*x), x)
```

3.77 $\int e^{x^2} \sin(a + cx^2)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 87

$$\int e^{x^2} \sin\left(a + cx^2\right)\, dx = \frac{ie^{-ia}\sqrt{\pi}\text{erfi}\left(\sqrt{1-ic}x\right)}{4\sqrt{1-ic}} - \frac{ie^{ia}\sqrt{\pi}\text{erfi}\left(\sqrt{1+ic}x\right)}{4\sqrt{1+ic}}$$

```
[Out] 1/4*I*erfi(x*(1-I*c)^(1/2))*Pi^(1/2)/exp(I*a)/(1-I*c)^(1/2)-1/4*I*exp(I*a)*
erfi(x*(1+I*c)^(1/2))*Pi^(1/2)/(1+I*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4560, 2235}

$$\int e^{x^2} \sin\left(a + cx^2\right)\, dx = \frac{i\sqrt{\pi}e^{-ia}\text{erfi}\left(\sqrt{1-ic}x\right)}{4\sqrt{1-ic}} - \frac{i\sqrt{\pi}e^{ia}\text{erfi}\left(\sqrt{1+ic}x\right)}{4\sqrt{1+ic}}$$

```
[In] Int[E^x^2*Sin[a + c*x^2],x]
```

```
[Out] ((I/4)*Sqrt[Pi]*Erfi[Sqrt[1 - I*c]*x])/(Sqrt[1 - I*c]*E^(I*a)) - ((I/4)*E^(
I*a)*Sqrt[Pi]*Erfi[Sqrt[1 + I*c]*x])/Sqrt[1 + I*c]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2}ie^{-ia+(1-ic)x^2} - \frac{1}{2}ie^{ia+(1+ic)x^2}\right) dx \\ &= \frac{1}{2}i\int e^{-ia+(1-ic)x^2}\,dx - \frac{1}{2}i\int e^{ia+(1+ic)x^2}\,dx \\ &= \frac{ie^{-ia}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1-ic}\,x\right)}{4\sqrt{1-ic}} - \frac{ie^{ia}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1+ic}\,x\right)}{4\sqrt{1+ic}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.48

$$\int e^{x^2}\sin\left(a+cx^2\right)\,dx = -\frac{\sqrt[4]{-1}\sqrt{\pi}\left(\sqrt{-i+c}(i+c)\mathrm{erfi}\left(\sqrt[4]{-1}\sqrt{-i+c}\,x\right)(\cos(a)+i\sin(a))+\sqrt{i+c}\left(\mathrm{erf}\left(\frac{(1+i)\sqrt{i+c}\,x}{\sqrt{2}}\right)\sin(a)+\mathrm{erfi}\left(\right.\right.\right.}{4\left(1+c^2\right)}$$

```
[In] Integrate[E^x^2*Sin[a + c*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[Pi]*(Sqrt[-I + c]*(I + c)*Erfi[(-1)^(1/4)*Sqrt[-I + c
]*x]*(Cos[a] + I*Sin[a]) + Sqrt[I + c]*(Erf[((1 + I)*Sqrt[I + c]*x)/Sqrt[2]
]*Sin[a] + Erfi[(-1)^(3/4)*Sqrt[I + c]*x]*(Cos[a] + I*c*Cos[a] + c*Sin[a]))
))/(1 + c^2)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(\sqrt{-ic-1}\,x\right)}{4\sqrt{-ic-1}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(\sqrt{ic-1}\,x\right)}{4\sqrt{ic-1}}$	62

```
[In] int(exp(x^2)*sin(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(I*a)/(-I*c-1)^(1/2)*erf((-I*c-1)^(1/2)*x)+1/4*I*Pi^(1/2
)*exp(-I*a)/(I*c-1)^(1/2)*erf((I*c-1)^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int e^{x^2} \sin \left(a + cx^2\right) \, dx$$
$$= \frac{\sqrt{\pi}((c-i)\cos(a) + (-i\,c-1)\sin(a))\sqrt{i\,c-1}\,\mathrm{erf}\left(\sqrt{i\,c-1}x\right) + \sqrt{\pi}((c+i)\cos(a) + (i\,c-1)\sin(a))\sqrt{-}}{4\left(c^2+1\right)}$$

```
[In] integrate(exp(x^2)*sin(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*((c - I)*cos(a) + (-I*c - 1)*sin(a))*sqrt(I*c - 1)*erf(sqrt(I
*c - 1)*x) + sqrt(pi)*((c + I)*cos(a) + (I*c - 1)*sin(a))*sqrt(-I*c - 1)*er
f(sqrt(-I*c - 1)*x))/(c^2 + 1)
```

Sympy [F]

$$\int e^{x^2} \sin \left(a + cx^2\right) \, dx = \int e^{x^2} \sin \left(a + cx^2\right) dx$$

```
[In] integrate(exp(x**2)*sin(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*sin(a + c*x**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(53) = 106.

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int e^{x^2} \sin \left(a + cx^2\right) \, dx$$
$$= \frac{\sqrt{\pi}\sqrt{2\,c^2+2}\left((\cos(a) - i\,\sin(a))\,\mathrm{erf}\left(\sqrt{i\,c-1}x\right) + (\cos(a) + i\,\sin(a))\,\mathrm{erf}\left(\sqrt{-i\,c-1}x\right)\right)\sqrt{\sqrt{c^2+1}+1}}{8\,(}$$

```
[In] integrate(exp(x^2)*sin(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2 + 2)*((cos(a) - I*sin(a))*erf(sqrt(I*c - 1)*x) + (
cos(a) + I*sin(a))*erf(sqrt(-I*c - 1)*x))*sqrt(sqrt(c^2 + 1) + 1) - sqrt(pi
)*sqrt(2*c^2 + 2)*((-I*cos(a) - sin(a))*erf(sqrt(I*c - 1)*x) + (I*cos(a) -
sin(a))*erf(sqrt(-I*c - 1)*x))*sqrt(sqrt(c^2 + 1) - 1))/(c^2 + 1)
```

Giac [F]

$$\int e^{x^2}\sin\left(a+cx^2\right)\,dx=\int e^{\left(x^2\right)}\sin\left(cx^2+a\right)\,dx$$

```
[In] integrate(exp(x^2)*sin(c*x^2+a),x, algorithm="giac")
[Out] integrate(e^(x^2)*sin(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\sin\left(a+cx^2\right)\,dx=\int \mathrm{e}^{x^2}\,\sin\left(c\,x^2+a\right)\,dx$$

```
[In] int(exp(x^2)*sin(a + c*x^2),x)
[Out] int(exp(x^2)*sin(a + c*x^2), x)
```

3.78 $\int e^{x^2} \sin\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 155

$$\int e^{x^2} \sin\left(a+bx+cx^2\right)\,dx=-\frac{ie^{-i\left(a-\frac{b^2}{4i+4c}\right)}\sqrt{\pi}\text{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}-\frac{ie^{ia+\frac{b^2}{4(1+ic)}}\sqrt{\pi}\text{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}$$

```
[Out] -1/4*I*erfi(1/2*(I*b-2*(1-I*c)*x)/(1-I*c)^(1/2))*Pi^(1/2)/exp(I*(a-b^2/(4*I
+4*c)))/(1-I*c)^(1/2)-1/4*I*exp(I*a+1/4*b^2/(1+I*c))*erfi(1/2*(I*b+2*(1+I*c
)*x)/(1+I*c)^(1/2))*Pi^(1/2)/(1+I*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4560, 2266, 2235}

$$\int e^{x^2} \sin\left(a+bx+cx^2\right)\,dx=-\frac{i\sqrt{\pi}e^{-i\left(a-\frac{b^2}{4c+4i}\right)}\text{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}-\frac{i\sqrt{\pi}e^{ia+\frac{b^2}{4(1+ic)}}\text{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}$$

```
[In] Int[E^x^2*Sin[a + b*x + c*x^2],x]
```

```
[Out] ((-1/4*I)*Sqrt[Pi]*Erfi[(I*b - 2*(1 - I*c)*x)/(2*Sqrt[1 - I*c])])/(Sqrt[1 -
 I*c]*E^(I*(a - b^2/(4*I + 4*c)))) - ((I/4)*E^(I*a + b^2/(4*(1 + I*c)))*Sqr
t[Pi]*Erfi[(I*b + 2*(1 + I*c)*x)/(2*Sqrt[1 + I*c])])/Sqrt[1 + I*c]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2} i e^{-ia-ibx+(1-ic)x^2} - \frac{1}{2} i e^{ia+ibx+(1+ic)x^2}\right) dx \\
&= \frac{1}{2} i \int e^{-ia-ibx+(1-ic)x^2}\, dx - \frac{1}{2} i \int e^{ia+ibx+(1+ic)x^2}\, dx \\
&= -\left(\frac{1}{2}\left(i e^{ia+\frac{b^2}{4(1+ic)}}\right) \int \exp\left(\frac{(ib+2(1+ic)x)^2}{4(1+ic)}\right) dx\right) \\
&\quad + \frac{1}{2}\left(i e^{-i\left(a-\frac{b^2}{4i+4c}\right)}\right) \int \exp\left(\frac{(-ib+2(1-ic)x)^2}{4(1-ic)}\right) dx \\
&= -\frac{i e^{-i\left(a-\frac{b^2}{4i+4c}\right)} \sqrt{\pi}\, \text{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}} - \frac{i e^{ia+\frac{b^2}{4(1+ic)}} \sqrt{\pi}\, \text{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06

$$\int e^{x^2} \sin\left(a+bx+cx^2\right) dx = -\frac{(-1)^{3/4} e^{\frac{ib^2}{4i-4c}} \sqrt{\pi}\left((-i+c)\sqrt{i+c}\, e^{\frac{ib^2c}{2+2c^2}}\, \text{erfi}\left(\frac{(-1)^{3/4}(b+2(i+c)x)}{2\sqrt{i+c}}\right)(\cos(a)-i\sin(a)) + \sqrt{-i+c}(i+c)\text{erfi}\left(\frac{4}{\sqrt{}}\right.\right.}{4\left(1+c^2\right)}$$

```
[In] Integrate[E^x^2*Sin[a + b*x + c*x^2],x]
[Out] -1/4*((-1)^(3/4)*E^((I*b^2)/(4*I - 4*c))*Sqrt[Pi]*((-I + c)*Sqrt[I + c]*E^(
(I*b^2*c)/(2 + 2*c^2))*Erfi[((-1)^(3/4)*(b + 2*(I + c)*x))/(2*Sqrt[I + c])]
*(Cos[a] - I*Sin[a]) + Sqrt[-I + c]*(I + c)*Erfi[((-1)^(1/4)*(b + 2*(-I + c
)*x))/(2*Sqrt[-I + c])]*((-I)*Cos[a] + Sin[a])))/(1 + c^2)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{i\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-4ia-b^{2}}{4(ic+1)}}\operatorname{erf}\left(-\sqrt{-ic-1}\,x+\frac{ib}{2\sqrt{-ic-1}}\right)}{4\sqrt{-ic-1}}+\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{4ac+4ia-b^{2}}{4ic-4}}\operatorname{erf}\left(\sqrt{ic-1}\,x+\frac{ib}{2\sqrt{ic-1}}\right)}{4\sqrt{ic-1}}$	129

```
[In] int(exp(x^2)*sin(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*exp(-1/4*(4*a*c-4*I*a-b^2)/(1+I*c))/(-I*c-1)^(1/2)*erf(-(-I*
c-1)^(1/2)*x+1/2*I*b/(-I*c-1)^(1/2))+1/4*I*Pi^(1/2)*exp(1/4*(4*a*c+4*I*a-b^
2)/(I*c-1))/(I*c-1)^(1/2)*erf((I*c-1)^(1/2)*x+1/2*I*b/(I*c-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.04

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}(c-i)\sqrt{i\,c-1}\operatorname{erf}\left(-\frac{\left(bc+2\,(c^2+1)x-i\,b\right)\sqrt{i\,c-1}}{2\,(c^2+1)}\right)e^{\left(\frac{i\,b^2c-4i\,ac^2+b^2-4i\,a}{4\,(c^2+1)}\right)}-\sqrt{\pi}(c+i)\sqrt{-i\,c-1}\operatorname{erf}\left(\frac{(bc+2\,(c^2+\cdots}{2\cdots}\right.}{4\,(c^2+1)}$$

```
[In] integrate(exp(x^2)*sin(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c - I)*sqrt(I*c - 1)*erf(-1/2*(b*c + 2*(c^2 + 1)*x - I*b)*s
qrt(I*c - 1)/(c^2 + 1))*e^(1/4*(I*b^2*c - 4*I*a*c^2 + b^2 - 4*I*a)/(c^2 + 1
)) - sqrt(pi)*(c + I)*sqrt(-I*c - 1)*erf(1/2*(b*c + 2*(c^2 + 1)*x + I*b)*sq
rt(-I*c - 1)/(c^2 + 1))*e^(1/4*(-I*b^2*c + 4*I*a*c^2 + b^2 + 4*I*a)/(c^2 +
1)))/(c^2 + 1)
```

Sympy [F]

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)\,dx = \int e^{x^2}\sin\left(a+bx+cx^2\right)dx$$

```
[In] integrate(exp(x**2)*sin(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sin(a + b*x + c*x**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(101) = 202.

Time = 0.24 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.06

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}\sqrt{2\,c^2+2}\left(\left(\left(\cos\left(-\frac{b^2c-4\,ac^2-4\,a}{4\,(c^2+1)}\right)e^{\left(\frac{b^2}{4\,(c^2+1)}\right)}-i\,e^{\left(\frac{b^2}{4\,(c^2+1)}\right)}\sin\left(-\frac{b^2c-4\,ac^2-4\,a}{4\,(c^2+1)}\right)\right)\operatorname{erf}\left(-\frac{2\,(-i\,c+1)x-i\,b}{2\,\sqrt{i\,c-1}}\right)-\right.\right.}{}$$

```
[In] integrate(exp(x^2)*sin(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2 + 2)*((cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))
*e^(1/4*b^2/(c^2 + 1)) - I*e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2
- 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c + 1)*x - I*b)/sqrt(I*c - 1)) - (cos(-1
/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)) + I*e^(1/4*b^2/
(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c -
 1)*x - I*b)/sqrt(-I*c - 1)))*sqrt(sqrt(c^2 + 1) + 1) - sqrt(pi)*sqrt(2*c^2
 + 2)*((-I*cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)
) - e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(
-1/2*(2*(-I*c + 1)*x - I*b)/sqrt(I*c - 1)) + (-I*cos(-1/4*(b^2*c - 4*a*c^2
- 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)) + e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b
^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c - 1)*x - I*b)/sqrt(-I*c
 - 1)))*sqrt(sqrt(c^2 + 1) - 1))/(c^2 + 1)
```

Giac [F]

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)\,dx=\int e^{(x^2)}\sin\left(cx^2+bx+a\right)\,dx$$

```
[In] integrate(exp(x^2)*sin(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(e^(x^2)*sin(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \sin\left(a+bx+cx^2\right)\,dx=\int \sin\left(c\,x^2+b\,x+a\right)\,\mathrm{e}^{x^2}\,dx$$

```
[In] int(sin(a + b*x + c*x^2)*exp(x^2),x)
[Out] int(sin(a + b*x + c*x^2)*exp(x^2), x)
```

3.79 $\int f^{a+bx} \sin(d+fx^2)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 142

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx = \frac{1}{4}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right) - \frac{1}{4}(-1)^{3/4}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(2ifx-b\log(f))}{2\sqrt{f}}\right)$$

```
[Out] 1/4*(-1)^(3/4)*exp(1/4*I*(4*d+b^2*ln(f)^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)
*(2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-1/4*(-1)^(3/4)*f^(-1/2+a)*erfi(1/2*(-1
)^(1/4)*(2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)/exp(1/4*I*(4*d+b^2*ln(f)^2/f))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4560, 2325, 2266, 2235, 2236}

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx = \frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right) - \frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+2ifx)}{2\sqrt{f}}\right)$$

```
[In] Int[f^(a + b*x)*Sin[d + f*x^2],x]
```

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[((
-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(3/4)*f^(-1/2 + a
)*Sqrt[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(4*E^((I/
4)*(4*d + (b^2*Log[f]^2)/f)))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2} i e^{-id-ifx^2} f^{a+bx} - \frac{1}{2} i e^{id+ifx^2} f^{a+bx}\right) dx \\
&= \frac{1}{2} i \int e^{-id-ifx^2} f^{a+bx}\, dx - \frac{1}{2} i \int e^{id+ifx^2} f^{a+bx}\, dx \\
&= \frac{1}{2} i \int e^{-id-ifx^2+a\log(f)+bx\log(f)}\, dx - \frac{1}{2} i \int e^{id+ifx^2+a\log(f)+bx\log(f)}\, dx \\
&= \frac{1}{2}\left(i e^{-\frac{1}{4} i \left(4d+\frac{b^2\log^2(f)}{f}\right)} f^a\right) \int e^{\frac{i(-2ifx+b\log(f))^2}{4f}}\, dx - \frac{1}{2}\left(i e^{\frac{1}{4} i \left(4d+\frac{b^2\log^2(f)}{f}\right)} f^a\right) \int e^{-\frac{i(2ifx+b\log(f))^2}{4f}}\, dx
\end{aligned}$$

$$= \frac{1}{4}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\mathrm{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right) - \frac{1}{4}(-1)^{3/4}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(2ifx-b\log(f))}{2\sqrt{f}}\right)$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx = -\frac{1}{4}\sqrt[4]{-1}e^{-\frac{ib^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(e^{\frac{ib^2\log^2(f)}{2f}}\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(2fx-ib\log(f))}{2\sqrt{f}}\right)(\cos(d)+i\sin(d)) + \mathrm{erfi}\left(\frac{(-1)^{3/4}(2fx+ib\log(f))}{2\sqrt{f}}\right)(i\cos(d)+\sin(d))\right)$$

```
[In] Integrate[f^(a + b*x)*Sin[d + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*f^(-1/2 + a)*Sqrt[Pi]*(E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[((-
1)^(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] + I*Sin[d]) + Erfi[((-1
)^(3/4)*(2*f*x + I*b*Log[f]))/(2*Sqrt[f])]*(I*Cos[d] + Sin[d])))/E^(((I/4)*
b^2*Log[f]^2)/f)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{i\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{i\left(\ln(f)^{2}b^{2}+4df\right)}{4f}}\mathrm{erf}\left(-\sqrt{-if}\,x+\frac{\ln(f)b}{2\sqrt{-if}}\right)}{4\sqrt{-if}}-\frac{i\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{i\left(\ln(f)^{2}b^{2}+4df\right)}{4f}}\mathrm{erf}\left(-\sqrt{if}\,x+\frac{\ln(f)b}{2\sqrt{if}}\right)}{4\sqrt{if}}$	116

```
[In] int(f^(b*x+a)*sin(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f
)^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*
b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(98) = 196.

Time = 0.25 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.87

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx$$
$$=\frac{i\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+4\,af\log(f)-4i\,df}{4\,f}\right)}\mathrm{C}\left(\frac{\sqrt{2}(2\,fx+i\,b\log(f))\sqrt{\frac{f}{\pi}}}{2\,f}\right)+i\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2+4\,af\log(f)+4i\,df}{4\,f}\right)}\mathrm{C}\left(-\frac{\sqrt{2}(2}{}\right.}{}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d
*f)/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) + I*sqrt(
2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresne
l_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) + sqrt(2)*pi*sqrt(f/p
i)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqr
t(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b
^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x -
I*b*log(f))*sqrt(f/pi)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx=\int f^{a+bx}\sin\left(d+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*sin(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + f*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx=$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\;f^a\cos\left(\frac{b^2\log(f)^2+4\,df}{4\,f}\right)+(i-1)\;f^a\sin\left(\frac{b^2\log(f)^2+4\,df}{4\,f}\right)\right)\mathrm{erf}\left(\frac{2i\,fx-b\log(f)}{2\sqrt{i\,f}}\right)+\left(-(i-1)\right.\right.}{8\sqrt{f}}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I + 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f)/f) + (I
 - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*f*x - b*log(f))/s
qrt(I*f)) + (-(I - 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f)/f) + (I + 1)*f^a*s
in(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*f*x + b*log(f))/sqrt(-I*f)))
/sqrt(f)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(98) = 196.

Time = 0.34 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.11

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx$$

$$= \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}i\sqrt{2}\left(4\,x-\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)}{f}\right)\left(\frac{i\,f}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\,\pi^2b^2\mathrm{sgn}(f)}{8\,f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4\,f}-\frac{i\,\pi^2b^2}{8\,f}-\frac{\pi b^2\log(|f|)}{4\,f}+\cdots}}{4\left(\frac{i\,f}{|f|}+1\right)\sqrt{|f|}}$$

$$+\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{8}i\sqrt{2}\left(4\,x+\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)}{f}\right)\left(-\frac{i\,f}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\,\pi^2b^2\mathrm{sgn}(f)}{8\,f}-\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4\,f}+\frac{i\,\pi^2b^2}{8\,f}+\frac{\pi b^2\log(|f|}{4\,f}\cdots}}{4\left(-\frac{i\,f}{|f|}+1\right)\sqrt{|f|}}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*
log(abs(f)))/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/f +
 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs(f)
)/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(ab
s(f)) + I*d)/((I*f/abs(f) + 1)*sqrt(abs(f))) + 1/4*sqrt(2)*sqrt(pi)*erf(1/8
*I*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(-I*f/abs(f)
+ 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*sgn
(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f))
^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) - I*d)/((-I*f/abs(f)
+ 1)*sqrt(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sin\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,\sin\left(f\,x^2+d\right)\,dx$$

```
[In] int(f^(a + b*x)*sin(d + f*x^2),x)
[Out] int(f^(a + b*x)*sin(d + f*x^2), x)
```

3.80 $\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 157

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx = \left(\frac{1}{16}+\frac{i}{16}\right)e^{2id+\frac{ib^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx+b\log(f))}{\sqrt{f}}\right) + \left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx-b\log(f))}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

```
[Out] 1/2*f^(b*x+a)/b/ln(f)+(1/16+1/16*I)*exp(2*I*d+1/8*I*b^2*ln(f)^2/f)*f^(-1/2+
a)*erf((1/4+1/4*I)*(4*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)+(1/16+1/16*I)*f^(-1/
2+a)*erfi((1/4+1/4*I)*(4*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)/exp(1/8*I*(16*d+b
^2*ln(f)^2/f))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4560, 2225, 2325, 2266, 2235, 2236}

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx = \left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{8f}+2id}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+4ifx)}{\sqrt{f}}\right) + \left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{8}i\left(\frac{b^2\log^2(f)}{f}+16d\right)}\operatorname{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\log(f)+4ifx)}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

```
[In] Int[f^(a + b*x)*Sin[d + f*x^2]^2,x]
```

```
[Out] (1/16 + I/16)*E^((2*I)*d + ((I/8)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Er
f[((1/4 + I/4)*((4*I)*f*x + b*Log[f]))/Sqrt[f]] + ((1/16 + I/16)*f^(-1/2 +
a)*Sqrt[Pi]*Erfi[((1/4 + I/4)*((4*I)*f*x - b*Log[f]))/Sqrt[f]])/E^((I/8)*(1
6*d + (b^2*Log[f]^2)/f)) + f^(a + b*x)/(2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2}f^{a+bx} - \frac{1}{4}e^{-2id-2ifx^2}f^{a+bx} - \frac{1}{4}e^{2id+2ifx^2}f^{a+bx}\right) dx \\ &= -\left(\frac{1}{4}\int e^{-2id-2ifx^2}f^{a+bx}\,dx\right) - \frac{1}{4}\int e^{2id+2ifx^2}f^{a+bx}\,dx + \frac{1}{2}\int f^{a+bx}\,dx\end{aligned}$$

$$= \frac{f^{a+bx}}{2b\log(f)} - \frac{1}{4}\int e^{-2id-2ifx^2+a\log(f)+bx\log(f)}\,dx - \frac{1}{4}\int e^{2id+2ifx^2+a\log(f)+bx\log(f)}\,dx$$

$$= \frac{f^{a+bx}}{2b\log(f)} - \frac{1}{4}\left(e^{2id+\frac{ib^2\log^2(f)}{8f}}f^a\right)\int e^{-\frac{i(4ifx+b\log(f))^2}{8f}}\,dx - \frac{1}{4}\left(e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-4ifx+b\log(f))^2}{8f}}\,dx$$

$$= \left(\frac{1}{16}+\frac{i}{16}\right)e^{2id+\frac{ib^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx+b\log(f))}{\sqrt{f}}\right) + \left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx-b\log(f))}{\sqrt{f}}\right) + \frac{f^{a+bx}}{2b\log(f)}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx = \frac{1}{16}f^a\left(\frac{8f^{bx}}{b\log(f)} - \frac{(1-i)e^{-\frac{ib^2\log^2(f)}{8f}}\sqrt{\pi}\text{erf}\left(\frac{(4+4i)fx-(1-i)b\log(f)}{4\sqrt{f}}\right)(\cos(d)-i\sin(d))^2}{\sqrt{f}} - \frac{(1-i)e^{\frac{ib^2\log^2(f)}{8f}}\sqrt{\pi}\text{erfi}\left(\frac{(4+4i)fx+(1-i)b\log(f)}{4\sqrt{f}}\right)(\cos(d)+i\sin(d))^2}{\sqrt{f}}\right)$$

```
[In] Integrate[f^(a + b*x)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^a*((8*f^(b*x))/(b*Log[f]) - ((1 - I)*Sqrt[Pi]*Erf[((4 + 4*I)*f*x - (1 -
I)*b*Log[f])/(4*Sqrt[f])]*(Cos[d] - I*Sin[d])^2)/(E^(((I/8)*b^2*Log[f]^2)/f
)*Sqrt[f]) - ((1 - I)*E^(((I/8)*b^2*Log[f]^2)/f)*Sqrt[Pi]*Erfi[((4 + 4*I)*f
*x + (1 - I)*b*Log[f])/(4*Sqrt[f])]*(Cos[d] + I*Sin[d])^2)/Sqrt[f]))/16
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{\sqrt{\pi}\, f^a \mathrm{e}^{-\frac{i\left(\ln(f)^2 b^2+16df\right)}{8f}}\sqrt{2}\,\text{erf}\left(-\sqrt{2}\,\sqrt{if}\,x+\frac{b\ln(f)\sqrt{2}}{4\sqrt{if}}\right)}{16\sqrt{if}} + \frac{\sqrt{\pi}\, f^a \mathrm{e}^{\frac{i\left(\ln(f)^2 b^2+16df\right)}{8f}}\,\text{erf}\left(-\sqrt{-2if}\,x+\frac{b\ln(f)}{2\sqrt{-2if}}\right)}{8\sqrt{-2if}} + \frac{f^{xb+a}}{2b\ln(f)}$	139

```
[In] int(f^(b*x+a)*sin(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-1/8*I*(ln(f)^2*b^2+16*d*f)/f)*2^(1/2)/(I*f)^(1/2)*er
f(-2^(1/2)*(I*f)^(1/2)*x+1/4*b*ln(f)*2^(1/2)/(I*f)^(1/2))+1/8*Pi^(1/2)*f^a*
exp(1/8*I*(ln(f)^2*b^2+16*d*f)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2*
b*ln(f)/(-2*I*f)^(1/2))+1/2*f^(b*x+a)/b/ln(f)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(103) = 206.

Time = 0.25 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.72

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx =$$

$$-\frac{\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+8\,af\log(f)-16i\,df}{8\,f}\right)}\,\mathrm{C}\left(\frac{(4\,fx+i\,b\log(f))\sqrt{\frac{f}{\pi}}}{2\,f}\right)\log(f)-\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2+8\,af\log(f)+16i\,df}{8\,f}\right)}\,\mathrm{C}\left(-\frac{(4}{}\right.}{}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)
*fresnel_cos(1/2*(4*f*x + I*b*log(f))*sqrt(f/pi)/f)*log(f) - pi*b*sqrt(f/pi
)*e^(1/8*(I*b^2*log(f)^2 + 8*a*f*log(f) + 16*I*d*f)/f)*fresnel_cos(-1/2*(4*
f*x - I*b*log(f))*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*l
og(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)*fresnel_sin(1/2*(4*f*x + I*b*log(f))*
sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 + 8*a*f*log
(f) + 16*I*d*f)/f)*fresnel_sin(-1/2*(4*f*x - I*b*log(f))*sqrt(f/pi)/f)*log(
f) - 4*f*f^(b*x + a))/(b*f*log(f))
```

Sympy [F]

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx = \int f^{a+bx}\sin^2\left(d+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*sin(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.18

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx = \frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left((i-1)\,bf^a\cos\left(\frac{b^2\log(f)^2+16\,df}{8\,f}\right)\log(f)+(i+1)\,bf^a\log(f)\sin\left(\frac{b^2\log(f)^2+16\,df}{8\,f}\right)\right)\operatorname{erf}\left(\frac{4i\,fx-b\log(f}{2\,\sqrt{2i\,f}}\right.\right.}{}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*b*f^a*cos(1/8*(b^2*log(f)^2 + 16*d
*f)/f)*log(f) + (I + 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*d*f)/f))*er
f(1/2*(4*I*f*x - b*log(f))/sqrt(2*I*f)) + ((I + 1)*b*f^a*cos(1/8*(b^2*log(f
)^2 + 16*d*f)/f)*log(f) + (I - 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*d
*f)/f))*erf(1/2*(4*I*f*x + b*log(f))/sqrt(-2*I*f)))*f^(3/2) + 16*f^(b*x)*f^
(a + 2))/(b*f^2*log(f))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(103) = 206.

Time = 0.32 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.32

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(
abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - pi*
b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2
*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(abs(f)
)) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I
*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I*e^(-1/2*I*pi*b*x*
sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) +
 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f))) - 1/8*I*s
qrt(pi)*erf(-1/8*I*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/
f)*(I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*log(abs(f))*s
gn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*b^2*log(abs(
f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 2*I*d)/(sqrt(f)*
(I*f/abs(f) + 1)) + 1/8*I*sqrt(pi)*erf(1/8*I*sqrt(f)*(8*x + (pi*b*sgn(f) -
pi*b + 2*I*b*log(abs(f)))/f)*(-I*f/abs(f) + 1))*e^(-1/16*I*pi^2*b^2*sgn(f)/
```

```
f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f + 1/8*pi*b^2*log(ab
s(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*lo
g(abs(f)) - 2*I*d)/(sqrt(f)*(-I*f/abs(f) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\sin\left(f\,x^2+d\right)^2\,dx$$

```
[In] int(f^(a + b*x)*sin(d + f*x^2)^2,x)
[Out] int(f^(a + b*x)*sin(d + f*x^2)^2, x)
```

3.81 $\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 298

$$\begin{aligned}
&\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx\\
&=\frac{3}{16}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right)\\
&\quad+\left(\frac{1}{16}-\frac{i}{16}\right)e^{3id+\frac{ib^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(6ifx+b\log(f))}{\sqrt{6}\sqrt{f}}\right)\\
&\quad-\frac{3}{16}(-1)^{3/4}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt[4]{-1}(2ifx-b\log(f))}{2\sqrt{f}}\right)\\
&\quad-\left(\frac{1}{16}-\frac{i}{16}\right)e^{-\frac{1}{12}i\left(36d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(6ifx-b\log(f))}{\sqrt{6}\sqrt{f}}\right)
\end{aligned}$$

```
[Out] (1/96-1/96*I)*exp(3*I*d+1/12*I*b^2*ln(f)^2/f)*f^(-1/2+a)*erf((1/12+1/12*I)*
(6*I*f*x+b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)+(-1/96+1/96*I)*f^(-1/2+
a)*erfi((1/12+1/12*I)*(6*I*f*x-b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)/e
xp(1/12*I*(36*d+b^2*ln(f)^2/f))+3/16*(-1)^(3/4)*exp(1/4*I*(4*d+b^2*ln(f)^2/
f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)*(2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*
(-1)^(3/4)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/
2)/exp(1/4*I*(4*d+b^2*ln(f)^2/f))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4560, 2325, 2266, 2235, 2236}

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx$$
$$=\frac{3}{16}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)$$
$$+\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{12f}+3id}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$
$$-\frac{3}{16}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+2ifx)}{2\sqrt{f}}\right)$$
$$-\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{-\frac{1}{12}i\left(\frac{b^2\log^2(f)}{f}+36d\right)}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-b\log(f)+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$

```
[In] Int[f^(a + b*x)*Sin[d + f*x^2]^3,x]
```

```
[Out] (3*(-1)^(3/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[
((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 + (1/16 - I/16)*E^((3*
I)*d + ((I/12)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]*Erf[((1/2 + I/2)*((
6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3*(-1)^(3/4)*f^(-1/2 + a)*Sqrt[
Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(16*E^((I/4)*(4*
d + (b^2*Log[f]^2)/f))) - ((1/16 - I/16)*f^(-1/2 + a)*Sqrt[Pi/6]*Erfi[((1/2
 + I/2)*((6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])])/E^((I/12)*(36*d + (b^2*
Log[f]^2)/f))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{8}ie^{-id-ifx^2}f^{a+bx}-\frac{3}{8}ie^{id+ifx^2}f^{a+bx}-\frac{1}{8}ie^{-3id-3ifx^2}f^{a+bx}+\frac{1}{8}ie^{3id+3ifx^2}f^{a+bx}\right)dx\\
&= -\left(\frac{1}{8}i\int e^{-3id-3ifx^2}f^{a+bx}\,dx\right)+\frac{1}{8}i\int e^{3id+3ifx^2}f^{a+bx}\,dx\\
&\quad +\frac{3}{8}i\int e^{-id-ifx^2}f^{a+bx}\,dx-\frac{3}{8}i\int e^{id+ifx^2}f^{a+bx}\,dx\\
&= -\left(\frac{1}{8}i\int e^{-3id-3ifx^2+a\log(f)+bx\log(f)}\,dx\right)+\frac{1}{8}i\int e^{3id+3ifx^2+a\log(f)+bx\log(f)}\,dx\\
&\quad +\frac{3}{8}i\int e^{-id-ifx^2+a\log(f)+bx\log(f)}\,dx-\frac{3}{8}i\int e^{id+ifx^2+a\log(f)+bx\log(f)}\,dx\\
&= \frac{1}{8}\left(ie^{3id+\frac{ib^2\log^2(f)}{12f}}f^a\right)\int e^{-\frac{i(6ifx+b\log(f))^2}{12f}}\,dx\\
&\quad +\frac{1}{8}\left(3ie^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-2ifx+b\log(f))^2}{4f}}\,dx\\
&\quad -\frac{1}{8}\left(3ie^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{-\frac{i(2ifx+b\log(f))^2}{4f}}\,dx\\
&\quad -\frac{1}{8}\left(ie^{-\frac{1}{12}i\left(36d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-6ifx+b\log(f))^2}{12f}}\,dx
\end{aligned}$$

$$= \frac{3}{16}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right)$$
$$+\left(\frac{1}{16}-\frac{i}{16}\right)e^{3id+\frac{ib^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(6ifx+b\log(f))}{\sqrt{6}\sqrt{f}}\right)$$
$$-\frac{3}{16}(-1)^{3/4}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(2ifx-b\log(f))}{2\sqrt{f}}\right)$$
$$-\left(\frac{1}{16}-\frac{i}{16}\right)e^{-\frac{1}{12}i\left(36d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(6ifx-b\log(f))}{\sqrt{6}\sqrt{f}}\right)$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.90

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx$$
$$=\frac{1}{48}(-1)^{3/4}e^{-\frac{ib^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(-9\operatorname{erfi}\left(\frac{(-1)^{3/4}(2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))+9ie^{\frac{ib^2\log^2(f)}{2f}}\operatorname{erfi}\left(\frac{\sqrt[4]{}}{}\right.\right.$$

```
[In] Integrate[f^(a + b*x)*Sin[d + f*x^2]^3,x]
```

```
[Out] ((-1)^(3/4)*f^(-1/2 + a)*Sqrt[Pi]*(-9*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f])
)/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) + (9*I)*E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[
((-1)^(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] + I*Sin[d]) + Sqrt[3
]*E^(((I/6)*b^2*Log[f]^2)/f)*(Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f]))/(2*Sqr
t[3]*Sqrt[f])]*(Cos[3*d] - I*Sin[3*d]) + E^(((I/6)*b^2*Log[f]^2)/f)*Erfi[((
6 + 6*I)*f*x + (1 - I)*b*Log[f])/(2*Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Sin[
3*d]))))/(48*E^(((I/4)*b^2*Log[f]^2)/f))
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{i\sqrt{\pi}\,f^a\mathrm{e}^{\frac{i\left(\ln(f)^2b^2+36df\right)}{12f}}\operatorname{erf}\left(-\sqrt{-3if}\,x+\frac{\ln(f)b}{2\sqrt{-3if}}\right)}{16\sqrt{-3if}}+\frac{i\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{i\left(\ln(f)^2b^2+36df\right)}{12f}}\sqrt{3}\,\operatorname{erf}\left(-\sqrt{3}\,\sqrt{if}\,x+\frac{\ln(f)b\sqrt{3}}{6\sqrt{if}}\right)}{48\sqrt{if}}-\frac{3i\sqrt{\pi}\,f^a\mathrm{e}}{}$

```
[In] int(f^(b*x+a)*sin(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(1/12*I*(ln(f)^2*b^2+36*d*f)/f)/(-3*I*f)^(1/2)*erf(
-(-3*I*f)^(1/2)*x+1/2*ln(f)*b/(-3*I*f)^(1/2))+1/48*I*Pi^(1/2)*f^a*exp(-1/12
*I*(ln(f)^2*b^2+36*d*f)/f)*3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*x+1
```

```
/6*ln(f)*b*3^(1/2)/(I*f)^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2
+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))+3/16*I*P
i^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f)^(1/2)
*x+1/2*ln(f)*b/(-I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(196) = 392.

Time = 0.26 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.76

$$\int f^{a+bx} \sin^3\left(d+fx^2\right)\,dx = \frac{-i\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+12\,af\log(f)-36i\,df}{12\,f}\right)}\mathrm{C}\left(\frac{\sqrt{6}(6\,fx+i\,b\log(f))\sqrt{\frac{f}{\pi}}}{6\,f}\right)-i\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2+12\,af\log(f)+36i\,df}{12\,f}\right)}\mathrm{C}\left(-\right.}{}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(-I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 12*a*f*log(f) - 3
6*I*d*f)/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f))*sqrt(f/pi)/f) - I*
sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 + 12*a*f*log(f) + 36*I*d*f)/f
)*fresnel_cos(-1/6*sqrt(6)*(6*f*x - I*b*log(f))*sqrt(f/pi)/f) + 9*I*sqrt(2)
*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel
_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) + 9*I*sqrt(2)*pi*sqrt(f
/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_cos(-1/2*s
qrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) - sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(
-I*b^2*log(f)^2 + 12*a*f*log(f) - 36*I*d*f)/f)*fresnel_sin(1/6*sqrt(6)*(6*f
*x + I*b*log(f))*sqrt(f/pi)/f) + sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f
)^2 + 12*a*f*log(f) + 36*I*d*f)/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*lo
g(f))*sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a
*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f
/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4
*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f))/f
```

Sympy [F]

$$\int f^{a+bx} \sin^3\left(d+fx^2\right)\,dx = \int f^{a+bx}\sin^3\left(d+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*sin(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx$$
$$=\frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\ f^{a}\cos\left(\frac{b^2\log(f)^2+36\,df}{12\,f}\right)+(i-1)\ f^{a}\sin\left(\frac{b^2\log(f)^2+36\,df}{12\,f}\right)\right)\operatorname{erf}\left(\frac{6i\,fx-b\log(f)}{2\,\sqrt{3i\,f}}\right)+\left(-(i-\right.\right.}{}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*((-(I + 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*d
*f)/f) + (I - 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f)/f))*erf(1/2*(6*I*f*x
- b*log(f))/sqrt(3*I*f)) + (-(I - 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*d*f)/f
) + (I + 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f)/f))*erf(1/2*(6*I*f*x + b*l
og(f))/sqrt(-3*I*f)))*f^(3/2) - 9*sqrt(2)*sqrt(pi)*((-(I + 1)*f^a*cos(1/4*(
b^2*log(f)^2 + 4*d*f)/f) + (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*e
rf(1/2*(2*I*f*x - b*log(f))/sqrt(I*f)) + (-(I - 1)*f^a*cos(1/4*(b^2*log(f)^
2 + 4*d*f)/f) + (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I
*f*x + b*log(f))/sqrt(-I*f)))*f^(3/2))/f^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(196) = 392.

Time = 0.39 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.00

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b
*log(abs(f)))/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/f
+ 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs(f
))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(a
bs(f)) + I*d)/((I*f/abs(f) + 1)*sqrt(abs(f))) - 1/48*sqrt(6)*sqrt(pi)*erf(-
1/24*I*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*
(I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*b^2*log(abs(f))*sgn
(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f + 1/12*I*b^2*log(abs(
f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 3*I*d)/(sqrt(f)*
(I*f/abs(f) + 1)) - 1/48*sqrt(6)*sqrt(pi)*erf(1/24*I*sqrt(6)*sqrt(f)*(12*x
+ (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(-I*f/abs(f) + 1))*e^(-1/24*I
```

```
*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*sgn(f)/f + 1/24*I*pi^2*b^2/f +
 1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f)
 + 1/2*I*pi*a + a*log(abs(f)) - 3*I*d)/(sqrt(f)*(-I*f/abs(f) + 1)) + 3/16*s
qrt(2)*sqrt(pi)*erf(1/8*I*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(ab
s(f)))/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4
*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f
- 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)
) - I*d)/((-I*f/abs(f) + 1)*sqrt(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,\sin\left(f\,x^2+d\right)^3\,dx$$

```
[In] int(f^(a + b*x)*sin(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*sin(d + f*x^2)^3, x)
```

3.82 $\int f^{a+bx} \sin(d+ex+fx^2)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 162

$$\begin{aligned}\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx &= \frac{1}{4}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)\\ &\quad-\frac{1}{4}(-1)^{3/4}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt[4]{-1}(ie+2ifx-b\log(f))}{2\sqrt{f}}\right)\end{aligned}$$

```
[Out] 1/4*(-1)^(3/4)*exp(1/4*I*(4*d+(I*e+b*ln(f))^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(
1/4)*(I*e+2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-1/4*(-1)^(3/4)*exp(-I*d+1/4*I*
(e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(I*e+2*I*f*x-b*ln(f))/f^(
1/2))*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4560, 2325, 2266, 2235, 2236}

$$\begin{aligned}\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx &= \frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)\\ &\quad-\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\text{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)\end{aligned}$$

```
[In] Int[f^(a + b*x)*Sin[d + e*x + f*x^2],x]
```

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]*Er
f[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(3/4)*E
^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Erfi[((-1)^(
1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2} i e^{-id-iex-ifx^2} f^{a+bx} - \frac{1}{2} i e^{id+iex+ifx^2} f^{a+bx}\right) dx \\
&= \frac{1}{2} i \int e^{-id-iex-ifx^2} f^{a+bx} \, dx - \frac{1}{2} i \int e^{id+iex+ifx^2} f^{a+bx} \, dx \\
&= \frac{1}{2} i \int \exp\left(-id - ifx^2 + a\log(f) - x(ie - b\log(f))\right) dx \\
&\quad - \frac{1}{2} i \int \exp\left(id + ifx^2 + a\log(f) + x(ie + b\log(f))\right) dx
\end{aligned}$$

$$= \frac{1}{2}\left(ie^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^a\right)\int e^{\frac{i(-ie-2ifx+b\log(f))^2}{4f}}\,dx - \frac{1}{2}\left(ie^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^a\right)\int e^{-\frac{i(ie+2ifx+b\log(f))^2}{4f}}\,dx$$

$$= \frac{1}{4}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right) - \frac{1}{4}(-1)^{3/4}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(ie+2ifx-b\log(f))}{2\sqrt{f}}\right)$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx = -\frac{1}{4}\sqrt[4]{-1}e^{-\frac{i\left(e^2+b^2\log^2(f)\right)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(e^{\frac{ib^2\log^2(f)}{2f}}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-ib\log(f))}{2\sqrt{f}}\right)(\cos(d)+i\sin(d))+e^{\frac{ie^2}{2f}}\operatorname{erfi}\left(\frac{(-1)^{3/4}(e+2fx+ib\log(f))}{2\sqrt{f}}\right)(i\cos(d)+\sin(d))\right)$$

```
[In] Integrate[f^(a + b*x)*Sin[d + e*x + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(E^(((I/2)*b^2*Log[f]^2)/
f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] + I*Sin[
d]) + E^(((I/2)*e^2)/f)*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f]))/(2*Sqrt[
f])]*(I*Cos[d] + Sin[d])))/E^(((I/4)*(e^2 + b^2*Log[f]^2))/f)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.96

method	result	s
risch	$\frac{i\sqrt{\pi}\,f^a f^{-\frac{be}{2f}}\,\mathrm{e}^{\frac{i\left(\ln(f)^2b^2+4df-e^2\right)}{4f}}\operatorname{erf}\left(-\sqrt{-if}\,x+\frac{ie+b\ln(f)}{2\sqrt{-if}}\right)}{4\sqrt{-if}}-\frac{i\sqrt{\pi}\,f^a f^{-\frac{be}{2f}}\,\mathrm{e}^{-\frac{i\left(\ln(f)^2b^2+4df-e^2\right)}{4f}}\operatorname{erf}\left(-\sqrt{if}\,x+\frac{b\ln(f)-ie}{2\sqrt{if}}\right)}{4\sqrt{if}}$	1

```
[In] int(f^(b*x+a)*sin(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(1/4*I*(ln(f)^2*b^2+4*d*f-e^2)/f)/(-I*
f)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))-1/4*I*Pi^(1/2)
*f^a*f^(-1/2/f*b*e)*exp(-1/4*I*(ln(f)^2*b^2+4*d*f-e^2)/f)/(I*f)^(1/2)*erf(-
(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(109) = 218.

Time = 0.25 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.93

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx$$
$$=\frac{i\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+i\,e^2-4i\,df-2\,(be-2\,af)\log(f)}{4\,f}\right)}\,\mathrm{C}\left(\frac{\sqrt{2}(2\,fx+i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)+i\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2-i\,e^2+4i\,df-2\,(be-2\,a}{4\,f}\right.}}{}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*e^2 - 4*I*d*f - 2*
(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f) + e)*s
qrt(f/pi)/f) + I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 - I*e^2 + 4*I
*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log
(f) + e)*sqrt(f/pi)/f) + sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*
e^2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x +
 I*b*log(f) + e)*sqrt(f/pi)/f) - sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)
^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*sqrt(2)*
(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\sin\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*sin(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + e*x + f*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.17

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx=$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left((i+1)\ f^a\cos\left(\frac{b^2\log(f)^2-e^2+4\,df}{4\,f}\right)-(i-1)\ f^a\sin\left(\frac{b^2\log(f)^2-e^2+4\,df}{4\,f}\right)\right)\mathrm{erf}\left(\frac{i\,(2i\,fx-b\log(f)+i\,e)\sqrt{i\,f}}{2\,f}\right)-}{8\sqrt{f}\,f}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*(((I + 1)*f^a*cos(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f)
 - (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))*erf(1/2*I*(2*I*f*x
- b*log(f) + I*e)*sqrt(I*f)/f) + (-(I - 1)*f^a*cos(1/4*(b^2*log(f)^2 - e^2
+ 4*d*f)/f) + (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))*erf(1/2*
I*(2*I*f*x + b*log(f) + I*e)*sqrt(-I*f)/f))/(sqrt(f)*f^(1/2*b*e/f))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(109) = 218$.

Time = 0.34 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.33

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx$$

$$= \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}i\sqrt{2}\left(4x-\frac{\pi b\mathrm{sgn}(f)-\pi b+2ib\log(|f|)-2e}{f}\right)\left(\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}-\frac{i\pi^2b^2}{8f}\right)}}{4\left(\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

$$+\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{8}i\sqrt{2}\left(4x+\frac{\pi b\mathrm{sgn}(f)-\pi b+2ib\log(|f|)+2e}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}-\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}+\frac{i\pi^2b^2}{8f}\right)}}{4\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*
log(abs(f)) - 2*e)/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(
f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(
abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sg
n(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/
f + I*d - 1/4*I*e^2/f)/((I*f/abs(f) + 1)*sqrt(abs(f))) + 1/4*sqrt(2)*sqrt(p
i)*erf(1/8*I*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 2*e)/
f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2
*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I
*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi
*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - I*d + 1/4*I*e
^2/f)/((-I*f/abs(f) + 1)*sqrt(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sin\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\,\sin\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int(f^(a + b*x)*sin(d + e*x + f*x^2),x)
[Out] int(f^(a + b*x)*sin(d + e*x + f*x^2), x)
```

3.83 $\int f^{a+bx} \sin^2(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\begin{aligned}
&\int f^{a+bx} \sin^2\left(d+ex+fx^2\right)\,dx \\
&= \left(\frac{1}{16}+\frac{i}{16}\right) e^{2id+\frac{i(2ie+b\log(f))^2}{8f}} f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx+b\log(f))}{\sqrt{f}}\right) \\
&\quad + \left(\frac{1}{16}+\frac{i}{16}\right) e^{-2id+\frac{i(2e+ib\log(f))^2}{8f}} f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx-b\log(f))}{\sqrt{f}}\right) \\
&\quad + \frac{f^{a+bx}}{2b\log(f)}
\end{aligned}$$

```
[Out] 1/2*f^(b*x+a)/b/ln(f)+(1/16+1/16*I)*exp(2*I*d+1/8*I*(2*I*e+b*ln(f))^2/f)*f^
(-1/2+a)*erf((1/4+1/4*I)*(2*I*e+4*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)+(1/16+1/
16*I)*exp(-2*I*d+1/8*I*(2*e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi((1/4+1/4*I)*(2*
I*e+4*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{4560, 2225, 2325, 2266, 2235, 2236\}$

$$\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx$$
$$=\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+2ie)^2}{8f}+2id}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)$$
$$+\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(2e+ib\log(f))^2}{8f}-2id}\operatorname{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)$$
$$+\frac{f^{a+bx}}{2b\log(f)}$$

```
[In] Int[f^(a + b*x)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (1/16 + I/16)*E^((2*I)*d + ((I/8)*((2*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*S
qrt[Pi]*Erf[((1/4 + I/4)*((2*I)*e + (4*I)*f*x + b*Log[f]))/Sqrt[f]] + (1/16
 + I/16)*E^((-2*I)*d + ((I/8)*(2*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi
]*Erfi[((1/4 + I/4)*((2*I)*e + (4*I)*f*x - b*Log[f]))/Sqrt[f]] + f^(a + b*x
)/(2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2} f^{a+bx} - \frac{1}{4} e^{-2id-2iex-2ifx^2} f^{a+bx} - \frac{1}{4} e^{2id+2iex+2ifx^2} f^{a+bx}\right) dx \\
&= -\left(\frac{1}{4} \int e^{-2id-2iex-2ifx^2} f^{a+bx}\, dx\right) - \frac{1}{4} \int e^{2id+2iex+2ifx^2} f^{a+bx}\, dx + \frac{1}{2} \int f^{a+bx}\, dx \\
&= \frac{f^{a+bx}}{2b\log(f)} - \frac{1}{4} \int \exp\left(-2id - 2ifx^2 + a\log(f) - x(2ie - b\log(f))\right) dx \\
&\quad - \frac{1}{4} \int \exp\left(2id + 2ifx^2 + a\log(f) + x(2ie + b\log(f))\right) dx \\
&= \frac{f^{a+bx}}{2b\log(f)} - \frac{1}{4} \exp\left(-2id + a\log(f) - \frac{i(-2ie + b\log(f))^2}{8f}\right) \int e^{\frac{i(-2ie-4ifx+b\log(f))^2}{8f}}\, dx \\
&\quad - \frac{1}{4}\left(e^{2id+\frac{i(2ie+b\log(f))^2}{8f}} f^a\right) \int e^{-\frac{i(2ie+4ifx+b\log(f))^2}{8f}}\, dx \\
&= \left(\frac{1}{16} + \frac{i}{16}\right) e^{2id+\frac{i(2ie+b\log(f))^2}{8f}} f^{-\frac{1}{2}+a} \sqrt{\pi}\,\text{erf}\left(\frac{\left(\frac{1}{4} + \frac{i}{4}\right)(2ie + 4ifx + b\log(f))}{\sqrt{f}}\right) \\
&\quad + \left(\frac{1}{16} + \frac{i}{16}\right) \exp\left(-\frac{1}{8} i\left(16d \right.\right. \\
&\qquad\qquad \left.\left. + \frac{(2ie - b\log(f))^2}{f}\right)\right) f^{-\frac{1}{2}+a} \sqrt{\pi}\,\text{erfi}\left(\frac{\left(\frac{1}{4} + \frac{i}{4}\right)(2ie + 4ifx - b\log(f))}{\sqrt{f}}\right) \\
&\quad + \frac{f^{a+bx}}{2b\log(f)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.36

$$\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{e^{-\frac{i\left(4e^2+b^2\log^2(f)\right)}{8f}}f^{a-\frac{be+f}{2f}}\left(8e^{\frac{i\left(4e^2+b^2\log^2(f)\right)}{8f}}f^{\frac{1}{2}+b\left(\frac{e}{2f}+x\right)}+\sqrt[4]{-1}be^{\frac{ib^2\log^2(f)}{4f}}\sqrt{2\pi}\text{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2i(e+2fx)+b\log(f))}{\sqrt{f}}\right)\log(f)\right.}{16b\log(f)}$$

[In] Integrate[f^(a + b*x)*Sin[d + e*x + f*x^2]^2,x]

[Out] (f^(a - (b*e + f)/(2*f))*(8*E^(((I/8)*(4*e^2 + b^2*Log[f]^2))/f)*f^(1/2 + b*(e/(2*f) + x)) + (-1)^(1/4)*b*E^(((I/4)*b^2*Log[f]^2)/f)*Sqrt[2*Pi]*Erf[((1/4 + I/4)*((2*I)*(e + 2*f*x) + b*Log[f]))/Sqrt[f]]*Log[f]*(Cos[2*d] + I*Sin[2*d]) + (-1)^(1/4)*b*E^((I*e^2)/f)*Sqrt[2*Pi]*Erf[((1/4 + I/4)*(2*e + 4*f*x + I*b*Log[f]))/Sqrt[f]]*Log[f]*(I*Cos[2*d] + Sin[2*d])))/(16*b*E^(((I/8)*(4*e^2 + b^2*Log[f]^2))/f)*Log[f])

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\sqrt{\pi}\,f^a f^{-\frac{be}{2f}}\,\mathrm{e}^{-\frac{i\left(\ln(f)^2b^2+16df-4e^2\right)}{8f}}\sqrt{2}\,\mathrm{erf}\left(-\sqrt{2}\,\sqrt{if}\,x+\frac{(b\ln(f)-2ie)\sqrt{2}}{4\sqrt{if}}\right)}{16\sqrt{if}}+\frac{\sqrt{\pi}\,f^a f^{-\frac{be}{2f}}\,\mathrm{e}^{\frac{i\left(\ln(f)^2b^2+16df-4e^2\right)}{8f}}\,\mathrm{erf}\left(-\sqrt{-2if}\,x+\frac{2i\ldots}{\ldots}\right.}{8\sqrt{-2if}}$

[In] int(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] 1/16*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/8*I*(ln(f)^2*b^2+16*d*f-4*e^2)/f)*2^(1/2)/(I*f)^(1/2)*erf(-2^(1/2)*(I*f)^(1/2)*x+1/4*(b*ln(f)-2*I*e)*2^(1/2)/(I*f)^(1/2))+1/8*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(1/8*I*(ln(f)^2*b^2+16*d*f-4*e^2)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-2*I*f)^(1/2))+1/2*f^(b*x+a)/b/ln(f)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(116) = 232.

Time = 0.26 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.82

$$\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+4i\,e^2-16i\,df-4\,(be-2\,af)\log(f)}{8\,f}\right)}\mathrm{C}\left(\frac{(4\,fx+i\,b\log(f)+2\,e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)\log\left(f\right)-\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2-4i\,e^2+16i\,df-}{8\,f}\right.}}{}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 4*I*e^2 - 16*I*d*f - 4*(b*e
 - 2*a*f)*log(f))/f)*fresnel_cos(1/2*(4*f*x + I*b*log(f) + 2*e)*sqrt(f/pi)/
f)*log(f) - pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 - 4*I*e^2 + 16*I*d*f - 4
*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/2*(4*f*x - I*b*log(f) + 2*e)*sqrt(
f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 4*I*e^2 - 16*
I*d*f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*(4*f*x + I*b*log(f) + 2*
e)*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 - 4*I*e^
2 + 16*I*d*f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*(4*f*x - I*b*log
(f) + 2*e)*sqrt(f/pi)/f)*log(f) - 4*f*f^(b*x + a))/(b*f*log(f))
```

Sympy [F]

$$\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx = \int f^{a+bx}\sin^2\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*sin(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + e*x + f*x**2)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(116) = 232.

Time = 0.33 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.34

$$\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx$$

$$=\frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-(i-1)\,bf^a\cos\left(\frac{b^2\log(f)^2-4\,e^2+16\,df}{8\,f}\right)\log\left(f\right)-(i+1)\,bf^a\log\left(f\right)\sin\left(\frac{b^2\log(f)^2-4\,e^2+16\,df}{8\,f}\right)\right)\mathrm{erf}}{}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*b*f^a*cos(1/8*(b^2*log(f)^2 - 4*e
^2 + 16*d*f)/f)*log(f) - (I + 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 - 4*e^2
 + 16*d*f)/f))*erf(1/4*I*(4*I*f*x - b*log(f) + 2*I*e)*sqrt(2*I*f)/f) + ((I
+ 1)*b*f^a*cos(1/8*(b^2*log(f)^2 - 4*e^2 + 16*d*f)/f)*log(f) + (I - 1)*b*f^
a*log(f)*sin(1/8*(b^2*log(f)^2 - 4*e^2 + 16*d*f)/f))*erf(1/4*I*(4*I*f*x + b
*log(f) + 2*I*e)*sqrt(-2*I*f)/f))*f^(3/2) + 16*f^(a + 2)*e^(b*x*log(f) + 1/
2*b*e*log(f)/f))/(b*f^2*f^(1/2*b*e/f)*log(f))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(116) = 232$.

Time = 0.37 (sec) , antiderivative size = 599, normalized size of antiderivative = 3.35

$$\int f^{a+bx} \sin^2\left(d + ex + fx^2\right) dx = \text{Too large to display}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(
abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - pi*
b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2
*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(abs(f)
)) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I
*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I*e^(-1/2*I*pi*b*x*
sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) +
 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f))) - 1/8*I*s
qrt(pi)*erf(-1/8*I*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) -
 4*e)/f)*(I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*log(abs
(f))*sgn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*b^2*lo
g(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/
4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 2*I*d - 1/2*I*e^2/f)
/(sqrt(f)*(I*f/abs(f) + 1)) + 1/8*I*sqrt(pi)*erf(1/8*I*sqrt(f)*(8*x + (pi*b
*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 4*e)/f)*(-I*f/abs(f) + 1))*e^(-1/16*I*
pi^2*b^2*sgn(f)/f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f + 1
/8*pi*b^2*log(abs(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1
/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*
e*log(abs(f))/f - 2*I*d + 1/2*I*e^2/f)/(sqrt(f)*(-I*f/abs(f) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx} \sin^2\left(d+ex+fx^2\right)\,dx = \int f^{a+b\,x}\sin\left(f\,x^2+e\,x+d\right)^2\,dx$$

```
[In] int(f^(a + b*x)*sin(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + b*x)*sin(d + e*x + f*x^2)^2, x)
```

3.84 $\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 340

$$\begin{aligned}\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx &= \frac{3}{16}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)\\ &+\left(\frac{1}{16}-\frac{i}{16}\right)e^{3id+\frac{i(3ie+b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3ie+6ifx+b\log(f))}{\sqrt{6}\sqrt{f}}\right)\\ &-\frac{3}{16}(-1)^{3/4}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(ie+2ifx-b\log(f))}{2\sqrt{f}}\right)\\ &-\left(\frac{1}{16}-\frac{i}{16}\right)e^{-3id+\frac{i(3e+ib\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3ie+6ifx-b\log(f))}{\sqrt{6}\sqrt{f}}\right)\end{aligned}$$

```
[Out] (1/96-1/96*I)*exp(3*I*d+1/12*I*(3*I*e+b*ln(f))^2/f)*f^(-1/2+a)*erf((1/12+1/
12*I)*(3*I*e+6*I*f*x+b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)+(-1/96+1/96
*I)*exp(-3*I*d+1/12*I*(3*e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi((1/12+1/12*I)*(3
*I*e+6*I*f*x-b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)+3/16*(-1)^(3/4)*exp
(1/4*I*(4*d+(I*e+b*ln(f))^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)*(I*e+2*I*f*x+
b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*(-1)^(3/4)*exp(-I*d+1/4*I*(e+I*b*ln(f))^2/f
)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(I*e+2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4560, 2325, 2266, 2235, 2236}

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx$$

$$= \frac{3}{16}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)$$
$$+\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+3ie)^2}{12f}+3id}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+3ie+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$
$$-\frac{3}{16}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\text{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)$$
$$-\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(3e+ib\log(f))^2}{12f}-3id}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-b\log(f)+3ie+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$

```
[In] Int[f^(a + b*x)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*(-1)^(3/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]*
Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 + (1/16 - I/
16)*E^((3*I)*d + ((I/12)*((3*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]
*Erf[((1/2 + I/2)*((3*I)*e + (6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3
*(-1)^(3/4)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]
*Erfi[((-1)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 -
I/16)*E^((-3*I)*d + ((I/12)*(3*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6
]*Erfi[((1/2 + I/2)*((3*I)*e + (6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{1}{8}ie^{-3i(d+ex+fx^2)}f^{a+bx} + \frac{3}{8}i\exp\left(2id+2iex+2ifx^2-3i(d+ex+fx^2)\right)f^{a+bx} - \frac{3}{8}i\exp\left(4id+4iex+4ifx^2-3i(d+ex+fx^2)\right)f^{a+bx} + \frac{1}{8}i\exp\left(6id+6iex+6ifx^2-3i(d+ex+fx^2)\right)f^{a+bx}\right)dx$$

$$= -\left(\frac{1}{8}i\int e^{-3i(d+ex+fx^2)}f^{a+bx}\,dx\right) + \frac{1}{8}i\int \exp\left(6id+6iex+6ifx^2-3i(d+ex+fx^2)\right)f^{a+bx}\,dx + \frac{3}{8}i\int \exp\left(2id+2iex+2ifx^2-3i(d+ex+fx^2)\right)f^{a+bx}\,dx - \frac{3}{8}i\int \exp\left(4id+4iex+4ifx^2-3i(d+ex+fx^2)\right)f^{a+bx}\,dx$$

$$= -\left(\frac{1}{8}i\int \exp\left(-3id-3ifx^2+a\log(f)-x(3ie-b\log(f))\right)\,dx\right) + \frac{1}{8}i\int \exp\left(3id+3ifx^2+a\log(f)+x(3ie+b\log(f))\right)\,dx + \frac{3}{8}i\int \exp\left(-id-ifx^2+a\log(f)-x(ie-b\log(f))\right)\,dx - \frac{3}{8}i\int \exp\left(id+ifx^2+a\log(f)+x(ie+b\log(f))\right)\,dx$$

$$= -\left(\frac{1}{8}\left(i\exp\left(-3id + a\log(f) - \frac{i(-3ie + b\log(f))^2}{12f}\right)\right)\int e^{\frac{i(-3ie-6ifx+b\log(f))^2}{12f}}\,dx\right)$$
$$+ \frac{1}{8}\left(3ie^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^a\right)\int e^{\frac{i(-ie-2ifx+b\log(f))^2}{4f}}\,dx$$
$$- \frac{1}{8}\left(3ie^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^a\right)\int e^{-\frac{i(ie+2ifx+b\log(f))^2}{4f}}\,dx$$
$$+ \frac{1}{8}\left(ie^{3id+\frac{i(3ie+b\log(f))^2}{12f}}f^a\right)\int e^{-\frac{i(3ie+6ifx+b\log(f))^2}{12f}}\,dx$$

$$= \frac{3}{16}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)$$
$$+ \left(\frac{1}{16} - \frac{i}{16}\right)e^{3id+\frac{i(3ie+b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\,\mathrm{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3ie+6ifx+b\log(f))}{\sqrt{6}\sqrt{f}}\right)$$
$$- \frac{3}{16}(-1)^{3/4}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(ie+2ifx-b\log(f))}{2\sqrt{f}}\right)$$
$$- \left(\frac{1}{16}-\frac{i}{16}\right)\exp\left(-\frac{1}{12}i\left(36d+\frac{(3ie-b\log(f))^2}{f}\right)\right)f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\,\mathrm{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3ie+6ifx-b\log}{\sqrt{6}\sqrt{f}}\right.$$

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.95

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx$$
$$= \frac{1}{48}(-1)^{3/4}e^{-\frac{i\left(3e^2+b^2\log^2(f)\right)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(9ie^{\frac{i\left(e^2+b^2\log^2(f)\right)}{2f}}\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-ib\log(f))}{2\sqrt{f}}\right)(\cos(d)+i\sin(d))\right.$$
$$+e^{\frac{ie^2}{f}}\left(-9\mathrm{erfi}\left(\frac{(-1)^{3/4}(e+2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))+\sqrt{3}e^{\frac{i\left(3e^2+b^2\log^2(f)\right)}{6f}}\mathrm{erfi}\left(\frac{(-1)^{3/4}(3e+6f}{2\sqrt{3}}\right.\right.$$

```
[In] Integrate[f^(a + b*x)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] ((-1)^(3/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*((9*I)*E^(((I/2)*(e^2 + b^2*Lo
g[f]^2))/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d]
 + I*Sin[d]) + E^((I*e^2)/f)*(-9*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f]))
/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) + Sqrt[3]*E^(((I/6)*(3*e^2 + b^2*Log[f]^2
))/f)*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f]))/(2*Sqrt[3]*Sqrt[f])]*(Co
s[3*d] - I*Sin[3*d])) + Sqrt[3]*E^(((I/3)*b^2*Log[f]^2)/f)*Erfi[((1/2 + I/2
)*(3*e + 6*f*x - I*b*Log[f]))/(Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Sin[3*d])
))/(48*E^(((I/4)*(3*e^2 + b^2*Log[f]^2))/f))
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+36df-9e^{2}\right)}{12f}} \operatorname{erf}\left(-\sqrt{-3if}\, x+\frac{3ie+b\ln(f)}{2\sqrt{-3if}}\right)}{16\sqrt{-3if}}+\frac{i\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+36df-9e^{2}\right)}{12f}} \sqrt{3}\, \operatorname{erf}\left(-\sqrt{3}\,\sqrt{if}\, x+\frac{(b}{}\right.}{48\sqrt{if}}$

```
[In] int(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(1/12*I*(ln(f)^2*b^2+36*d*f-9*e^2)/f
)/(-3*I*f)^(1/2)*erf(-(-3*I*f)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-3*I*f)^(1/2))+
1/48*I*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/12*I*(ln(f)^2*b^2+36*d*f-9*e^2)/f
)*3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*x+1/6*(b*ln(f)-3*I*e)*3^(1/2
)/(I*f)^(1/2))-3/16*I*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/4*I*(ln(f)^2*b^2+4
*d*f-e^2)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))+
3/16*I*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(1/4*I*(ln(f)^2*b^2+4*d*f-e^2)/f)/(-I
*f)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(220) = 440.

Time = 0.26 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.85

$$\int f^{a+bx}\sin^{3}\left(d+ex+fx^{2}\right)\,dx = \frac{-i\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-ib^{2}\log(f)^{2}+9ie^{2}-36idf-6(be-2af)\log(f)}{12f}\right)}\mathrm{C}\left(\frac{\sqrt{6}(6fx+ib\log(f)+3e)\sqrt{\frac{f}{\pi}}}{6f}\right)-i\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{ib^{2}\log(f)^{2}-9ie^{2}+36idf-}{12f}\right.}}{}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(-I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 9*I*e^2 - 36*I*d*
f - 6*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f)
+ 3*e)*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 - 9*
I*e^2 + 36*I*d*f - 6*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/6*sqrt(6)*(6*f
*x - I*b*log(f) + 3*e)*sqrt(f/pi)/f) + 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I
*b^2*log(f)^2 + I*e^2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/
2*sqrt(2)*(2*f*x + I*b*log(f) + e)*sqrt(f/pi)/f) + 9*I*sqrt(2)*pi*sqrt(f/pi
)*e^(1/4*(I*b^2*log(f)^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fre
snel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f) - sqrt(6)*pi*s
qrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 9*I*e^2 - 36*I*d*f - 6*(b*e - 2*a*f)*l
og(f))/f)*fresnel_sin(1/6*sqrt(6)*(6*f*x + I*b*log(f) + 3*e)*sqrt(f/pi)/f)
```

```
+ sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 - 9*I*e^2 + 36*I*d*f - 6*(b
*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*log(f) + 3*e)*
sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*e^2 - 4
*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*lo
g(f) + e)*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 -
I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*
x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx = \int f^{a+bx}\sin^3\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*sin(f*x**2+e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + e*x + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.11

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx$$
$$= \frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(\left(i+1\right)\,f^a\cos\left(\frac{b^2\log(f)^2-9\,e^2+36\,df}{12\,f}\right)-\left(i-1\right)\,f^a\sin\left(\frac{b^2\log(f)^2-9\,e^2+36\,df}{12\,f}\right)\right)\operatorname{erf}\left(\frac{i\,(6i\,fx-b\log(f)+3i\,e)}{6\,f}\right.\right.}{}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*f^a*cos(1/12*(b^2*log(f)^2 - 9*e^2
 + 36*d*f)/f) - (I - 1)*f^a*sin(1/12*(b^2*log(f)^2 - 9*e^2 + 36*d*f)/f))*er
f(1/6*I*(6*I*f*x - b*log(f) + 3*I*e)*sqrt(3*I*f)/f) + (-(I - 1)*f^a*cos(1/1
2*(b^2*log(f)^2 - 9*e^2 + 36*d*f)/f) + (I + 1)*f^a*sin(1/12*(b^2*log(f)^2 -
 9*e^2 + 36*d*f)/f))*erf(1/6*I*(6*I*f*x + b*log(f) + 3*I*e)*sqrt(-3*I*f)/f)
)*f^(3/2) - 9*sqrt(2)*sqrt(pi)*(((I + 1)*f^a*cos(1/4*(b^2*log(f)^2 - e^2 +
4*d*f)/f) - (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))*erf(1/2*I*
(2*I*f*x - b*log(f) + I*e)*sqrt(I*f)/f) + (-(I - 1)*f^a*cos(1/4*(b^2*log(f)
^2 - e^2 + 4*d*f)/f) + (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))
*erf(1/2*I*(2*I*f*x + b*log(f) + I*e)*sqrt(-I*f)/f))*f^(3/2))/(f^2*f^(1/2*b
*e/f))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(220) = 440$.

Time = 0.46 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.21

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b
*log(abs(f)) - 2*e)/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn
(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log
(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*s
gn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))
/f + I*d - 1/4*I*e^2/f)/((I*f/abs(f) + 1)*sqrt(abs(f))) - 1/48*sqrt(6)*sqrt
(pi)*erf(-1/24*I*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(ab
s(f)) - 6*e)/f)*(I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*b^2
*log(abs(f))*sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f + 1/1
2*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I
*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 3*I*d - 3/
4*I*e^2/f)/(sqrt(f)*(I*f/abs(f) + 1)) - 1/48*sqrt(6)*sqrt(pi)*erf(1/24*I*sq
rt(6)*sqrt(f)*(12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 6*e)/f)*(-I
*f/abs(f) + 1))*e^(-1/24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*sgn(
f)/f + 1/24*I*pi^2*b^2/f + 1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(abs(f
))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*
b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - 3*I*d + 3/4*I*e^2/f)/(sqrt(
f)*(-I*f/abs(f) + 1)) + 3/16*sqrt(2)*sqrt(pi)*erf(1/8*I*sqrt(2)*(4*x + (pi*
b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 2*e)/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)
))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi
^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*
a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(
f)) - 1/2*b*e*log(abs(f))/f - I*d + 1/4*I*e^2/f)/((-I*f/abs(f) + 1)*sqrt(ab
s(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx = \int f^{a+b\,x}\,{\sin\left(f\,x^2+e\,x+d\right)}^3\,dx$$

```
[In] int(f^(a + b*x)*sin(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*sin(d + e*x + f*x^2)^3, x)
```

3.85 $\int f^{a+cx^2}\sin(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 151

$$\int f^{a+cx^2}\sin(d+ex)\,dx=-\frac{ie^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{ie^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 1/4*I*exp(-I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-I*e+2*c*x*ln(f))/c^(1/2)/ln(
f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/4*I*exp(I*d+1/4*e^2/c/ln(f))*f^a*e
rfi(1/2*(I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4560, 2325, 2266, 2235}

$$\int f^{a+cx^2}\sin(d+ex)\,dx=-\frac{i\sqrt{\pi}f^ae^{\frac{e^2}{4c\log(f)}-id}\operatorname{erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{i\sqrt{\pi}f^ae^{\frac{e^2}{4c\log(f)}+id}\operatorname{erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x],x]
```

```
[Out] ((-1/4*I)*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[
f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) - ((I/4)*E^(I*d + e^2
```

```
/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f
]])])/(Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2} i e^{-id-iex} f^{a+cx^2} - \frac{1}{2} i e^{id+iex} f^{a+cx^2}\right) dx \\
&= \frac{1}{2} i \int e^{-id-iex} f^{a+cx^2} \, dx - \frac{1}{2} i \int e^{id+iex} f^{a+cx^2} \, dx \\
&= \frac{1}{2} i \int e^{-id-iex+a\log(f)+cx^2\log(f)} \, dx - \frac{1}{2} i \int e^{id+iex+a\log(f)+cx^2\log(f)} \, dx \\
&= \frac{1}{2} \left(i e^{-id+\frac{e^2}{4c\log(f)}} f^a\right) \int e^{\frac{(-ie+2cx\log(f))^2}{4c\log(f)}} \, dx - \frac{1}{2} \left(i e^{id+\frac{e^2}{4c\log(f)}} f^a\right) \int e^{\frac{(ie+2cx\log(f))^2}{4c\log(f)}} \, dx \\
&= -\frac{i e^{-id+\frac{e^2}{4c\log(f)}} f^a \sqrt{\pi} \operatorname{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{i e^{id+\frac{e^2}{4c\log(f)}} f^a \sqrt{\pi} \operatorname{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.79

$$\int f^{a+cx^2}\sin(d+ex)\,dx = \frac{e^{\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(i\text{erfi}\left(\frac{-ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))+\text{erfi}\left(\frac{-ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(i\cos(d)+\sin(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x],x]
```

```
[Out] (E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*(I*Erfi[((-I)*e - 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) + Erfi[((-I)*e + 2*c*x*Log[f])/(2*Sqr
t[c]*Sqrt[Log[f]])]*(I*Cos[d] + Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{4 i d \ln(f) c+e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{i e}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 i d \ln(f) c-e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c\ln(f)}\, x+\frac{i e}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}$	123

```
[In] int(f^(c*x^2+a)*sin(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*er
f(-(-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))+1/4*I*Pi^(1/2)*f^a*exp(-1/4
*(4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*I
*e/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95

$$\int f^{a+cx^2}\sin(d+ex)\,dx = \frac{i\,\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(2\,cx\log\left(f\right)+i\,e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)e^{\left(\frac{4\,ac\log\left(f\right)^{2}+4i\,cd\log\left(f\right)+e^{2}}{4\,c\log\left(f\right)}\right)}-i\,\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(2\,cx\log\left(f\right)-i\,e\right)}{2\,c\log(}\right.}{4\,c\log\left(f\right)}$$

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*log(f)
)/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))) -
I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f))/(c*
log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))))/(c*log
(f))
```

Sympy [F]

$$\int f^{a+cx^2}\sin(d+ex)\,dx=\int f^{a+cx^2}\sin\left(d+ex\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sin(e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.36

$$\int f^{a+cx^2}\sin(d+ex)\,dx=$$

$$-\frac{\sqrt{\pi}\left(f^a(i\,\cos\left(d\right)+\sin\left(d\right))\,\mathrm{erf}\left(x\overline{\sqrt{-c\log\left(f\right)}}+\frac{1}{2}i\,e\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^2}{4\,c\log(f)}\right)}+f^a(-i\,\cos\left(d\right)+\sin\left(d\right))\,\mathrm{erf}\left(\right.\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(f^a*(I*cos(d) + sin(d))*erf(x*conjugate(sqrt(-c*log(f))) + 1
/2*I*e*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(-I*cos(d
) + sin(d))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*I*e*conjugate(1/sqrt(-c*
log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(I*cos(d) - sin(d))*erf(1/2*(2*c*x*l
og(f) + I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))) + f^a*(-I*cos(d) - sin
(d))*erf(1/2*(2*c*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))))*
sqrt(-c*log(f))/(c*log(f))
```

Giac [F]

$$\int f^{a+cx^2} \sin(d+ex)\,dx = \int f^{cx^2+a} \sin(ex+d)\,dx$$

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d),x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*sin(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \sin(d+ex)\,dx = \int f^{c\,x^2+a} \sin(d+e\,x)\,dx$$

```
[In] int(f^(a + c*x^2)*sin(d + e*x),x)
[Out] int(f^(a + c*x^2)*sin(d + e*x), x)
```

3.86 $\int f^{a+cx^2}\sin^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 171

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx=\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{e^{2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -1/8*exp(-2*I*d+e^2/c/ln(f))*f^a*erfi((-I*e+c*x*ln(f))/c^(1/2)/ln(f)^(1/2))
*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/8*exp(2*I*d+e^2/c/ln(f))*f^a*erfi((I*e+c*x*
ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^a*erfi(x*c^(
1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4560, 2235, 2325, 2266}

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx=\frac{\sqrt{\pi}f^ae^{\frac{e^2}{c\log(f)}-2id}\operatorname{erfi}\left(\frac{-cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^ae^{\frac{e^2}{c\log(f)}+2id}\operatorname{erfi}\left(\frac{cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(
(-2*I)*d + e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - c*x*Log[f])/(Sqrt[c]*Sq
rt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) - (E^((2*I)*d + e^2/(c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[L
og[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}f^{a+cx^2} - \frac{1}{4}e^{-2id-2iex}f^{a+cx^2} - \frac{1}{4}e^{2id+2iex}f^{a+cx^2}\right) dx \\
&= -\left(\frac{1}{4}\int e^{-2id-2iex}f^{a+cx^2}\,dx\right) - \frac{1}{4}\int e^{2id+2iex}f^{a+cx^2}\,dx + \frac{1}{2}\int f^{a+cx^2}\,dx \\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{1}{4}\int e^{-2id-2iex+a\log(f)+cx^2\log(f)}\,dx \\
&\quad - \frac{1}{4}\int e^{2id+2iex+a\log(f)+cx^2\log(f)}\,dx \\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{1}{4}\left(e^{-2id+\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(-2ie+2cx\log(f))^2}{4c\log(f)}}\,dx \\
&\quad - \frac{1}{4}\left(e^{2id+\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(2ie+2cx\log(f))^2}{4c\log(f)}}\,dx
\end{aligned}$$

$$= \frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{e^{-2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} - \frac{e^{2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.77

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx = \frac{f^a\sqrt{\pi}\left(2\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right) - e^{\frac{e^2}{c\log(f)}}\left(\text{erfi}\left(\frac{-ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)-i\sin(2d)) + \text{erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)+\right.\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] - E^(e^2/(c*Log[f]))*(Erfi[((
-I)*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + Erfi[
(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] + I*Sin[2*d]))))/(8*Sq
rt[c]*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\sqrt{\pi}\,f^a \mathrm{e}^{-\frac{2id\ln(f)c-e^2}{\ln(f)c}}\,\mathrm{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{ie}{\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}} + \frac{\sqrt{\pi}\,f^a \mathrm{e}^{\frac{2id\ln(f)c+e^2}{\ln(f)c}}\,\mathrm{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{ie}{\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}} + \frac{f^a\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\right.}}{4\sqrt{-c\ln}}$

```
[In] int(f^(c*x^2+a)*sin(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-
c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp((2*I*d*ln(f)*c+
e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2)
)+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx =$$
$$-\frac{2\sqrt{\pi}\sqrt{-c\log(f)}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)-\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(cx\log(f)+ie)\sqrt{-c\log(f)}}{c\log(f)}\right)e^{\left(\frac{ac\log(f)^2+2icd\log(f}{c\log(f)}\right.}}{8\,c\log(f)}$$

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log(f))*x) - sqrt(pi)*sqrt
(-c*log(f))*erf((c*x*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(
f)^2 + 2*I*c*d*log(f) + e^2)/(c*log(f))) - sqrt(pi)*sqrt(-c*log(f))*erf((c*
x*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(f)^2 - 2*I*c*d*log(
f) + e^2)/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx = \int f^{a+cx^2}\sin^2(d+ex)\,dx$$

```
[In] integrate(f**(c*x**2+a)*sin(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.38

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx =$$
$$-\frac{\sqrt{\pi}\left(f^a(\cos(2d)-i\sin(2d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}+ie\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^2}{c\log(f)}\right)}+f^a(\cos(2d)+i\sin(2d))e\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(pi)*(f^a*(cos(2*d) - I*sin(2*d))*erf(x*conjugate(sqrt(-c*log(f))
) + I*e*conjugate(1/sqrt(-c*log(f))))*e^(e^2/(c*log(f))) + f^a*(cos(2*d) +
```

```
I*sin(2*d))*erf(x*conjugate(sqrt(-c*log(f))) - I*e*conjugate(1/sqrt(-c*log(
f))))*e^(e^2/(c*log(f))) - f^a*(cos(2*d) + I*sin(2*d))*erf((c*x*log(f) + I*
e)/sqrt(-c*log(f)))*e^(e^2/(c*log(f))) - f^a*(cos(2*d) - I*sin(2*d))*erf((c
*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(e^2/(c*log(f))) - 2*f^a*erf(x*conjugat
e(sqrt(-c*log(f)))) - 2*f^a*erf(sqrt(-c*log(f))*x))/sqrt(-c*log(f))
```

Giac [F]

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx=\int f^{cx^2+a}\sin\left(ex+d\right)^2\,dx$$

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx=\int f^{c\,x^2+a}\,\sin\left(d+e\,x\right)^2\,dx$$

```
[In] int(f^(a + c*x^2)*sin(d + e*x)^2,x)
```

```
[Out] int(f^(a + c*x^2)*sin(d + e*x)^2, x)
```

3.87 $\int f^{a+cx^2} \sin^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 301

$$\int f^{a+cx^2} \sin^3(d+ex)\,dx = -\frac{3ie^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{ie^{-3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3ie^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{ie^{3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 3/16*I*exp(-I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-I*e+2*c*x*ln(f))/c^(1/2)/ln
(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/16*I*exp(-3*I*d+9/4*e^2/c/ln(f))*
f^a*erfi(1/2*(-3*I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(
f)^(1/2)-3/16*I*exp(I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(I*e+2*c*x*ln(f))/c^(
1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*I*exp(3*I*d+9/4*e^2/c/l
n(f))*f^a*erfi(1/2*(3*I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4560, 2325, 2266, 2235}

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx = -\frac{3i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\text{erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{i\sqrt{\pi}f^a e^{\frac{9e^2}{4c\log(f)}-3id}\text{erfi}\left(\frac{-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\text{erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{i\sqrt{\pi}f^a e^{\frac{9e^2}{4c\log(f)}+3id}\text{erfi}\left(\frac{2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x]^3,x]
```

```
[Out] (((-3*I)/16)*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((-3*I
)*d + (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e - 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) - (((3*I)/16)*E^(I*d + e^2/(4
*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])
])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((3*I)*d + (9*e^2)/(4*c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*
Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{8}ie^{-id-iex}f^{a+cx^2} - \frac{3}{8}ie^{id+iex}f^{a+cx^2} - \frac{1}{8}ie^{-3id-3iex}f^{a+cx^2} + \frac{1}{8}ie^{3id+3iex}f^{a+cx^2}\right) dx \\
&= -\left(\frac{1}{8}i\int e^{-3id-3iex}f^{a+cx^2}\,dx\right) + \frac{1}{8}i\int e^{3id+3iex}f^{a+cx^2}\,dx \\
&\quad + \frac{3}{8}i\int e^{-id-iex}f^{a+cx^2}\,dx - \frac{3}{8}i\int e^{id+iex}f^{a+cx^2}\,dx \\
&= -\left(\frac{1}{8}i\int e^{-3id-3iex+a\log(f)+cx^2\log(f)}\,dx\right) + \frac{1}{8}i\int e^{3id+3iex+a\log(f)+cx^2\log(f)}\,dx \\
&\quad + \frac{3}{8}i\int e^{-id-iex+a\log(f)+cx^2\log(f)}\,dx - \frac{3}{8}i\int e^{id+iex+a\log(f)+cx^2\log(f)}\,dx \\
&= \frac{1}{8}\left(3ie^{-id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-ie+2cx\log(f))^2}{4c\log(f)}}\,dx - \frac{1}{8}\left(3ie^{id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(ie+2cx\log(f))^2}{4c\log(f)}}\,dx \\
&\quad - \frac{1}{8}\left(ie^{-3id+\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-3ie+2cx\log(f))^2}{4c\log(f)}}\,dx + \frac{1}{8}\left(ie^{3id+\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(3ie+2cx\log(f))^2}{4c\log(f)}}\,dx \\
&= -\frac{3ie^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{ie^{-3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{3ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} \\
&\quad - \frac{3ie^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{ie^{3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{3ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.74

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx$$

$$= \frac{e^{\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(3i\,\text{erfi}\left(\frac{-ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d)) + 3\,\text{erfi}\left(\frac{-ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(i\cos(d)+\sin(d)) - ie^{\frac{2e^2}{c\log(f)}}\right.}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x]^3,x]
```

```
[Out] (E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*((3*I)*Erfi[((-I)*e - 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) + 3*Erfi[((-I)*e + 2*c*x*Log[f])/
(2*Sqrt[c]*Sqrt[Log[f]])]*(I*Cos[d] + Sin[d]) - I*E^((2*e^2)/(c*Log[f]))*(E
rfi[((-3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] - I*Sin[3
*d]) - Erfi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] +
I*Sin[3*d]))))/(16*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{3 i d \ln(f) c+\frac{9 e^{2}}{4}}{c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{3 i e}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 i d \ln(f) c-3 e^{2}\right)}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{3 i e}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}+\frac{3 i\sqrt{\pi}}{}$

```
[In] int(f^(c*x^2+a)*sin(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2
)*erf(-(-c*ln(f))^(1/2)*x+3/2*I*e/(-c*ln(f))^(1/2))-1/16*I*Pi^(1/2)*f^a*exp
(-3/4*(4*I*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*
x+3/2*I*e/(-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c-e^2
)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2)
)+3/16*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)
*erf(-(-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.94

$$\int f^{a+cx^2} \sin^3(d+ex)\,dx = \frac{-i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2\,cx\log(f)+3i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(\frac{4\,ac\log(f)^2+12i\,cd\log(f)+9\,e^2}{4\,c\log(f)}\right)}+3i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2\,cx\log(f}{2}\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(-I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + 3*I*e)*sqrt(-c*lo
g(f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 12*I*c*d*log(f) + 9*e^2)/(c*log(
f))) + 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*lo
g(f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))
) - 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f
))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))) +
```

```
 I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - 3*I*e)*sqrt(-c*log(f))/
(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 - 12*I*c*d*log(f) + 9*e^2)/(c*log(f))))/
(c*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx=\int f^{a+cx^2}\sin^3(d+ex)\,dx$$

```
[In] integrate(f**(c*x**2+a)*sin(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.37

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx$$
$$=\frac{\sqrt{\pi}\left(f^a(i\cos(3d)+\sin(3d))\operatorname{erf}\left(x\sqrt{-c\log(f)}+\frac{3}{2}ie\frac{1}{\sqrt{-c\log(f)}}\right)e^{\left(\frac{9e^2}{4c\log(f)}\right)}+f^a(-i\cos(3d)+\sin(3d)\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(pi)*(f^a*(I*cos(3*d) + sin(3*d))*erf(x*conjugate(sqrt(-c*log(f)))
 + 3/2*I*e*conjugate(1/sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(-I*c
os(3*d) + sin(3*d))*erf(x*conjugate(sqrt(-c*log(f))) - 3/2*I*e*conjugate(1/
sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(I*cos(3*d) - sin(3*d))*erf(
1/2*(2*c*x*log(f) + 3*I*e)/sqrt(-c*log(f)))*e^(9/4*e^2/(c*log(f))) + f^a*(-
I*cos(3*d) - sin(3*d))*erf(1/2*(2*c*x*log(f) - 3*I*e)/sqrt(-c*log(f)))*e^(9
/4*e^2/(c*log(f))) - 3*f^a*(I*cos(d) + sin(d))*erf(x*conjugate(sqrt(-c*log(
f))) + 1/2*I*e*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) - 3*f^a
*(-I*cos(d) + sin(d))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*I*e*conjugate(
1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(I*cos(d) - sin(d))*erf(
1/2*(2*c*x*log(f) + I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(-
I*cos(d) - sin(d))*erf(1/2*(2*c*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(1/4*e^2
/(c*log(f))))*sqrt(-c*log(f))/(c*log(f))
```

Giac [F]

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx = \int f^{cx^2+a}\sin\left(ex+d\right)^3\,dx$$

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*sin(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sin^3(d+ex)\,dx = \int f^{c\,x^2+a}\sin\left(d+e\,x\right)^3\,dx$$

```
[In] int(f^(a + c*x^2)*sin(d + e*x)^3,x)
[Out] int(f^(a + c*x^2)*sin(d + e*x)^3, x)
```

3.88 $\int f^{a+cx^2} \sin(d+fx^2)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int f^{a+cx^2}\sin(d+fx^2)\,dx=\frac{ie^{-id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{if-c\log(f)}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{if+c\log(f)}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] 1/4*I*f^a*erf(x*(I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(I*d)/(I*f-c*ln(f))^(1/2)-
1/4*I*exp(I*d)*f^a*erfi(x*(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4560, 2325, 2236, 2235}

$$\int f^{a+cx^2}\sin(d+fx^2)\,dx=\frac{i\sqrt{\pi}e^{-id}f^a\operatorname{erf}\left(x\sqrt{-c\log(f)+if}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}e^{id}f^a\operatorname{erfi}\left(x\sqrt{c\log(f)+if}\right)}{4\sqrt{c\log(f)+if}}$$

```
[In] Int[f^(a + c*x^2)*Sin[d + f*x^2],x]
```

```
[Out] ((I/4)*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(E^(I*d)*Sqrt[I*f - c*Log[
f]]) - ((I/4)*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])/Sqrt[I*f +
 c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}ie^{-id-ifx^2}f^{a+cx^2}-\frac{1}{2}ie^{id+ifx^2}f^{a+cx^2}\right)\,dx \\
&= \frac{1}{2}i\int e^{-id-ifx^2}f^{a+cx^2}\,dx-\frac{1}{2}i\int e^{id+ifx^2}f^{a+cx^2}\,dx \\
&= \frac{1}{2}i\int e^{-id+a\log(f)-x^2(if-c\log(f))}\,dx-\frac{1}{2}i\int e^{id+a\log(f)+x^2(if+c\log(f))}\,dx \\
&= \frac{ie^{-id}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{if-c\log(f)}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id}f^a\sqrt{\pi}\,\text{erfi}\left(x\sqrt{if+c\log(f)}\right)}{4\sqrt{if+c\log(f)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.59

$$\int f^{a+cx^2}\sin\left(d+fx^2\right)\,dx =$$
$$-\frac{\sqrt[4]{-1}f^a\sqrt{\pi}\left(\operatorname{erfi}\left(\sqrt[4]{-1}x\sqrt{f-ic\log(f)}\right)\sqrt{f-ic\log(f)}(f+ic\log(f))(\cos(d)+i\sin(d))+\sqrt{f+ic\log}\right.}{4\left(f^2\right.}$$

[In] Integrate[f^(a + c*x^2)*Sin[d + f*x^2],x]

[Out] -1/4*((-1)^(1/4)*f^a*Sqrt[Pi]*(Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt [f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] + I*Sin[d]) + Sqrt[f + I*c*Log[f]]*(c*Erf[((1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]]*Log[f]*Sin[d] + Erfi[(-1)^(3/4)*x*Sqrt[f + I*c*Log[f]]]*(Cos[d]*(I*f + c*Log[f]) + f*Sin[d]))))/(f^ 2 + c^2*Log[f]^2)

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{i\sqrt{\pi}\,f^{a}\mathrm{e}^{id}\operatorname{erf}\left(\sqrt{-c\ln(f)-if}\,x\right)}{4\sqrt{-c\ln(f)-if}}+\frac{i\sqrt{\pi}\,f^{a}\mathrm{e}^{-id}\operatorname{erf}\left(x\sqrt{if-c\ln(f)}\right)}{4\sqrt{if-c\ln(f)}}$	84

[In] int(f^(c*x^2+a)*sin(f*x^2+d),x,method=_RETURNVERBOSE)

[Out] -1/4*I*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-c*ln(f)-I*f)^(1/2)* x)+1/4*I*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/ 2))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int f^{a+cx^2}\sin\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}(i\,c\log\left(f\right)+f)\sqrt{-c\log\left(f\right)-i\,f}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-i\,f}x\right)e^{\left(a\log(f)+i\,d\right)}+\sqrt{\pi}(-i\,c\log\left(f\right)+f)\sqrt{-c\log}}{4\left(c^2\log\left(f\right)^2+f^2\right)}$$

[In] integrate(f^(c*x^2+a)*sin(f*x^2+d),x, algorithm="fricas")

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) - I
*f)*x)*e^(a*log(f) + I*d) + sqrt(pi)*(-I*c*log(f) + f)*sqrt(-c*log(f) + I*f
)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d))/(c^2*log(f)^2 + f^2)
```

Sympy [F]

$$\int f^{a+cx^2}\sin\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\sin\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+d),x)
[Out] Integral(f**(a + c*x**2)*sin(d + f*x**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(73) = 146.

Time = 0.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.95

$$\int f^{a+cx^2}\sin\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+2\,f^2}\left(f^a(\cos\left(d\right)-i\,\sin\left(d\right))\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+i\,fx}\right)+f^a(\cos\left(d\right)+i\,\sin\left(d\right))\operatorname{erf}\left(\sqrt{-}\right.\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d),x, algorithm="maxima")
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(f^a*(cos(d) - I*sin(d))*erf(sqr
t(-c*log(f) + I*f)*x) + f^a*(cos(d) + I*sin(d))*erf(sqrt(-c*log(f) - I*f)*x
))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^2*log(f)^2
 + 2*f^2)*(f^a*(-I*cos(d) - sin(d))*erf(sqrt(-c*log(f) + I*f)*x) + f^a*(I*c
os(d) - sin(d))*erf(sqrt(-c*log(f) - I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*log
(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Giac [F]

$$\int f^{a+cx^2}\sin\left(d+fx^2\right)\,dx=\int f^{cx^2+a}\sin\left(fx^2+d\right)\,dx$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d),x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \sin \left(d+fx^2\right) \, dx = \int f^{c\,x^2+a} \,\sin \left(f\,x^2+d\right) \, dx$$

```
[In] int(f^(a + c*x^2)*sin(d + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*sin(d + f*x^2), x)
```

3.89 $\int f^{a+cx^2} \sin^2\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int f^{a+cx^2}\sin^2\left(d+fx^2\right)dx=\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{2if-c\log(f)}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{2if+c\log(f)}\right)}{8\sqrt{2if+c\log(f)}}$$

```
[Out] 1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/8*f^a*er
f(x*(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(2*I*d)/(2*I*f-c*ln(f))^(1/2)-1/8*ex
p(2*I*d)*f^a*erfi(x*(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4560, 2235, 2325, 2236}

$$\int f^{a+cx^2}\sin^2\left(d+fx^2\right)dx=-\frac{\sqrt{\pi}e^{-2id}f^a\,\mathrm{erf}\left(x\sqrt{-c\log(f)+2if}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}e^{2id}f^a\,\mathrm{erfi}\left(x\sqrt{c\log(f)+2if}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (f^a
*Sqrt[Pi]*Erf[x*Sqrt[(2*I)*f - c*Log[f]]])/(8*E^((2*I)*d)*Sqrt[(2*I)*f - c*
Log[f]]) - (E^((2*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(2*I)*f + c*Log[f]]])/(8*S
qrt[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}f^{a+cx^2} - \frac{1}{4}e^{-2id-2ifx^2}f^{a+cx^2} - \frac{1}{4}e^{2id+2ifx^2}f^{a+cx^2}\right) dx \\
&= -\left(\frac{1}{4}\int e^{-2id-2ifx^2}f^{a+cx^2}\,dx\right) - \frac{1}{4}\int e^{2id+2ifx^2}f^{a+cx^2}\,dx + \frac{1}{2}\int f^{a+cx^2}\,dx \\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{1}{4}\int \exp\left(-2id + a\log(f) - x^2(2if - c\log(f))\right) dx \\
&\quad - \frac{1}{4}\int \exp\left(2id + a\log(f) + x^2(2if + c\log(f))\right) dx \\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-2id}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{2if - c\log(f)}\right)}{8\sqrt{2if - c\log(f)}} \\
&\quad - \frac{e^{2id}f^a\sqrt{\pi}\,\text{erfi}\left(x\sqrt{2if + c\log(f)}\right)}{8\sqrt{2if + c\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.34

$$\int f^{a+cx^2}\sin^2\left(d+fx^2\right)\,dx=\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}\left(\text{erf}\left(\sqrt[4]{-1}x\sqrt{2f+ic\log(f)}\right)\sqrt{2f+ic\log(f)}(2if+c\log(f))(\cos(2d)-i\sin(2d))+\text{erf}\left((-1)^{3/4}x\right.\right.}{4f^2+c^2\log^2(f)}\right.$$

```
[In] Integrate[f^(a + c*x^2)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + ((
-1)^(1/4)*(Erf[(-1)^(1/4)*x*Sqrt[2*f + I*c*Log[f]]]*Sqrt[2*f + I*c*Log[f]]*
((2*I)*f + c*Log[f])*(Cos[2*d] - I*Sin[2*d]) + Erf[(-1)^(3/4)*x*Sqrt[2*f -
I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]]*(2*f + I*c*Log[f])*(Cos[2*d] + I*Sin[2*
d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{\sqrt{\pi}\,f^{a}{\mathrm e}^{-2 i d} \operatorname{erf}\left(x \sqrt{2 i f -c \ln \left(f \right)}\right)}{8 \sqrt{2 i f -c \ln \left(f \right)}}-\frac{\sqrt{\pi}\,f^{a}{\mathrm e}^{2 i d} \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)-2 i f}\,x \right)}{8 \sqrt{-c \ln \left(f \right)-2 i f}}+\frac{f^{a} \sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c \ln \left(f \right)}\,x \right)}{4 \sqrt{-c \ln \left(f \right)}}$	107

```
[In] int(f^(c*x^2+a)*sin(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-2*I*d)/(2*I*f-c*ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(
1/2))-1/8*Pi^(1/2)*f^a*exp(2*I*d)/(-c*ln(f)-2*I*f)^(1/2)*erf((-c*ln(f)-2*I*
f)^(1/2)*x)+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.21

$$\int f^{a+cx^2}\sin^2\left(d+fx^2\right)\,dx=-\frac{2\sqrt{\pi}\left(c^2\log\left(f\right)^2+4\,f^2\right)\sqrt{-c\log\left(f\right)}f^a\,\text{erf}\left(\sqrt{-c\log\left(f\right)}x\right)-\sqrt{\pi}\left(c^2\log\left(f\right)^2-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right.}}{}$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log
(f))*x) - sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*
erf(sqrt(-c*log(f) - 2*I*f)*x)*e^(a*log(f) + 2*I*d) - sqrt(pi)*(c^2*log(f)^
2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(sqrt(-c*log(f) + 2*I*f)*x)*
e^(a*log(f) - 2*I*d))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

Sympy [F]

$$\int f^{a+cx^2} \sin^2\left(d+fx^2\right)\,dx = \int f^{a+cx^2} \sin^2\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + f*x**2)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.25

$$\int f^{a+cx^2} \sin^2\left(d+fx^2\right)\,dx$$
$$= \frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+8\,f^2}\left(f^a(i\,\cos\left(2\,d\right)+\sin\left(2\,d\right))\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+2i\,fx}\right)+f^a(-i\,\cos\left(2\,d\right)+\sin\left(2\,d\right.\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(f^a*(I*cos(2*d) + sin(2*d))*er
f(sqrt(-c*log(f) + 2*I*f)*x) + f^a*(-I*cos(2*d) + sin(2*d))*erf(sqrt(-c*log
(f) - 2*I*f)*x))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)
) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(f^a*(cos(2*d) - I*sin(2*d))*erf(
sqrt(-c*log(f) + 2*I*f)*x) + f^a*(cos(2*d) + I*sin(2*d))*erf(sqrt(-c*log(f)
 - 2*I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f))
+ 2*sqrt(pi)*((c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(x*conjugate(sqrt(-c*log(
f)))) + (c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(sqrt(-c*log(f))*x)))/((c^2*log
(f)^2 + 4*f^2)*sqrt(-c*log(f)))
```

Giac [F]

$$\int f^{a+cx^2}\sin^2\left(d+fx^2\right)\,dx=\int f^{cx^2+a}\sin\left(fx^2+d\right)^2\,dx$$

[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^2,x, algorithm="giac")

[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + d)^2, x)

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sin^2\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\sin\left(f\,x^2+d\right)^2\,dx$$

[In] int(f^(a + c*x^2)*sin(d + f*x^2)^2,x)

[Out] int(f^(a + c*x^2)*sin(d + f*x^2)^2, x)

3.90 $\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 213

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx=\frac{3ie^{-id}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{if-c\log(f)}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{3if-c\log(f)}\right)}{16\sqrt{3if-c\log(f)}}-\frac{3ie^{id}f^a\sqrt{\pi}\text{erfi}\left(x\sqrt{if+c\log(f)}\right)}{16\sqrt{if+c\log(f)}}+\frac{ie^{3id}f^a\sqrt{\pi}\text{erfi}\left(x\sqrt{3if+c\log(f)}\right)}{16\sqrt{3if+c\log(f)}}$$

```
[Out] 3/16*I*f^a*erf(x*(I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(I*d)/(I*f-c*ln(f))^(1/2)
-1/16*I*f^a*erf(x*(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(3*I*d)/(3*I*f-c*ln(f)
)^(1/2)-3/16*I*exp(I*d)*f^a*erfi(x*(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(
f))^(1/2)+1/16*I*exp(3*I*d)*f^a*erfi(x*(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I
*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4560, 2325, 2236, 2235}

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx=\frac{3i\sqrt{\pi}e^{-id}f^a\text{erf}\left(x\sqrt{-c\log(f)+if}\right)}{16\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}e^{-3id}f^a\text{erf}\left(x\sqrt{-c\log(f)+3if}\right)}{16\sqrt{-c\log(f)+3if}}-\frac{3i\sqrt{\pi}e^{id}f^a\text{erfi}\left(x\sqrt{c\log(f)+if}\right)}{16\sqrt{c\log(f)+if}}+\frac{i\sqrt{\pi}e^{3id}f^a\text{erfi}\left(x\sqrt{c\log(f)+3if}\right)}{16\sqrt{c\log(f)+3if}}$$

```
[In] Int[f^(a + c*x^2)*Sin[d + f*x^2]^3,x]
```

```
[Out] (((3*I)/16)*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(E^(I*d)*Sqrt[I*f - c
*Log[f]]) - ((I/16)*f^a*Sqrt[Pi]*Erf[x*Sqrt[(3*I)*f - c*Log[f]]])/(E^((3*I)
*d)*Sqrt[(3*I)*f - c*Log[f]]) - (((3*I)/16)*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqr
t[I*f + c*Log[f]]])/Sqrt[I*f + c*Log[f]] + ((I/16)*E^((3*I)*d)*f^a*Sqrt[Pi]
*Erfi[x*Sqrt[(3*I)*f + c*Log[f]]])/Sqrt[(3*I)*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{8}ie^{-id-ifx^2}f^{a+cx^2}-\frac{3}{8}ie^{id+ifx^2}f^{a+cx^2}-\frac{1}{8}ie^{-3id-3ifx^2}f^{a+cx^2}+\frac{1}{8}ie^{3id+3ifx^2}f^{a+cx^2}\right)dx\\
&= -\left(\frac{1}{8}i\int e^{-3id-3ifx^2}f^{a+cx^2}\,dx\right)+\frac{1}{8}i\int e^{3id+3ifx^2}f^{a+cx^2}\,dx+\frac{3}{8}i\int e^{-id-ifx^2}f^{a+cx^2}\,dx-\frac{3}{8}i\int e^{id+ifx^2}f^{a+cx^2}\,dx\\
&= -\left(\frac{1}{8}i\int \exp\left(-3id+a\log(f)-x^2(3if-c\log(f))\right)dx\right)+\frac{1}{8}i\int \exp\left(3id+a\log(f)+x^2(3if+c\log(f))\right)dx\\
&\quad+\frac{3}{8}i\int e^{-id+a\log(f)-x^2(if-c\log(f))}\,dx-\frac{3}{8}i\int e^{id+a\log(f)+x^2(if+c\log(f))}\,dx\\
&= \frac{3ie^{-id}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{if-c\log(f)}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{3if-c\log(f)}\right)}{16\sqrt{3if-c\log(f)}}\\
&\quad-\frac{3ie^{id}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{if+c\log(f)}\right)}{16\sqrt{if+c\log(f)}}+\frac{ie^{3id}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{3if+c\log(f)}\right)}{16\sqrt{3if+c\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.81

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)dx$$
$$=\frac{\sqrt[4]{-1}f^a\sqrt{\pi}\left(-3\mathrm{erfi}\left(\sqrt[4]{-1}x\sqrt{f-ic\log(f)}\right)\sqrt{f-ic\log(f)}\left(9f^3+9icf^2\log(f)+c^2f\log^2(f)+ic^3\log^3(f\right.\right.}{}$$

```
[In] Integrate[f^(a + c*x^2)*Sin[d + f*x^2]^3,x]
[Out] ((-1)^(1/4)*f^a*Sqrt[Pi]*(-3*Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt[f
 - I*c*Log[f]]*(9*f^3 + (9*I)*c*f^2*Log[f] + c^2*f*Log[f]^2 + I*c^3*Log[f]^
3)*(Cos[d] + I*Sin[d]) + (f - I*c*Log[f])*(Erfi[(-1)^(1/4)*x*Sqrt[3*f - I*c
*Log[f]]]*Sqrt[3*f - I*c*Log[f]]*(3*f^2 + (4*I)*c*f*Log[f] - c^2*Log[f]^2)*
```

```
(Cos[3*d] + I*Sin[3*d]) + (3*f - I*c*Log[f])*(3*Erfi[(-1)^(3/4)*x*Sqrt[f +
I*c*Log[f]]]*Sqrt[f + I*c*Log[f]]*(c*Cos[d]*Log[f] - 3*f*Sin[d]) + 3*Erf[((
1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]]*Sqrt[f + I*c*Log[f]]*(3*f*Cos[d] +
c*Log[f]*Sin[d]) + Erfi[(-1)^(3/4)*x*Sqrt[3*f + I*c*Log[f]]]*(f + I*c*Log[f
])*Sqrt[3*f + I*c*Log[f]]*(I*Cos[3*d] + Sin[3*d])))))/(16*(9*f^4 + 10*c^2*f
^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.78

method	result
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{3 i d} \operatorname{erf}\left(\sqrt{-c\ln\left(f\right)-3 i f}\, x\right)}{16 \sqrt{-c\ln\left(f\right)-3 i f}}-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-3 i d} \operatorname{erf}\left(x\sqrt{3 i f-c\ln\left(f\right)}\right)}{16 \sqrt{3 i f-c\ln\left(f\right)}}+\frac{3 i\sqrt{\pi}\, f^{a} \mathrm{e}^{-i d} \operatorname{erf}\left(x\sqrt{i f-c\ln\left(f\right)}\right)}{16 \sqrt{i f-c\ln\left(f\right)}}-\frac{3 i\sqrt{\pi}\, f^{a} \mathrm{e}^{i d} \operatorname{erf}\left(\sqrt{\right.}\right.}{16 \sqrt{-c\,\mathrm{l}}}$

```
[In] int(f^(c*x^2+a)*sin(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*Pi^(1/2)*f^a*exp(3*I*d)/(-c*ln(f)-3*I*f)^(1/2)*erf((-c*ln(f)-3*I*f)^
(1/2)*x)-1/16*I*Pi^(1/2)*f^a*exp(-3*I*d)/(3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f
-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I
*f-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-
c*ln(f)-I*f)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(145) = 290.

Time = 0.27 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.49

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx$$

$$=\frac{\sqrt{\pi}\left(-i\,c^3\log\left(f\right)^3-3\,c^2f\log\left(f\right)^2-i\,cf^2\log\left(f\right)-3\,f^3\right)\sqrt{-c\log\left(f\right)-3i\,f}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-3i\,f}x\right)e^{(a\,\mathrm{l}}}{}$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(sqrt(-c*log(f) - 3*I*f)*x)*e^(a*log(f) + 3*I*
d) - 3*sqrt(pi)*(-I*c^3*log(f)^3 - c^2*f*log(f)^2 - 9*I*c*f^2*log(f) - 9*f^
3)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) - I*f)*x)*e^(a*log(f) + I*d) -
3*sqrt(pi)*(I*c^3*log(f)^3 - c^2*f*log(f)^2 + 9*I*c*f^2*log(f) - 9*f^3)*sqr
t(-c*log(f) + I*f)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d) + sqrt(p
i)*(I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log
(f) + 3*I*f)*erf(sqrt(-c*log(f) + 3*I*f)*x)*e^(a*log(f) - 3*I*d))/(c^4*log(
f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F]

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\sin^3\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+d)**3,x)
[Out] Integral(f**(a + c*x**2)*sin(d + f*x**2)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(145) = 290.

Time = 0.24 (sec) , antiderivative size = 661, normalized size of antiderivative = 3.10

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx=$$

$$-\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+18\,f^2}\Big(\left(\left(c^2\cos\left(3\,d\right)-i\,c^2\sin\left(3\,d\right)\right)f^a\log\left(f\right)^2+f^{a+2}\left(\cos\left(3\,d\right)-i\,\sin\left(3\,d\right)\right)\right)\operatorname{erf}\Big(\sqrt{}$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^3,x, algorithm="maxima")
[Out] -1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*cos(3*d) - I*c^2*sin(3
*d))*f^a*log(f)^2 + f^(a + 2)*(cos(3*d) - I*sin(3*d)))*erf(sqrt(-c*log(f) +
 3*I*f)*x) + ((c^2*cos(3*d) + I*c^2*sin(3*d))*f^a*log(f)^2 + f^(a + 2)*(cos
(3*d) + I*sin(3*d)))*erf(sqrt(-c*log(f) - 3*I*f)*x))*sqrt(c*log(f) + sqrt(c
^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((c^2*cos(
d) - I*c^2*sin(d))*f^a*log(f)^2 + 9*f^(a + 2)*(cos(d) - I*sin(d)))*erf(sqrt
(-c*log(f) + I*f)*x) + ((c^2*cos(d) + I*c^2*sin(d))*f^a*log(f)^2 + 9*f^(a +
 2)*(cos(d) + I*sin(d)))*erf(sqrt(-c*log(f) - I*f)*x))*sqrt(c*log(f) + sqrt
(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*cos
(3*d) + c^2*sin(3*d))*f^a*log(f)^2 + f^(a + 2)*(I*cos(3*d) + sin(3*d)))*erf
(sqrt(-c*log(f) + 3*I*f)*x) + ((-I*c^2*cos(3*d) + c^2*sin(3*d))*f^a*log(f)^
2 + f^(a + 2)*(-I*cos(3*d) + sin(3*d)))*erf(sqrt(-c*log(f) - 3*I*f)*x))*sqr
t(-c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) + 3*sqrt(pi)*sqrt(2*c^2*log(f)^2
+ 2*f^2)*(((-I*c^2*cos(d) - c^2*sin(d))*f^a*log(f)^2 + 9*f^(a + 2)*(-I*cos(
d) - sin(d)))*erf(sqrt(-c*log(f) + I*f)*x) + ((I*c^2*cos(d) - c^2*sin(d))*f
^a*log(f)^2 + 9*f^(a + 2)*(I*cos(d) - sin(d)))*erf(sqrt(-c*log(f) - I*f)*x)
)*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*lo
g(f)^2 + 9*f^4)
```

Giac [F]

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx=\int f^{cx^2+a}\sin\left(fx^2+d\right)^3\,dx$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\sin\left(f\,x^2+d\right)^3\,dx$$

```
[In] int(f^(a + c*x^2)*sin(d + f*x^2)^3,x)
[Out] int(f^(a + c*x^2)*sin(d + f*x^2)^3, x)
```

3.91 $\int f^{a+cx^2} \sin(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 187

$$\int f^{a+cx^2}\sin(d+ex+fx^2)\,dx = \frac{ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}} - \frac{ie^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] 1/4*I*exp(-I*d-e^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e+2*x*(I*f-c*ln(f)))/(
I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/4*I*exp(I*d+e^2/(4*I*f+4
*c*ln(f)))*f^a*erfi(1/2*(I*e+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/
2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4560, 2325, 2266, 2236, 2235}

$$\int f^{a+cx^2}\sin(d+ex+fx^2)\,dx = \frac{i\sqrt{\pi}f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\mathrm{erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}} - \frac{i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)+4if}+id}\mathrm{erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x + f*x^2],x]
```

```
[Out] ((I/4)*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*
(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*Log[f]] - ((I/4)*
E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*x*(I*f + c*
Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2}ie^{-id-iex-ifx^2}f^{a+cx^2}-\frac{1}{2}ie^{id+iex+ifx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{2}i\int e^{-id-iex-ifx^2}f^{a+cx^2}\,dx-\frac{1}{2}i\int e^{id+iex+ifx^2}f^{a+cx^2}\,dx\\ &= \frac{1}{2}i\int \exp\left(-id-iex+a\log(f)-x^2(if-c\log(f))\right)dx\\ &\quad -\frac{1}{2}i\int \exp\left(id+iex+a\log(f)+x^2(if+c\log(f))\right)dx\end{aligned}$$

$$= \frac{1}{2}\left(ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(-ie+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx$$
$$-\frac{1}{2}\left(ie^{id+\frac{e^2}{4if+4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+2x(if+c\log(f)))^2}{4(if+c\log(f))}\right)dx$$
$$= \frac{ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\text{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}} - \frac{ie^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.16

$$\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)dx =$$
$$-\frac{(-1)^{3/4}e^{\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\left(e^{\frac{ie^2f}{2(f^2+c^2\log^2(f))}}\text{erfi}\left(\frac{(-1)^{3/4}(e+2fx+2icx\log(f))}{2\sqrt{f+ic\log(f)}}\right)(f-ic\log(f))\sqrt{f+ic\log(f)}(\cos(\right.}{4\left(f^2+c^2\text{lo}\right.}$$

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x + f*x^2],x]
```

```
[Out] -1/4*((-1)^(3/4)*E^(e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*(E^(((I/2)*e^2
*f)/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/
(2*Sqrt[f + I*c*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(Cos[d] - I
*Sin[d]) + Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c
*Log[f]])]*Sqrt[f - I*c*Log[f]]*((-I)*f + c*Log[f])*(Cos[d] + I*Sin[d])))/(
f^2 + c^2*Log[f]^2)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.90

method	result
risch	$\frac{i\sqrt{\pi}\,f^a \mathrm{e}^{\frac{4id\ln(f)c-4df+e^2}{4if+4c\ln(f)}}\,\text{erf}\left(-\sqrt{-c\ln(f)-if}\,x+\frac{ie}{2\sqrt{-c\ln(f)-if}}\right)}{4\sqrt{-c\ln(f)-if}} + \frac{i\sqrt{\pi}\,f^a \mathrm{e}^{-\frac{4id\ln(f)c+4df-e^2}{4(c\ln(f)-if)}}\,\text{erf}\left(x\sqrt{if-c\ln(f)}+\frac{ie}{2\sqrt{if-c\ln(f}}\right.}{4\sqrt{if-c\ln(f)}}$

```
[In] int(f^(c*x^2+a)*sin(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(f)))/(-c*ln(
f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f)^(1/2))+1/4
*I*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-I*f))/(I*f-c*ln
(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(135) = 270.

Time = 0.26 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.60

$$\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)\,dx$$

$$=\frac{\sqrt{\pi}(i\,c\log\left(f\right)+f)\sqrt{-c\log\left(f\right)-i\,f}\,\mathrm{erf}\left(\frac{\left(2\,c^2x\log(f)^2+2\,f^2x+i\,ce\log(f)+ef\right)\sqrt{-c\log(f)-i\,f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(\frac{4\,ac^2\log(f)^3+4i\,c^2d\log(f)^2\cdots}{4\left(c^2\mathrm{l}\cdots\right.}\right.}}{\cdots}$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(f
)^2 + 2*f^2*x + I*c*e*log(f) + e*f)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f
^2))*e^(1/4*(4*a*c^2*log(f)^3 + 4*I*c^2*d*log(f)^2 - I*e^2*f + 4*I*d*f^2 +
(c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-I*c*log(f) + f
)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x - I*c*e*log(f)
+ e*f)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*log(f)^3
 - 4*I*c^2*d*log(f)^2 + I*e^2*f - 4*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^
2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Sympy [F]

$$\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x + f*x**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 760 vs. 2(135) = 270.

Time = 0.23 (sec) , antiderivative size = 760, normalized size of antiderivative = 4.06

$$\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)\,dx=$$

$$-\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+2\,f^2}\left(\left(\left(f^{\frac{ce^2}{4\left(c^2\log(f)^2+f^2\right)}}f^a\cos\left(\frac{4\,c^2d\log(f)^2-e^2f+4\,df^2}{4\left(c^2\log(f)^2+f^2\right)}\right)-i\,f^{\frac{ce^2}{4\left(c^2\log(f)^2+f^2\right)}}f^a\sin\left(\frac{4\,c^2d\log(\cdots}{4\left(c^2\mathrm{l}\cdots\right.}\right.\right.\right.\right.}{\cdots}$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((f^(1/4*c*e^2/(c^2*log(f)^2 +
f^2))*f^a*cos(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)
) - I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 - e^
2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*e)/
sqrt(-c*log(f) + I*f)) + (f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(1/4*(4
*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) + I*f^(1/4*c*e^2/(
c^2*log(f)^2 + f^2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*
log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f
)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^
2 + 2*f^2)*((I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(1/4*(4*c^2*d*log(
f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) + f^(1/4*c*e^2/(c^2*log(f)^2
+ f^2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^
2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*e)/sqrt(-c*log(f) + I*f)) + (-I*f^(1
/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*
f^2)/(c^2*log(f)^2 + f^2)) + f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(1/4
*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*
log(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*lo
g(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Giac [F]

$$\int f^{a+cx^2} \sin\left(d+ex+fx^2\right) \, dx = \int f^{cx^2+a} \sin\left(fx^2+ex+d\right) \, dx$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \sin\left(d+ex+fx^2\right) \, dx = \int f^{c\,x^2+a} \sin\left(f\,x^2+e\,x+d\right) \, dx$$

```
[In] int(f^(a + c*x^2)*sin(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*sin(d + e*x + f*x^2), x)
```

3.92 $\int f^{a+cx^2} \sin^2(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 211

$$\int f^{a+cx^2} \sin^2\left(d+ex+fx^2\right)\,dx = \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{ie+x(2if-c\log(f))}{\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}} - \frac{e^{2id+\frac{e^2}{2if+c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie+x(2if+c\log(f))}{\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}$$

```
[Out] 1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/8*exp(-2
*I*d-e^2/(2*I*f-c*ln(f)))*f^a*erf((I*e+x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(
1/2))*Pi^(1/2)/(2*I*f-c*ln(f))^(1/2)-1/8*exp(2*I*d+e^2/(2*I*f+c*ln(f)))*f^a
*erfi((I*e+x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f
))^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used

= {4560, 2235, 2325, 2266, 2236}

$$\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx = -\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{-c\log(f)+2if}-2id}\operatorname{erf}\left(\frac{x(-c\log(f)+2if)+ie}{\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}} - \frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)+2if}+2id}\operatorname{erfi}\left(\frac{x(c\log(f)+2if)+ie}{\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}} + \frac{\sqrt{\pi}f^a\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(
(-2*I)*d - e^2/((2*I)*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + x*((2*I)*f - c
*Log[f]))/Sqrt[(2*I)*f - c*Log[f]]])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*
I)*d + e^2/((2*I)*f + c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + x*((2*I)*f + c*Lo
g[f]))/Sqrt[(2*I)*f + c*Log[f]]])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
```

```
x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}f^{a+cx^2}-\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+cx^2}-\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+cx^2}\right)dx\\
&= -\left(\frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{1}{4}\int \exp\left(-2id-2iex+a\log(f)-x^2(2if-c\log(f))\right)dx\\
&\quad -\frac{1}{4}\int \exp\left(2id+2iex+a\log(f)+x^2(2if+c\log(f))\right)dx\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}\\
&\quad -\frac{1}{4}\left(e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\right)\int \exp\left(\frac{(-2ie+2x(-2if+c\log(f)))^2}{4(-2if+c\log(f))}\right)dx\\
&\quad -\frac{1}{4}\left(e^{2id+\frac{e^2}{2if+c\log(f)}}f^a\right)\int \exp\left(\frac{(2ie+2x(2if+c\log(f)))^2}{4(2if+c\log(f))}\right)dx\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{ie+x(2if-c\log(f))}{\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}\\
&\quad -\frac{e^{2id+\frac{e^2}{2if+c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie+x(2if+c\log(f))}{\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.19

$$\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)dx=\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{\sqrt{c}\sqrt{\log(f)}}\right.$$
$$+\frac{\sqrt[4]{-1}\left(e^{\frac{e^2}{2if+c\log(f)}}\text{erf}\left(\frac{(-1)^{3/4}(e+2fx-icx\log(f))}{\sqrt{2f-ic\log(f)}}\right)\sqrt{2f-ic\log(f)}(2f+ic\log(f))(\cos(2d)+i\sin(2d))+e^{\frac{-2if}{}}\cdots\right.}{4f^2+c^2\log^2(f)\cdots}$$

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

[Out] (f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + ((-1)^(1/4)*(E^(e^2/((2*I)*f + c*Log[f]))*Erf[((-1)^(3/4)*(e + 2*f*x - I*c*x*Log[f]))/Sqrt[2*f - I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]]*(2*f + I*c*Log[f])*(Cos[2*d] + I*Sin[2*d]) + E^(e^2/((-2*I)*f + c*Log[f]))*Erf[((-1)^(1/4)*(e + 2*f*x + I*c*x*Log[f]))/Sqrt[2*f + I*c*Log[f]]]*(2*f - I*c*Log[f])*Sqrt[2*f + I*c*Log[f]]*(I*Cos[2*d] + Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/8

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2 i d \ln(f) c+4 d f-e^{2}}{c \ln(f)-2 i f}} \operatorname{erf}\left(x \sqrt{2 i f-c \ln(f)}+\frac{i e}{\sqrt{2 i f-c \ln(f)}}\right)}{8 \sqrt{2 i f-c \ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2 i d \ln(f) c-4 d f+e^{2}}{2 i f+c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-2 i f}\, x+\frac{i e}{\sqrt{-c \ln(f)}}\right.}{8 \sqrt{-c \ln(f)-2 i f}}$

[In] int(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-2*I*f))/(2*I*f-c*ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(1/2)+I*e/(2*I*f-c*ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp((2*I*d*ln(f)*c-4*d*f+e^2)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+I*e/(-c*ln(f)-2*I*f)^(1/2))+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(155) = 310.

Time = 0.26 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.72

$$\int f^{a+cx^2} \sin^2\left(d+ex+fx^2\right)\, dx =$$

$$-\frac{2\sqrt{\pi}\left(c^2\log\left(f\right)^2+4\,f^2\right)\sqrt{-c\log\left(f\right)}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)-\sqrt{\pi}\left(c^2\log\left(f\right)^2-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(\right.}}{}$$

[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x, algorithm="fricas")

[Out] -1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log(f))*x) - sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf((c^2*x*log(f)^2 + 4*f^2*x + I*c*e*log(f) + 2*e*f)*sqrt(-c*log(f) - 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^((a*c^2*log(f)^3 + 2*I*c^2*d*log(f)^2 - 2*I*e^2*f + 8*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) - sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf((c^2*x*log(f

```
f)^2 + 4*f^2*x - I*c*e*log(f) + 2*e*f)*sqrt(-c*log(f) + 2*I*f)/(c^2*log(f)^
2 + 4*f^2))*e^((a*c^2*log(f)^3 - 2*I*c^2*d*log(f)^2 + 2*I*e^2*f - 8*I*d*f^2
 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)))/(c^3*log(f)^3 + 4*c*f
^2*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+e*x+d)**2,x)
[Out] Integral(f**(a + c*x**2)*sin(d + e*x + f*x**2)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 863, normalized size of antiderivative = 4.09

$$\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x, algorithm="maxima")
[Out] -1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^(c*e^2/(c^2*log(f)^2 + 4
*f^2))*f^a*cos(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2))
 + f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*sin(2*(c^2*d*log(f)^2 - e^2*f + 4*d
*f^2)/(c^2*log(f)^2 + 4*f^2)))*erf(((c*log(f) - 2*I*f)*x - I*e)/sqrt(-c*log
(f) + 2*I*f)) + (-I*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*cos(2*(c^2*d*log(f
)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)) + f^(c*e^2/(c^2*log(f)^2 + 4
*f^2))*f^a*sin(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2))
)*erf(((c*log(f) + 2*I*f)*x + I*e)/sqrt(-c*log(f) - 2*I*f)))*sqrt(c*log(f)
+ sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^
2 + 8*f^2)*((f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*cos(2*(c^2*d*log(f)^2 - e
^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)) - I*f^(c*e^2/(c^2*log(f)^2 + 4*f^2)
)*f^a*sin(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)))*erf
(((c*log(f) - 2*I*f)*x - I*e)/sqrt(-c*log(f) + 2*I*f)) + (f^(c*e^2/(c^2*log
(f)^2 + 4*f^2))*f^a*cos(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2
+ 4*f^2)) + I*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*sin(2*(c^2*d*log(f)^2 -
e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)))*erf(((c*log(f) + 2*I*f)*x + I*e)/
sqrt(-c*log(f) - 2*I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt
(-c*log(f)) - 2*sqrt(pi)*((c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(x*conjugate(
sqrt(-c*log(f)))) + (c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(sqrt(-c*log(f))*x)
))/((c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f)))
```

Giac [F]

$$\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+a}\sin\left(fx^2+ex+d\right)^2\,dx$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\sin\left(f\,x^2+e\,x+d\right)^2\,dx$$

```
[In] int(f^(a + c*x^2)*sin(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + c*x^2)*sin(d + e*x + f*x^2)^2, x)
```

3.93 $\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 377

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx=\frac{3ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\text{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\text{erf}\left(\frac{3ie+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}}-\frac{3ie^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}}+\frac{ie^{3id+\frac{9e^2}{4(3if+c\log(f))}}f^a\sqrt{\pi}\text{erfi}\left(\frac{3ie+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

```
[Out] 3/16*I*exp(-I*d-e^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e+2*x*(I*f-c*ln(f)))/
(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/16*I*exp(-3*I*d-9/4*e^2
/(3*I*f-c*ln(f)))*f^a*erf(1/2*(3*I*e+2*x*(3*I*f-c*ln(f)))/(3*I*f-c*ln(f))^(
1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)-3/16*I*exp(I*d+e^2/(4*I*f+4*c*ln(f)))*
f^a*erfi(1/2*(I*e+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*l
n(f))^(1/2)+1/16*I*exp(3*I*d+9/4*e^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(3*I*e+2
*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4560, 2325, 2266, 2236, 2235}

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx$$
$$=-\frac{i\sqrt{\pi}f^a\exp\left(-\frac{9e^2}{4(-c\log(f)+3if)}-3id\right)\operatorname{erf}\left(\frac{2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}$$
$$+\frac{3i\sqrt{\pi}f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\operatorname{erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}$$
$$-\frac{3i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)+4if}+id}\operatorname{erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}$$
$$+\frac{i\sqrt{\pi}f^a e^{\frac{9e^2}{4(c\log(f)+3if)}+3id}\operatorname{erfi}\left(\frac{2x(c\log(f)+3if)+3ie}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}$$

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] (((3*I)/16)*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e +
 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*Log[f]] - ((
I/16)*E^((-3*I)*d - (9*e^2)/(4*((3*I)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*
I)*e + 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/Sqrt[(3*I)*
f - c*Log[f]] - (((3*I)/16)*E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[P
i]*Erfi[(I*e + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f +
c*Log[f]] + ((I/16)*E^((3*I)*d + (9*e^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt
[Pi]*Erfi[((3*I)*e + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])
])/Sqrt[(3*I)*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8} i e^{-3i(d+ex+fx^2)} f^{a+cx^2} + \frac{3}{8} i \exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx^2\right)\right) f^{a+cx^2} \right. \\
&\quad - \frac{3}{8} i \exp\left(4id+4iex+4ifx^2-3i\left(d+ex+fx^2\right)\right) f^{a+cx^2} \\
&\quad \left. + \frac{1}{8} i \exp\left(6id+6iex+6ifx^2-3i\left(d+ex+fx^2\right)\right) f^{a+cx^2} \right) dx \\
&= -\left(\frac{1}{8} i \int e^{-3i(d+ex+fx^2)} f^{a+cx^2} \, dx\right) \\
&\quad + \frac{1}{8} i \int \exp\left(6id+6iex+6ifx^2-3i\left(d+ex+fx^2\right)\right) f^{a+cx^2} \, dx \\
&\quad + \frac{3}{8} i \int \exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx^2\right)\right) f^{a+cx^2} \, dx \\
&\quad - \frac{3}{8} i \int \exp\left(4id+4iex+4ifx^2-3i\left(d+ex+fx^2\right)\right) f^{a+cx^2} \, dx \\
&= -\left(\frac{1}{8} i \int \exp\left(-3id-3iex+a\log(f)-x^2(3if-c\log(f))\right) \, dx\right) \\
&\quad + \frac{1}{8} i \int \exp\left(3id+3iex+a\log(f)+x^2(3if+c\log(f))\right) \, dx \\
&\quad + \frac{3}{8} i \int \exp\left(-id-iex+a\log(f)-x^2(if-c\log(f))\right) \, dx \\
&\quad - \frac{3}{8} i \int \exp\left(id+iex+a\log(f)+x^2(if+c\log(f))\right) \, dx
\end{aligned}$$

$$= \frac{1}{8}\left(3ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(-ie+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx - \frac{1}{8}\left(ie^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a\right)\int \exp\left(\frac{(-3ie+2x(-3if+c\log(f)))^2}{4(-3if+c\log(f))}\right)dx + \frac{1}{8}\left(ie^{3id+\frac{9e^2}{4(3if+c\log(f))}}f^a\right)\int \exp\left(\frac{(3ie+2x(3if+c\log(f)))^2}{4(3if+c\log(f))}\right)dx - \frac{1}{8}\left(3ie^{id+\frac{e^2}{4if+4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+2x(if+c\log(f)))^2}{4(if+c\log(f))}\right)dx$$

$$= \frac{3ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} - \frac{ie^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{3ie+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}} - \frac{3ie^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} + \frac{ie^{3id+\frac{9e^2}{4(3if+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

Mathematica [A] (verified)

Time = 5.14 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.30

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx$$

$$= \frac{\sqrt[4]{-1}f^a\sqrt{\pi}\left(-3e^{\frac{e^2}{4if+4c\log(f)}}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-2icx\log(f))}{2\sqrt{f-ic\log(f)}}\right)\sqrt{f-ic\log(f)}\left(9f^3+9icf^2\log(f)+c^2f\log^2(f)\right.\right.}{}$$

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^a*Sqrt[Pi]*(-3*E^(e^2/((4*I)*f + 4*c*Log[f]))*Erfi[((-1)^(1/4
)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Lo
g[f]]*(9*f^3 + (9*I)*c*f^2*Log[f] + c^2*f*Log[f]^2 + I*c^3*Log[f]^3)*(Cos[d
] + I*Sin[d]) + (f - I*c*Log[f])*(E^((9*e^2)/(4*((3*I)*f + c*Log[f])))*Erfi
[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*
Sqrt[3*f - I*c*Log[f]]*(3*f^2 + (4*I)*c*f*Log[f] - c^2*Log[f]^2)*(Cos[3*d]
+ I*Sin[3*d]) + (3*f - I*c*Log[f])*(3*E^(e^2/((-4*I)*f + 4*c*Log[f]))*Erfi[
((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[
f + I*c*Log[f]]*((-3*I)*f + c*Log[f])*(Cos[d] - I*Sin[d]) + E^((9*e^2)/(4*(
(-3*I)*f + c*Log[f])))*Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(
2*Sqrt[3*f + I*c*Log[f]])]*(f + I*c*Log[f])*Sqrt[3*f + I*c*Log[f]]*(I*Cos[3
*d] + Sin[3*d])))))/(16*(9*f^4 + 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{3id\ln(f)c-9df+\frac{9e^{2}}{4}}{3if+c\ln(f)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-3if}\, x+\frac{3ie}{2\sqrt{-c\ln(f)-3if}}\right)}{16\sqrt{-c\ln(f)-3if}}-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4id\ln(f)c+12df-3e^{2}\right)}{4(c\ln(f)-3if)}} \operatorname{erf}\left(x\sqrt{3if-c\ln(f)}+\right.}{16\sqrt{3if-c\ln(f)}}$

```
[In] int(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c-12*d*f+3*e^2)/(3*I*f+c*ln(f)))/
(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+3/2*I*e/(-c*ln(f)-3*I*
f)^(1/2))-1/16*I*Pi^(1/2)*f^a*exp(-3/4*(4*I*d*ln(f)*c+12*d*f-3*e^2)/(c*ln(f
)-3*I*f))/(3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f-c*ln(f))^(1/2)+3/2*I*e/(3*I*f-
c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(c*l
n(f)-I*f))/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(
f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(
f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f
)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 713 vs. 2(269) = 538.

Time = 0.29 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.89

$$\int f^{a+cx^2} \sin^3\left(d+ex+fx^2\right)\, dx$$

$$=\frac{\sqrt{\pi}\left(-i\,c^{3}\log(f)^{3}-3\,c^{2}f\log(f)^{2}-i\,cf^{2}\log(f)-3\,f^{3}\right)\sqrt{-c\log(f)-3i\,f}\,\operatorname{erf}\left(\frac{\left(2\,c^{2}x\log(f)^{2}+18\,f^{2}x+3i\,ce\log(f\right.}{2\left(c^{2}\log(f)^{2}+\right.}\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 18*f^2*x + 3*I*c*e*lo
g(f) + 9*e*f)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c
^2*log(f)^3 + 12*I*c^2*d*log(f)^2 - 27*I*e^2*f + 108*I*d*f^2 + 9*(c*e^2 + 4
*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(I*c^3*log(f)^3 - 3*c^2*
f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(2*c^2
*x*log(f)^2 + 18*f^2*x - 3*I*c*e*log(f) + 9*e*f)*sqrt(-c*log(f) + 3*I*f)/(c
^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c^2*log(f)^3 - 12*I*c^2*d*log(f)^2 + 27*I
*e^2*f - 108*I*d*f^2 + 9*(c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^2))
```

```
- 3*sqrt(pi)*(-I*c^3*log(f)^3 - c^2*f*log(f)^2 - 9*I*c*f^2*log(f) - 9*f^3)*
sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x + I*c*e*log(f) +
e*f)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*log(f)^3 +
 4*I*c^2*d*log(f)^2 - I*e^2*f + 4*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*
log(f)^2 + f^2)) - 3*sqrt(pi)*(I*c^3*log(f)^3 - c^2*f*log(f)^2 + 9*I*c*f^2*
log(f) - 9*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x -
 I*c*e*log(f) + e*f)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*
a*c^2*log(f)^3 - 4*I*c^2*d*log(f)^2 + I*e^2*f - 4*I*d*f^2 + (c*e^2 + 4*a*f^
2)*log(f))/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f
^4)
```

Sympy [F]

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+e*x+d)**3,x)
[Out] Integral(f**(a + c*x**2)*sin(d + e*x + f*x**2)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2175 vs. 2(269) = 538.

Time = 0.28 (sec) , antiderivative size = 2175, normalized size of antiderivative = 5.77

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x, algorithm="maxima")
[Out] 1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^(9/4*c*e^2/(c^2*log(f
)^2 + 9*f^2))*f^a*log(f)^2 + f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^(a + 2)
)*cos(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)) +
 (-I*c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^a*log(f)^2 - I*f^(9/4*c*e^2
/(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*sin(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 3
6*d*f^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*f)*x - 3*I*e)/
sqrt(-c*log(f) + 3*I*f)) + ((c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^a*l
og(f)^2 + f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*cos(3/4*(4*c^2*d*
log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)) + (I*c^2*f^(9/4*c*e^
2/(c^2*log(f)^2 + 9*f^2))*f^a*log(f)^2 + I*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f
^2))*f^(a + 2))*sin(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)
^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + 3*I*e)/sqrt(-c*log(f) - 3*I
```

```
*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*l
og(f)^2 + 2*f^2)*(((c^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*log(f)^2 + 9
*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^(a + 2))*cos(1/4*(4*c^2*d*log(f)^2 -
e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) + (-I*c^2*f^(1/4*c*e^2/(c^2*log(f)^2
 + f^2))*f^a*log(f)^2 - 9*I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^(a + 2))*s
in(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*
(2*(c*log(f) - I*f)*x - I*e)/sqrt(-c*log(f) + I*f)) + ((c^2*f^(1/4*c*e^2/(c
^2*log(f)^2 + f^2))*f^a*log(f)^2 + 9*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^(
a + 2))*cos(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2))
+ (I*c^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*log(f)^2 + 9*I*f^(1/4*c*e^2
/(c^2*log(f)^2 + f^2))*f^(a + 2))*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f
^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log
(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^
2*log(f)^2 + 18*f^2)*(((I*c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^a*log(
f)^2 + I*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*cos(3/4*(4*c^2*d*l
og(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^(9/4*c*e^2/(
c^2*log(f)^2 + 9*f^2))*f^a*log(f)^2 + f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*
f^(a + 2))*sin(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 +
9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*f)*x - 3*I*e)/sqrt(-c*log(f) + 3*I*f))
+ ((-I*c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^a*log(f)^2 - I*f^(9/4*c*e
^2/(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*cos(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f +
 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^
2))*f^a*log(f)^2 + f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*sin(3/4*
(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2
*(c*log(f) + 3*I*f)*x + 3*I*e)/sqrt(-c*log(f) - 3*I*f)))*sqrt(-c*log(f) + s
qrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((I*c
^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*log(f)^2 + 9*I*f^(1/4*c*e^2/(c^2*
log(f)^2 + f^2))*f^(a + 2))*cos(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c
^2*log(f)^2 + f^2)) + (c^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*log(f)^2
+ 9*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^(a + 2))*sin(1/4*(4*c^2*d*log(f)^2
 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x -
I*e)/sqrt(-c*log(f) + I*f)) + ((-I*c^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f
^a*log(f)^2 - 9*I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^(a + 2))*cos(1/4*(4*
c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) + (c^2*f^(1/4*c*e^2
/(c^2*log(f)^2 + f^2))*f^a*log(f)^2 + 9*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*
f^(a + 2))*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2
)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f)))*sqrt(-c*lo
g(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f
^4)
```

Giac [F]

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+a}\sin\left(fx^2+ex+d\right)^3\,dx$$

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\sin\left(f\,x^2+e\,x+d\right)^3\,dx$$

```
[In] int(f^(a + c*x^2)*sin(d + e*x + f*x^2)^3,x)
[Out] int(f^(a + c*x^2)*sin(d + e*x + f*x^2)^3, x)
```

3.94 $\int f^{a+bx+cx^2} \sin(d+ex)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 176

$$\int f^{a+bx+cx^2} \sin(d+ex)\, dx = -\frac{ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}} f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}} f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 1/4*I*exp(-I*d+1/4*(e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-I*e+b*ln(f)+2*c*
x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/4*I*exp(I*d+1/
4*(e-I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/l
n(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {4560, 2325, 2266, 2235}

$$\int f^{a+bx+cx^2} \sin(d+ex)\, dx = -\frac{i\sqrt{\pi} f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\,\mathrm{erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{i\sqrt{\pi} f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\,\mathrm{erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x],x]
```

```
[Out] ((-1/4*I)*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I
*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[
f]]) - ((I/4)*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(
I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log
[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2} i e^{-id-iex} f^{a+bx+cx^2} - \frac{1}{2} i e^{id+iex} f^{a+bx+cx^2}\right) dx \\
&= \frac{1}{2} i \int e^{-id-iex} f^{a+bx+cx^2}\, dx - \frac{1}{2} i \int e^{id+iex} f^{a+bx+cx^2}\, dx \\
&= \frac{1}{2} i \int \exp\left(-id + a\log(f) + cx^2\log(f) - x(ie - b\log(f))\right) dx \\
&\quad - \frac{1}{2} i \int \exp\left(id + a\log(f) + cx^2\log(f) + x(ie + b\log(f))\right) dx \\
&= -\left(\frac{1}{2}\left(i e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}} f^a\right) \int \exp\left(\frac{(ie + b\log(f) + 2cx\log(f))^2}{4c\log(f)}\right) dx\right) \\
&\quad + \frac{1}{2}\left(i e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}} f^a\right) \int \exp\left(\frac{(-ie + b\log(f) + 2cx\log(f))^2}{4c\log(f)}\right) dx \\
&= -\frac{i e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}} f^a \sqrt{\pi}\, \text{erfi}\left(\frac{ie - b\log(f) - 2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{i e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}} f^a \sqrt{\pi}\, \text{erfi}\left(\frac{ie + b\log(f) + 2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx$$
$$= \frac{e^{\frac{e(e-2ib\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(i\operatorname{erfi}\left(\frac{-ie-(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))+e^{\frac{ibe}{c}}\operatorname{erfi}\left(\frac{-ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(i\cos(d)+\sin(\right.}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x],x]
```

```
[Out] (E^((e*(e - (2*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*(I*Er
fi[((-I)*e - (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[
d]) + E^((I*b*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[
f]])]*(I*Cos[d] + Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98

method	result
risch	$\frac{i\sqrt{\pi}\,f^a f^{-\frac{b^2}{4c}}\,\mathrm{e}^{-\frac{2i\ln(f)be-4id\ln(f)c-e^2}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{ie+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}-\frac{i\sqrt{\pi}\,f^a f^{-\frac{b^2}{4c}}\,\mathrm{e}^{\frac{2i\ln(f)be-4id\ln(f)c+e^2}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x\right.}{4\sqrt{-c\ln(f)}}$

```
[In] int(f^(c*x^2+b*x+a)*sin(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c-e^2
)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*l
n(f))^(1/2))-1/4*I*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*I*ln(f)*b*e-4*I*d
*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f
)-I*e)/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx$$
$$= \frac{-i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2cx+b)\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{\left(b^2-4ac\right)\log(f)^2-e^2+2(2icd-ibe)\log(f)}{4c\log(f)}\right)}+i\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{er}}{4c\log(f)}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(-I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - I*e)*sqrt(-c
*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 + 2*(2*I*c*d - I
*b*e)*log(f))/(c*log(f))) + I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)
*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2
- e^2 + 2*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx = \int f^{a+bx+cx^2}\sin\left(d+ex\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sin(e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + e*x), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.01

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx = \frac{\sqrt{\pi}\left(f^a\left(i\cos\left(-\frac{2\,cd-be}{2\,c}\right)+\sin\left(-\frac{2\,cd-be}{2\,c}\right)\right)\operatorname{erf}\left(x\sqrt{-c\log\left(f\right)}-\frac{1}{2}\left(b\log\left(f\right)+i\,e\right)\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^2}{4\,c\log(f)}\right)}+f\cdots}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*(f^a*(I*cos(-1/2*(2*c*d - b*e)/c) + sin(-1/2*(2*c*d - b*e)/c))
*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + I*e)*conjugate(1/sqrt(-
c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(-I*cos(-1/2*(2*c*d - b*e)/c) + si
n(-1/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) -
 I*e)*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(I*cos(-1/
2*(2*c*d - b*e)/c) + sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*l
og(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))) + f^a*(-I*c
os(-1/2*(2*c*d - b*e)/c) + sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f)
 + b*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))))*sqrt
(-c*log(f))/(c*f^(1/4*b^2/c)*log(f))
```

Giac [F]

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx=\int f^{cx^2+bx+a}\sin(ex+d)\ dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d),x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*sin(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\sin(d+e\,x)\ dx$$

```
[In] int(f^(a + b*x + c*x^2)*sin(d + e*x),x)
[Out] int(f^(a + b*x + c*x^2)*sin(d + e*x), x)
```

3.95 $\int f^{a+bx+cx^2} \sin^2(d+ex)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 231

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{2ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{e^{2id-\frac{(2ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{2ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -1/8*exp(-2*I*d+1/4*(2*e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-2*I*e+b*ln(f)
+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/8*exp(2*I
*d-1/4*(2*I*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(2*I*e+b*ln(f)+2*c*x*ln(f))/
c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^(a-1/4*b^2/c)*erfi(
1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

$= \{4560, 2266, 2235, 2325\}$

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx = \frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi}f^a e^{\frac{(2e+ib\log(f))^2}{4c\log(f)}-2id}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi}f^a e^{2id-\frac{(b\log(f)+2ie)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d + (2*e + I*b*Log[f])^2/(4*c*Log[f]))
*f^a*Sqrt[Pi]*Erfi[((2*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[
f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) - (E^((2*I)*d - ((2*I)*e + b*Log[f])^2/(4*c
*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}f^{a+bx+cx^2}-\frac{1}{4}e^{-2id-2iex}f^{a+bx+cx^2}-\frac{1}{4}e^{2id+2iex}f^{a+bx+cx^2}\right)\,dx\\
&= -\left(\frac{1}{4}\int e^{-2id-2iex}f^{a+bx+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2iex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\
&= -\left(\frac{1}{4}\int \exp\left(-2id+a\log(f)+cx^2\log(f)-x(2ie-b\log(f))\right)\,dx\right)-\frac{1}{4}\int \exp\left(2id\right.\\
&\quad \left.+a\log(f)+cx^2\log(f)+x(2ie+b\log(f))\right)\,dx+\frac{1}{2}f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}\,dx\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{1}{4}\left(\exp\left(-2id\right.\right.\\
&\quad \left.\left.+\frac{(2e+ib\log(f))^2}{4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(-2ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)\,dx\\
&\quad -\frac{1}{4}\left(e^{2id-\frac{(2ie+b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(2ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)\,dx\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\\
&\quad +\frac{\exp\left(-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{2ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}\\
&\quad -\frac{e^{2id-\frac{(2ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{2ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.88

$$
\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx =
$$

$$
-\frac{e^{-\frac{ibe}{c}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(-2e^{\frac{ibe}{c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+e^{\frac{e(e+2ib\log(f))}{c\log(f)}}\operatorname{erfi}\left(\frac{-2ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)-i\sin(2d))+e\right.}{8\sqrt{c}\sqrt{\log(f)}}
$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x]^2,x]
```

```
[Out] -1/8*(f^(a - b^2/(4*c))*Sqrt[Pi]*(-2*E^((I*b*e)/c)*Erfi[((b + 2*c*x)*Sqrt[L
og[f]])/(2*Sqrt[c])] + E^((e*(e + (2*I)*b*Log[f]))/(c*Log[f]))*Erfi[((-2*I)
*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d])
+ E^(e^2/(c*Log[f]))*Erfi[((2*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Lo
g[f]])]*(Cos[2*d] + I*Sin[2*d])))/(Sqrt[c]*E^((I*b*e)/c)*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.94

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{i \ln(f) b e-2 i d \ln(f) c+e^{2}}{\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{b \ln(f)-2 i e}{2 \sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}}+\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{-\frac{i \ln(f) b e-2 i d \ln(f) c-e^{2}}{\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{2}{2}\right.}{8 \sqrt{-c \ln(f)}}$

```
[In] int(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp((I*ln(f)*b*e-2*I*d*ln(f)*c+e^2)/ln(f)/c
)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*I*e)/(-c*ln(f))^(
1/2))+1/8*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-(I*ln(f)*b*e-2*I*d*ln(f)*c-e^2)/
ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-c*l
n(f))^(1/2))-1/4*Pi^(1/2)*f^(-1/4*b^2/c)*f^a/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.97

$$\int f^{a+bx+cx^2} \sin^2(d+ex)\, dx = \frac{\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2\,cx+b)\log(f)-2i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-4\,e^2+4\,(2i\,cd-i\,be)\log(f)}{4\,c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\right.}{8\,c\log(f)}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - 2*I*e)*sqrt(-c*
log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*e^2 + 4*(2*I*c*d -
I*b*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*
log(f) + 2*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2
 - 4*e^2 + 4*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))) - 2*sqrt(pi)*sqrt(-c*lo
g(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c*lo
g(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx=\int f^{a+bx+cx^2}\sin^2{(d+ex)}\,dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sin(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + e*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.73

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx=$$

$$-\frac{\sqrt{\pi}\left(f^a\left(\cos\left(-\frac{2\,cd-be}{c}\right)-i\,\sin\left(-\frac{2\,cd-be}{c}\right)\right)\operatorname{erf}\left(x\sqrt{-c\log\left(f\right)}-\frac{1}{2}\left(b\log\left(f\right)+2i\,e\right)\frac{1}{\sqrt{-c\log(f)}}\right)e^{\left(\frac{e^2}{c\log(f)}\right)}+}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(pi)*(f^a*(cos(-(2*c*d - b*e)/c) - I*sin(-(2*c*d - b*e)/c))*erf(x
*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + 2*I*e)*conjugate(1/sqrt(-c*lo
g(f))))*e^(e^2/(c*log(f))) + f^a*(cos(-(2*c*d - b*e)/c) + I*sin(-(2*c*d - b
*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) - 2*I*e)*conjugate
(1/sqrt(-c*log(f))))*e^(e^2/(c*log(f))) + f^a*(cos(-(2*c*d - b*e)/c) - I*si
n(-(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) + 2*I*e)*sqrt(-c*log(
f))/(c*log(f)))*e^(e^2/(c*log(f))) + f^a*(cos(-(2*c*d - b*e)/c) + I*sin(-(2
*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) - 2*I*e)*sqrt(-c*log(f))/(
c*log(f)))*e^(e^2/(c*log(f))) - 2*f^a*erf(-1/2*b*conjugate(1/sqrt(-c*log(f)
))*log(f) + x*conjugate(sqrt(-c*log(f)))) + 2*f^a*erf(1/2*(2*c*x*log(f) + b
*log(f))/sqrt(-c*log(f))))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Giac [F]

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx=\int f^{cx^2+bx+a}\sin{(ex+d)}^2\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\sin\left(d+e\,x\right)^2\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sin(d + e*x)^2,x)
[Out] int(f^(a + b*x + c*x^2)*sin(d + e*x)^2, x)
```

3.96 $\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 354

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx = -\frac{3ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{ie^{-3id+\frac{(3e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{ie^{3id-\frac{(3ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 3/16*I*exp(-I*d+1/4*(e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-I*e+b*ln(f)+2*c
*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/16*I*exp(-3*I
*d+1/4*(3*e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-3*I*e+b*ln(f)+2*c*x*ln(f))
/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-3/16*I*exp(I*d+1/4*(e-I*
b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1
/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*I*exp(3*I*d-1/4*(3*I*e+b*ln(f))^2/c/
ln(f))*f^a*erfi(1/2*(3*I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/
2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4560, 2325, 2266, 2235}

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx = -\frac{3i\sqrt{\pi}f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{i\sqrt{\pi}f^a e^{\frac{(3e+ib\log(f))^2}{4c\log(f)}-3id}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3i\sqrt{\pi}f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{i\sqrt{\pi}f^a e^{3id-\frac{(b\log(f)+3ie)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x]^3,x]
```

```
[Out] (((-3*I)/16)*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi
[(I*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[L
og[f]]) + ((I/16)*E^((-3*I)*d + (3*e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt
[Pi]*Erfi[((3*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(S
qrt[c]*Sqrt[Log[f]]) - (((3*I)/16)*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f])
)*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]]
)])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4
*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c
]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{3}{8}ie^{-id-iex}f^{a+bx+cx^2} - \frac{3}{8}ie^{id+iex}f^{a+bx+cx^2} - \frac{1}{8}ie^{-3id-3iex}f^{a+bx+cx^2} + \frac{1}{8}ie^{3id+3iex}f^{a+bx+cx^2}\right)dx$$

$$= -\left(\frac{1}{8}i\int e^{-3id-3iex}f^{a+bx+cx^2}\,dx\right) + \frac{1}{8}i\int e^{3id+3iex}f^{a+bx+cx^2}\,dx + \frac{3}{8}i\int e^{-id-iex}f^{a+bx+cx^2}\,dx - \frac{3}{8}i\int e^{id+iex}f^{a+bx+cx^2}\,dx$$

$$= -\left(\frac{1}{8}i\int \exp\left(-3id + a\log(f) + cx^2\log(f) - x(3ie - b\log(f))\right)dx\right) + \frac{1}{8}i\int \exp\left(3id + a\log(f) + cx^2\log(f) + x(3ie + b\log(f))\right)dx + \frac{3}{8}i\int \exp\left(-id + a\log(f) + cx^2\log(f) - x(ie - b\log(f))\right)dx - \frac{3}{8}i\int \exp\left(id + a\log(f) + cx^2\log(f) + x(ie + b\log(f))\right)dx$$

$$= -\left(\frac{1}{8}\left(3ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\right) + \frac{1}{8}\left(3ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx - \frac{1}{8}\left(i\exp\left(-3id+\frac{(3e+ib\log(f))^2}{4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(-3ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx + \frac{1}{8}\left(ie^{3id-\frac{(3ie+b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(3ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx$$

$$= -\frac{3ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\mathrm{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{i\exp\left(-3id+\frac{(3e+ib\log(f))^2}{4c\log(f)}\right)f^a\sqrt{\pi}\mathrm{erfi}\left(\frac{3ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\mathrm{erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{ie^{3id-\frac{(3ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\mathrm{erfi}\left(\frac{3ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.10

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx = \frac{e^{\frac{e(e-6ib\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(-ie^{\frac{e(2e+3ib\log(f))}{c\log(f)}}\cos(3d)\mathrm{erfi}\left(\frac{-3ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+ie^{\frac{2e^2}{c\log(f)}}\cos(3d)\mathrm{erfi}\left(\frac{3ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)\right.}{}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x]^3,x]
```

```
[Out] (E^((e*(e - (6*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*((-I)
*E^((e*(2*e + (3*I)*b*Log[f]))/(c*Log[f]))*Cos[3*d]*Erfi[((-3*I)*e + (b + 2
*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + I*E^((2*e^2)/(c*Log[f]))*Cos[3*d]
*Erfi[((3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + (3*I)*E^((
I*b*e)/c)*Erfi[((-I)*e - (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos
[d] + I*Sin[d]) + 3*E^(((2*I)*b*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2
*Sqrt[c]*Sqrt[Log[f]])]*(I*Cos[d] + Sin[d]) - E^((e*(2*e + (3*I)*b*Log[f]))
/(c*Log[f]))*Erfi[((-3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]
*Sin[3*d] - E^((2*e^2)/(c*Log[f]))*Erfi[((3*I)*e + (b + 2*c*x)*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])]*Sin[3*d]))/(16*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{-\frac{3\left(2i\ln(f)be-4id\ln(f)c-3e^{2}\right)}{4\ln(f)c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{3ie+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}+\frac{i\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{\frac{3i\ln(f)be}{2}-3id\ln(f)c+\frac{9e^{2}}{4}}{c\ln(f)}} \operatorname{erf}\left(-\right.}{16\sqrt{-c\ln(f)}}$

```
[In] int(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-3/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c-3
*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*I*e+b*ln(f))
/(-c*ln(f))^(1/2))+1/16*I*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(3/4*(2*I*ln(f)*b*
e-4*I*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/
2*(b*ln(f)-3*I*e)/(-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(
1/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln
(f))^(1/2)*x+1/2*(b*ln(f)-I*e)/(-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*f^(-1/
4*b^2/c)*exp(-1/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/
2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.98

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx$$
$$=\frac{-3i\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2\,cx+b)\log(f)-i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-e^2+2\,(2i\,cd-i\,be)\log(f)}{4\,c\log(f)}\right)}+3i\sqrt{\pi}\sqrt{-c\log(f}}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(-3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - I*e)*sqrt
(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 + 2*(2*I*c*d
- I*b*e)*log(f))/(c*log(f))) + 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x
 + b)*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(
f)^2 - e^2 + 2*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))) + I*sqrt(pi)*sqrt(-c*
log(f))*erf(1/2*((2*c*x + b)*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^
(-1/4*((b^2 - 4*a*c)*log(f)^2 - 9*e^2 + 6*(2*I*c*d - I*b*e)*log(f))/(c*log(
f))) - I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + 3*I*e)*sqrt
(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 9*e^2 + 6*(-2*I*c
*d + I*b*e)*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx=\int f^{a+bx+cx^2}\sin^3{(d+ex)}\,dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sin(e*x+d)**3,x)
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + e*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.92

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x, algorithm="maxima")
[Out] -1/32*sqrt(pi)*(f^a*(I*cos(-3/2*(2*c*d - b*e)/c) + sin(-3/2*(2*c*d - b*e)/c
))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + 3*I*e)*conjugate(1/sq
rt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(-I*cos(-3/2*(2*c*d - b*e)/c)
+ sin(-3/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(
f) - 3*I*e)*conjugate(1/sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(I*c
os(-3/2*(2*c*d - b*e)/c) + sin(-3/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f)
 + b*log(f) + 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(9/4*e^2/(c*log(f))) + f
^a*(-I*cos(-3/2*(2*c*d - b*e)/c) + sin(-3/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*
x*log(f) + b*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(9/4*e^2/(c*log(
f))) - 3*f^a*(I*cos(-1/2*(2*c*d - b*e)/c) + sin(-1/2*(2*c*d - b*e)/c))*erf(
x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + I*e)*conjugate(1/sqrt(-c*log
(f))))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(-I*cos(-1/2*(2*c*d - b*e)/c) + sin(-
1/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) - I*
e)*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(I*cos(-1/2
*(2*c*d - b*e)/c) + sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*lo
g(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(-I*
cos(-1/2*(2*c*d - b*e)/c) + sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f
) + b*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))))*sqr
t(-c*log(f))/(c*f^(1/4*b^2/c)*log(f))
```

Giac [F]

$$\int f^{a+bx+cx^2} \sin^3(d+ex)\,dx = \int f^{cx^2+bx+a} \sin\left(ex+d\right)^3\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*sin(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \sin^3(d+ex)\,dx = \int f^{c\,x^2+b\,x+a} \sin\left(d+e\,x\right)^3\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sin(d + e*x)^3,x)
[Out] int(f^(a + b*x + c*x^2)*sin(d + e*x)^3, x)
```

3.97 $\int f^{a+bx+cx^2}\sin(d+fx^2)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 193

$$\int f^{a+bx+cx^2}\sin(d+fx^2)\,dx=-\frac{ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\text{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] -1/4*I*exp(-I*d+b^2*ln(f)^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(I*
f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/4*I*exp(I*d
-b^2*ln(f)^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(I*f+c*ln(f)))/(I
*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4560, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\sin(d+fx^2)\,dx=-\frac{i\sqrt{\pi}f^ae^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\text{erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^ae^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + f*x^2],x]
```

```
[Out] ((-1/4*I)*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*E
rf[(b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f -
c*Log[f]] - ((I/4)*E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt
[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt
[I*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2} i e^{-id-ifx^2} f^{a+bx+cx^2} - \frac{1}{2} i e^{id+ifx^2} f^{a+bx+cx^2}\right) dx \\ &= \frac{1}{2} i \int e^{-id-ifx^2} f^{a+bx+cx^2}\, dx - \frac{1}{2} i \int e^{id+ifx^2} f^{a+bx+cx^2}\, dx \\ &= \frac{1}{2} i \int \exp\left(-id + a\log(f) + bx\log(f) - x^2(if - c\log(f))\right) dx \\ &\quad - \frac{1}{2} i \int \exp\left(id + a\log(f) + bx\log(f) + x^2(if + c\log(f))\right) dx\end{aligned}$$

$$= \frac{1}{2}\left(ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx$$
$$-\frac{1}{2}\left(ie^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(if+c\log(f)))^2}{4(if+c\log(f))}\right)dx$$
$$= -\frac{ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}$$
$$-\frac{ie^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.19

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)dx =$$
$$-\frac{\sqrt[4]{-1}e^{\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\left(\operatorname{erfi}\left(\frac{(-1)^{3/4}(2fx+i(b+2cx)\log(f))}{2\sqrt{f+ic\log(f)}}\right)\sqrt{f+ic\log(f)}(if+c\log(f))(\cos(d)-i\sin(d))+\right.}{4\left(f^2+c^2\log^2\right.}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*(Er
fi[((-1)^(3/4)*(2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sq
rt[f + I*c*Log[f]]*(I*f + c*Log[f])*(Cos[d] - I*Sin[d]) + E^(((I/2)*b^2*f*L
og[f]^2)/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(1/4)*(2*f*x - I*(b + 2*c*x)*Log[
f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d
] + I*Sin[d])))/(f^2 + c^2*Log[f]^2)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.93

method	result
risch	$\frac{i\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2-4id\ln(f)c+4df}{4(if+c\ln(f))}}\operatorname{erf}\left(-\sqrt{-c\ln(f)-if}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-if}}\right)}{4\sqrt{-c\ln(f)-if}}-\frac{i\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2+4id\ln(f)c+4df}{4(c\ln(f)-if)}}\operatorname{erf}\left(-x\sqrt{if-c\ln(f)}\right.}{4\sqrt{if-c\ln(f)}}$

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*d*ln(f)*c+4*d*f)/(I*f+c*ln(f))
)/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-I*
f)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*d*ln(f)*c+4*d*f)/(c*
```

```
ln(f)-I*f))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(I*f
-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(145) = 290.

Time = 0.27 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.60

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)\,dx = \frac{\sqrt{\pi}(i\,c\log\left(f\right)+f)\sqrt{-c\log\left(f\right)-i\,f}\,\mathrm{erf}\left(\frac{\left(2\,f^2x-i\,bf\log(f)+\left(2\,c^2x+bc\right)\log(f)^2\right)\sqrt{-c\log(f)-i\,f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(\frac{4\,af^2\log(f)-\left(b^2c-4\,ac^2 \cdots}{4 \cdots}\right.}\cdots}{\cdots}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*b
*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 +
 f^2))*e^(1/4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (4
*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-I*c*log(f)
 + f)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*log(f) + (2*c^2*x + b*
c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^2*lo
g(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I*c^2*d - I*b^2*f)*log(
f)^2)/(c^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)\,dx = \int f^{a+bx+cx^2}\sin\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + f*x**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(145) = 290.

Time = 0.23 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.35

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+2\,f^2}\left(\left(\left(f^a\cos\left(\frac{4\,df^2+\left(4\,c^2d+b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2+f^2\right)}\right)-i\,f^a\sin\left(\frac{4\,df^2+\left(4\,c^2d+b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2+f^2\right)}\right)\right)\operatorname{erf}\left(\frac{2\left(c\log\right.}{2\,\sqrt{}}\right.\right.\right.}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((f^a*cos(1/4*(4*d*f^2 + (4*c^2
*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - I*f^a*sin(1/4*(4*d*f^2 + (4*c
^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*
x + b*log(f))/sqrt(-c*log(f) + I*f)) + (f^a*cos(1/4*(4*d*f^2 + (4*c^2*d + b
^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + I*f^a*sin(1/4*(4*d*f^2 + (4*c^2*d +
 b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*
log(f))/sqrt(-c*log(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) +
 sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*f^a*cos(1/4*(4*d*f^2 + (4*c^2*d
+ b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^a*sin(1/4*(4*d*f^2 + (4*c^2*d
+ b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b
*log(f))/sqrt(-c*log(f) + I*f)) + (-I*f^a*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2
*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^a*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2
*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(
f))/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^
2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^2*e^(1/4*b^2*c*l
og(f)^3/(c^2*log(f)^2 + f^2)))
```

Giac [F]

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)\,dx=\int f^{cx^2+bx+a}\sin\left(fx^2+d\right)\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\,\sin\left(f\,x^2+d\right)\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sin(d + f*x^2),x)
[Out] int(f^(a + b*x + c*x^2)*sin(d + f*x^2), x)
```

3.98 $\int f^{a+bx+cx^2} \sin^2(d+fx^2)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 245

$$\int f^{a+bx+cx^2}\sin^2(d+fx^2)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id-\frac{b^2\log^2(f)}{8if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(2if+c\log(f))}{2\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}$$

```
[Out] 1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)+1/8*exp(-2*I*d+b^2*ln(f)^2/(8*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*
ln(f)-2*x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f-c*ln(f))^
(1/2)-1/8*exp(2*I*d-b^2*ln(f)^2/(8*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*
x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

= {4560, 2266, 2235, 2325, 2236}

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)\,dx=\frac{\sqrt{\pi}f^ae^{\frac{b^2\log^2(f)}{-4c\log(f)+8if}-2id}\mathrm{erf}\left(\frac{b\log(f)-2x(-c\log(f)+2if)}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}f^ae^{2id-\frac{b^2\log^2(f)}{4c\log(f)+8if}}\mathrm{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\mathrm{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d + (b^2*Log[f]^2)/((8*I)*f - 4*c*Log[
f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((2*I)*f - c*Log[f]))/(2*Sqrt[(2*I)*f
 - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*I)*d - (b^2*Log[f]^2)
/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((2*I)*f + c*Log
[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
```

```
x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}f^{a+bx+cx^2}-\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx+cx^2}-\frac{1}{4}e^{2id+2ifx^2}f^{a+bx+cx^2}\right)dx\\
&= -\left(\frac{1}{4}\int e^{-2id-2ifx^2}f^{a+bx+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\
&= -\left(\frac{1}{4}\int \exp\left(-2id+a\log(f)+bx\log(f)-x^2(2if-c\log(f))\right)dx\right)-\frac{1}{4}\int \exp\left(2id\right.\\
&\qquad \left.+a\log(f)+bx\log(f)+x^2(2if+c\log(f))\right)dx+\frac{1}{2}f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}\,dx\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\\
&\quad -\frac{1}{4}\left(e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-2if+c\log(f)))^2}{4(-2if+c\log(f))}\right)dx\\
&\quad -\frac{1}{4}\left(e^{2id-\frac{b^2\log^2(f)}{8if+4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(2if+c\log(f)))^2}{4(2if+c\log(f))}\right)dx\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log(f)-2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}\\
&\quad -\frac{e^{2id-\frac{b^2\log^2(f)}{8if+4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{b\log(f)+2x(2if+c\log(f))}{2\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 2.23 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.22

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)dx=\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2f^{-\frac{b^2}{4c}}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}\right.$$
$$+\frac{\sqrt[4]{-1}e^{\frac{b^2\log^2(f)}{8if-4c\log(f)}}\left(\text{erf}\left(\frac{\sqrt[4]{-1}(4fx+i(b+2cx)\log(f))}{2\sqrt{2f+ic\log(f)}}\right)\sqrt{2f+ic\log(f)}(2if+c\log(f))(\cos(2d)-i\sin(2d))+e\right.}{4f^2+c^2\log^2}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f^
(b^2/(4*c))*Sqrt[Log[f]]) + ((-1)^(1/4)*E^((b^2*Log[f]^2)/((8*I)*f - 4*c*Lo
g[f]))*(Erf[((-1)^(1/4)*(4*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f + I*c*L
og[f]])]*Sqrt[2*f + I*c*Log[f]]*((2*I)*f + c*Log[f])*(Cos[2*d] - I*Sin[2*d]
) + E^((I*b^2*f*Log[f]^2)/(4*f^2 + c^2*Log[f]^2))*Erf[((-1)^(3/4)*(4*f*x -
I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Sqrt[2*f - I*c*Log[f]]*(
2*f + I*c*Log[f])*(Cos[2*d] + I*Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+8 i d \ln(f) c+16 d f}{4(c\ln(f)-2 i f)}} \operatorname{erf}\left(-x\sqrt{2 i f-c\ln(f)}+\frac{\ln(f) b}{2\sqrt{2 i f-c\ln(f)}}\right)}{8\sqrt{2 i f-c\ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-8 i d \ln(f) c+16 d f}{4(2 i f+c\ln(f))}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-2 i}\right.}{8\sqrt{-c\ln(f)-2 i f}}$

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+8*I*d*ln(f)*c+16*d*f)/(c*ln(f)-2*I*f
))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*I*f-c*
ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-8*I*d*ln(f)*c+16*d*f)/
(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2*l
n(f)*b/(-c*ln(f)-2*I*f)^(1/2))-1/4*Pi^(1/2)*f^(-1/4*b^2/c)*f^a/(-c*ln(f))^(
1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(185) = 370.

Time = 0.27 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.64

$$\int f^{a+bx+cx^2} \sin^2\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}\left(c^2\log\left(f\right)^2-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-2i\,f}\,\operatorname{erf}\left(\frac{\left(8\,f^2x-2i\,bf\log(f)+\left(2\,c^2x+bc\right)\log(f)^2\right)\sqrt{-c\log(f)-2i\,f}}{2\left(c^2\log(f)^2+4\,f^2\right)}\right)e^{\left(\frac{16}{\cdots}\right.}}{\cdots}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(1
/2*(8*f^2*x - 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) - 2
*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c^2)*l
og(f)^3 + 32*I*d*f^2 - 2*(-4*I*c^2*d - I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4
```

```
*f^2)) + sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*e
rf(1/2*(8*f^2*x + 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f)
 + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c^
2)*log(f)^3 - 32*I*d*f^2 - 2*(4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2
+ 4*f^2)) - 2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c*
x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2*
log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + f*x**2)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 997, normalized size of antiderivative = 4.07

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*f^(1/4*b^2/c)*cos(1/2*
(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)) + f^a*f^(1/
4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*
f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f))/sqrt(-c*log(f) + 2*I*f))
 + (-I*f^a*f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c
^2*log(f)^2 + 4*f^2)) + f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^
2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) + 2*I*f)*x + b
*log(f))/sqrt(-c*log(f) - 2*I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 4*f^
2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((f^a*f^(1/4*b^
2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)
) - I*f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^
2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f))/sqrt(-c*l
og(f) + 2*I*f)) + (f^a*f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*
log(f)^2)/(c^2*log(f)^2 + 4*f^2)) + I*f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 +
 (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f)
+ 2*I*f)*x + b*log(f))/sqrt(-c*log(f) - 2*I*f)))*sqrt(-c*log(f) + sqrt(c^2*
```

```
log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - 2*sqrt(pi)*((c^2*f^a*e^(1/4*b^2*c*log(
f)^3/(c^2*log(f)^2 + 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(
c^2*log(f)^2 + 4*f^2)))*erf(-1/2*b*conjugate(1/sqrt(-c*log(f)))*log(f) + x*
conjugate(sqrt(-c*log(f)))) - (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2
+ 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^
2)))*erf(1/2*(2*c*x*log(f) + b*log(f))/sqrt(-c*log(f)))))/((c^2*e^(1/4*b^2*
c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*log(f)^2 + 4*f^2*e^(1
/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c))*sqrt(-c*log(f
)))
```

Giac [F]

$$\int f^{a+bx+cx^2} \sin^2\left(d+fx^2\right)\,dx = \int f^{cx^2+bx+a} \sin\left(fx^2+d\right)^2\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \sin^2\left(d+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\,\sin\left(f\,x^2+d\right)^2\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sin(d + f*x^2)^2,x)
[Out] int(f^(a + b*x + c*x^2)*sin(d + f*x^2)^2, x)
```

3.99 $\int f^{a+bx+cx^2} \sin^3(d+fx^2)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 386

$$\int f^{a+bx+cx^2} \sin^3(d+fx^2)\,dx = -\frac{3ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}} f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} + \frac{ie^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}} f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}} - \frac{3ie^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}} f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} + \frac{ie^{3id-\frac{b^2\log^2(f)}{4(3if+c\log(f))}} f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

```
[Out] -3/16*I*exp(-I*d+b^2*ln(f)^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(I
*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/16*I*exp(-
3*I*d+b^2*ln(f)^2/(12*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(3*I*f-c*ln(
f)))/(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)-3/16*I*exp(I*d-b
^2*ln(f)^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(I*f+c*ln(f)))/(I*f
+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)+1/16*I*exp(3*I*d-1/4*b^2*ln(f
)^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln
(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4560, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx$$
$$=-\frac{3i\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\operatorname{erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{i\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+12if}-3id}\operatorname{erf}\left(\frac{b\log(f)-2x(-c\log(f)+3if)}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{i\sqrt{\pi}f^a\exp\left(3id-\frac{b^2\log^2(f)}{4(c\log(f)+3if)}\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3if)}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}-\frac{3i\sqrt{\pi}f^a e^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^3,x]
```

```
[Out] (((-3*I)/16)*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi
]*Erf[(b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f
 - c*Log[f]] + ((I/16)*E^((-3*I)*d + (b^2*Log[f]^2)/((12*I)*f - 4*c*Log[f])
)*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f -
c*Log[f]])])/Sqrt[(3*I)*f - c*Log[f]] - (((3*I)/16)*E^(I*d - (b^2*Log[f]^2)
/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f])
)/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f + c*Log[f]] + ((I/16)*E^((3*I)*d - (b
^2*Log[f]^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((
3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])])/Sqrt[(3*I)*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{8}ie^{-id-ifx^2}f^{a+bx+cx^2}-\frac{3}{8}ie^{id+ifx^2}f^{a+bx+cx^2}-\frac{1}{8}ie^{-3id-3ifx^2}f^{a+bx+cx^2}\right.\\
&\qquad\left.+\frac{1}{8}ie^{3id+3ifx^2}f^{a+bx+cx^2}\right)dx\\
&= -\left(\frac{1}{8}i\int e^{-3id-3ifx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}i\int e^{3id+3ifx^2}f^{a+bx+cx^2}\,dx\\
&\quad+\frac{3}{8}i\int e^{-id-ifx^2}f^{a+bx+cx^2}\,dx-\frac{3}{8}i\int e^{id+ifx^2}f^{a+bx+cx^2}\,dx\\
&= -\left(\frac{1}{8}i\int \exp\left(-3id+a\log(f)+bx\log(f)-x^2(3if-c\log(f))\right)dx\right)\\
&\quad+\frac{1}{8}i\int \exp\left(3id+a\log(f)+bx\log(f)+x^2(3if+c\log(f))\right)dx\\
&\quad+\frac{3}{8}i\int \exp\left(-id+a\log(f)+bx\log(f)-x^2(if-c\log(f))\right)dx\\
&\quad-\frac{3}{8}i\int \exp\left(id+a\log(f)+bx\log(f)+x^2(if+c\log(f))\right)dx\\
&= \frac{1}{8}\left(3ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx\\
&\quad-\frac{1}{8}\left(ie^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-3if+c\log(f)))^2}{4(-3if+c\log(f))}\right)dx\\
&\quad+\frac{1}{8}\left(i\exp\left(3id-\frac{b^2\log^2(f)}{4(3if+c\log(f))}\right)f^a\right)\int \exp\left(\frac{(b\log(f)+2x(3if+c\log(f)))^2}{4(3if+c\log(f))}\right)dx\\
&\quad-\frac{1}{8}\left(3ie^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(if+c\log(f)))^2}{4(if+c\log(f))}\right)dx
\end{aligned}$$

$$= -\frac{3ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} + \frac{ie^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}} - \frac{3ie^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} + \frac{i\exp\left(3id-\frac{b^2\log^2(f)}{4(3if+c\log(f))}\right)f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3291 vs. $2(386) = 772$.

Time = 6.73 (sec) , antiderivative size = 3291, normalized size of antiderivative = 8.53

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx = \text{Result too large to show}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^3
*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f
 - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*(-1)^(1/4)*c*E^(((I/4)*b^2*Log[f
]^2)/(f - I*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2
*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*
(-1)^(3/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f*Cos[d]*Erfi[((-1
)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*
Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(f
 - I*c*Log[f]))*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log
[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] + 3*(-1)^(3/4
)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[((-1)^(1/4)
*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Sqrt[
3*f - I*c*Log[f]] - (-1)^(1/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f])
)*f^2*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2
*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]] + 3*(-1)^(3/4)*c^2*
E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(1/4)*(6*
f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^2*
Sqrt[3*f - I*c*Log[f]] - (-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*
Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))
/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]] + (27*(-1)^(1/
4)*f^3*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*
```

```
Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I
*c*Log[f])) - (27*(-1)^(3/4)*c*f^2*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log
[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[
f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) + (3*(-1)^(1/4)*c^2*f*Cos[d]
*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*
Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*L
og[f])) - (3*(-1)^(3/4)*c^3*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (
2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]])/
E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (3*(-1)^(1/4)*f^3*Cos[3*d]*Erfi
[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log
[f]])]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f]))
+ ((-1)^(3/4)*c*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c
*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]])/E^((
(I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) - (3*(-1)^(1/4)*c^2*f*Cos[3*d]*Erfi
[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log
[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*
Log[f])) + ((-1)^(3/4)*c^3*Cos[3*d]*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] +
(2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[
f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + 27*(-1)^(1/4)*E^(((I/4)*
b^2*Log[f]^2)/(f - I*c*Log[f]))*f^3*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] -
(2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*Sin[d] +
27*(-1)^(3/4)*c*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^2*Erfi[((-1)^(1
/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[
f]*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(
f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f])
)/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(
3/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(2*f*x
- I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f
 - I*c*Log[f]]*Sin[d] - (27*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(2*f*x + I*b*Lo
g[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*Si
n[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (27*(-1)^(1/4)*c*f^2*Erfi
[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f
]])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*L
og[f])) - (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*
c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d]
)/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (3*(-1)^(1/4)*c^3*Erfi[((-1)^
(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Lo
g[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f]
)) - 3*(-1)^(1/4)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Erfi[((-1
)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])
]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - (-1)^(3/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3
*f - I*c*Log[f]))*f^2*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - 3
*(-1)^(1/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Erfi[((-1)^(1
/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Lo
```

```
g[f]^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - (-1)^(3/4)*c^3*E^(((I/4)*b^2*Log[f
]^2)/(3*f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*L
og[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]]*Sin[3*d
] + (3*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)
*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4)*(6*
f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sq
rt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f]))
+ (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/
E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c^3*Erfi[((-1)^(3
/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Lo
g[f]^3*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*
Log[f]))))/(16*(I*f - c*Log[f])*(f - I*c*Log[f])*(3*f - I*c*Log[f])*(3*f +
I*c*Log[f]))
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-12 i d \ln(f) c+36 d f}{4(3 i f+c\ln(f))}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-3 i f}\, x+\frac{\ln(f) b}{2\sqrt{-c\ln(f)-3 i f}}\right)}{16\sqrt{-c\ln(f)-3 i f}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+12 i d \ln(f) c+36 d f}{4(c\ln(f)-3 i f)}} \operatorname{erf}\left(-x\sqrt{3\right.}{16\sqrt{3 i f-c\ln(f)}}$

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-12*I*d*ln(f)*c+36*d*f)/(3*I*f+c*
ln(f)))/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+1/2*ln(f)*b/(-
c*ln(f)-3*I*f)^(1/2))+1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+12*I*d*ln(f
)*c+36*d*f)/(c*ln(f)-3*I*f))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(
1/2)+1/2*ln(f)*b/(3*I*f-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)
^2*b^2+4*I*d*ln(f)*c+4*d*f)/(c*ln(f)-I*f))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-
c*ln(f))^(1/2)+1/2*ln(f)*b/(I*f-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-1/
4*(ln(f)^2*b^2-4*I*d*ln(f)*c+4*d*f)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf
(-(-c*ln(f)-I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(289) = 578$.

Time = 0.29 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.89

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx$$

$$= \frac{\sqrt{\pi}\left(-i\,c^3\log\left(f\right)^3 - 3\,c^2f\log\left(f\right)^2 - i\,cf^2\log\left(f\right) - 3\,f^3\right)\sqrt{-c\log\left(f\right) - 3i\,f}\,\mathrm{erf}\left(\frac{\left(18\,f^2x - 3i\,bf\log(f) + (2\,c^2x + bc)\right.\cdots}{2\left(c^2\log(f)^2 + \cdots\right.}\cdots\right.}{\cdots}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x - 3*I*b*f*log(f) + (2*c^2*x + b
*c)*log(f)^2)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*
f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 108*I*d*f^2 - 3*(-4*I*c^2*d - I*b
^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(I*c^3*log(f)^3 - 3*c^2*
f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(18*f^
2*x + 3*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + 3*I*f)/(c
^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3
- 108*I*d*f^2 - 3*(4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) -
 3*sqrt(pi)*(-I*c^3*log(f)^3 - c^2*f*log(f)^2 - 9*I*c*f^2*log(f) - 9*f^3)*s
qrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*b*f*log(f) + (2*c^2*x + b*c)*log(
f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^2*log(f) -
(b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (4*I*c^2*d + I*b^2*f)*log(f)^2)/(c
^2*log(f)^2 + f^2)) - 3*sqrt(pi)*(I*c^3*log(f)^3 - c^2*f*log(f)^2 + 9*I*c*f
^2*log(f) - 9*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*log(f) +
(2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/
4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I*c^2*d -
I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)
^2 + 9*f^4)
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx = \int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + f*x**2)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2451 vs. 2(289) = 578.

Time = 0.31 (sec) , antiderivative size = 2451, normalized size of antiderivative = 6.35

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f)
^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*lo
g(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)
^2 + 9*f^2)) + (-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(
f)^2 - I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/4*(36
*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c
*log(f) - 3*I*f)*x + b*log(f))/sqrt(-c*log(f) + 3*I*f)) + ((c^2*f^a*e^(1/4*
b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(
f)^3/(c^2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)
/(c^2*log(f)^2 + 9*f^2)) + (I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 +
 f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*
sin(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*er
f(1/2*(2*(c*log(f) + 3*I*f)*x + b*log(f))/sqrt(-c*log(f) - 3*I*f)))*sqrt(c*
log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f
^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f
^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1/4*(4*d*f^2 +
(4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (-I*c^2*f^a*e^(1/4*b^2*
c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - 9*I*f^(a + 2)*e^(1/4*b^2*c*lo
g(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)
^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f))/sqrt(-
c*log(f) + I*f)) + ((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*
log(f)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1
/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (I*c^2*f^
a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*I*f^(a + 2)*e^
(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d +
b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*l
og(f))/sqrt(-c*log(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) +
sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(
c^2*log(f)^2 + f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(
f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2
 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2
+ f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/4*(36*d*f^2
+ (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f)
 - 3*I*f)*x + b*log(f))/sqrt(-c*log(f) + 3*I*f)) + ((-I*c^2*f^a*e^(1/4*b^2*
```

```
c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 - I*f^(a + 2)*e^(1/4*b^2*c*log(f)
^3/(c^2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(
c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2
))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/
4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*
(2*(c*log(f) + 3*I*f)*x + b*log(f))/sqrt(-c*log(f) - 3*I*f)))*sqrt(-c*log(f
) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(
((I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*I*f^
(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1/4*(4*d*f^2 + (
4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (c^2*f^a*e^(1/4*b^2*c*lo
g(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*log(f)^3
/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c
^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f))/sqrt(-c*log(
f) + I*f)) + ((-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log
(f)^2 - 9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1/
4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (c^2*f^a*e
^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a + 2)*e^(1/4*
b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f
)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f)
)/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*
e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(
f)^2 + f^2))*log(f)^4 + 10*c^2*f^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*
f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^4*e^(1/4*b^2
*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2
)))
```

Giac [F]

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx=\int f^{cx^2+bx+a}\sin\left(fx^2+d\right)^3\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\,\sin\left(f\,x^2+d\right)^3\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sin(d + f*x^2)^3,x)
[Out] int(f^(a + b*x + c*x^2)*sin(d + f*x^2)^3, x)
```

3.100 $\int f^{a+bx+cx^2} \sin(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 212

$$\int f^{a+bx+cx^2}\sin\left(d+ex+fx^2\right)\,dx=\frac{ie^{-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] 1/4*I*exp(-I*d-(e+I*b*ln(f))^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e-b*ln(f)+
2*x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/4*I*
exp(I*d+(e-I*b*ln(f))^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*e+b*ln(f)+2*x*(I
*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4560, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\sin\left(d+ex+fx^2\right)\,dx$$
$$=\frac{i\sqrt{\pi}f^a\exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\mathrm{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^a\exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\mathrm{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2],x]
```

```
[Out] ((I/4)*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*
Erf[(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt
[I*f - c*Log[f]] - ((I/4)*E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]
))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f +
c*Log[f]])])/Sqrt[I*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2} i e^{-id-iex-ifx^2} f^{a+bx+cx^2} - \frac{1}{2} i e^{id+iex+ifx^2} f^{a+bx+cx^2}\right) dx \\
&= \frac{1}{2} i \int e^{-id-iex-ifx^2} f^{a+bx+cx^2} \, dx - \frac{1}{2} i \int e^{id+iex+ifx^2} f^{a+bx+cx^2} \, dx \\
&= \frac{1}{2} i \int \exp\left(-id + a\log(f) - x(ie - b\log(f)) - x^2(if - c\log(f))\right) dx \\
&\quad - \frac{1}{2} i \int \exp\left(id + a\log(f) + x(ie + b\log(f)) + x^2(if + c\log(f))\right) dx
\end{aligned}$$

$$= \frac{1}{2}\left(i\exp\left(-id - \frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(-ie+b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx - \frac{1}{2}\left(i\exp\left(id + \frac{(e-ib\log(f))^2}{4if+4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2x(if+c\log(f)))^2}{4(if+c\log(f))}\right)dx$$

$$= \frac{i\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}} - \frac{i\exp\left(id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

Mathematica [A] (warning: unable to verify)

Time = 2.02 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.64

$$\int f^{a+bx+cx^2}\sin\left(d+ex+fx^2\right)dx = -\frac{\sqrt[4]{-1}e^{-\frac{1}{4}i\left(\frac{e^2}{f-ic\log(f)}+\frac{b^2\log^2(f)}{f+ic\log(f)}\right)}f^{\frac{f(-be+af)+ac^2\log^2(f)}{f^2+c^2\log^2(f)}}\sqrt{\pi}\left(e^{\frac{ib^2f\log^2(f)}{2\left(f^2+c^2\log^2(f)\right)}}f^{\frac{be}{2f+2ic\log(f)}}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-i(b+2cx)\log}{2\sqrt{f-ic\log(f)}}\right.\right.}{}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*f^((f*(-(b*e) + a*f) + a*c^2*Log[f]^2)/(f^2 + c^2*Log[f]^2
))*Sqrt[Pi]*(E^(((I/2)*b^2*f*Log[f]^2)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f
+ (2*I)*c*Log[f]))*Erfi[((-1)^(1/4)*(e + 2*f*x - I*(b + 2*c*x)*Log[f]))/(2*
Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] + I*Si
n[d]) + E^(((I/2)*e^2*f)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f - (2*I)*c*Log[
f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c*L
og[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(I*Cos[d] + Sin[d])))/(E^((I
/4)*(e^2/(f - I*c*Log[f]) + (b^2*Log[f]^2)/(f + I*c*Log[f])))*(f^2 + c^2*Lo
g[f]^2))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.02

method	result
risch	$\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+2 i \ln(f) b e-4 i d \ln(f) c+4 d f-e^{2}}{4(i f+c \ln(f))}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-i f}\, x+\frac{i e+b \ln(f)}{2 \sqrt{-c \ln(f)-i f}}\right)}{4 \sqrt{-c \ln(f)-i f}}-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-2 i \ln(f) b e+4 i d \ln(f) c+4}{4(c \ln(f)-i f)}}}{4\sqrt{i f}}$

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c+4*d*f-
e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*(I
*e+b*ln(f))/(-c*ln(f)-I*f)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-
2*I*ln(f)*b*e+4*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-I*f))/(I*f-c*ln(f))^(1/2)*e
rf(-x*(I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-I*e)/(I*f-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(155) = 310.

Time = 0.26 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.77

$$\int f^{a+bx+cx^2} \sin\left(d+ex+fx^2\right)\,dx$$

$$= \frac{\sqrt{\pi}(i\,c\log\left(f\right)+f)\sqrt{-c\log\left(f\right)-i\,f}\operatorname{erf}\left(\frac{\left(2\,f^{2}x+\left(2\,c^{2}x+bc\right)\log(f)^{2}+ef+\left(i\,ce-i\,bf\right)\log(f)\right)\sqrt{-c\log(f)-i\,f}}{2\left(c^{2}\log(f)^{2}+f^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)}{}\right.}}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x + (2*
c^2*x + b*c)*log(f)^2 + e*f + (I*c*e - I*b*f)*log(f))*sqrt(-c*log(f) - I*f)
/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*e^2*f - 4*I*
d*f^2 - (4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f + 4*a
*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-I*c*log(f) + f)*sqrt(-c*lo
g(f) + I*f)*erf(1/2*(2*f^2*x + (2*c^2*x + b*c)*log(f)^2 + e*f + (-I*c*e + I
*b*f)*log(f))*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c -
 4*a*c^2)*log(f)^3 - I*e^2*f + 4*I*d*f^2 - (-4*I*c^2*d + 2*I*b*c*e - I*b^2*
f)*log(f)^2 - (c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)))/(c
^2*log(f)^2 + f^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2} \sin \left(d+ex+fx^2\right) \, dx = \int f^{a+bx+cx^2} \sin \left(d+ex+fx^2\right) dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + e*x + f*x**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1007 vs. 2(155) = 310.

Time = 0.25 (sec) , antiderivative size = 1007, normalized size of antiderivative = 4.75

$$\int f^{a+bx+cx^2} \sin \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((f^(1/4*c*e^2/(c^2*log(f)^2 + f
^2))*f^a*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/
(c^2*log(f)^2 + f^2)) - I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(-1/4*(
e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2
)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I*f)/(
c*log(f) - I*f)) + (f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(-1/4*(e^2*f
- 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + I
*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*
d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f)
+ I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f) - I*f)/(c*log(f) + I*f)))*sqrt(c*
log(f) + sqrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*
((I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c
^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^(1/4*c*e^2/(c^2
*log(f)^2 + f^2))*f^a*sin(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*
f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f
) - I*e)*sqrt(-c*log(f) + I*f)/(c*log(f) - I*f)) + (-I*f^(1/4*c*e^2/(c^2*lo
g(f)^2 + f^2))*f^a*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*
log(f)^2)/(c^2*log(f)^2 + f^2)) + f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*si
n(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)
^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f)
- I*f)/(c*log(f) + I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^2*
e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2
 + f^2))*log(f)^2 + f^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*
e*f*log(f)/(c^2*log(f)^2 + f^2)))
```

Giac [F]

$$\int f^{a+bx+cx^2}\sin\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+bx+a}\sin\left(fx^2+ex+d\right)\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\sin\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2),x)
[Out] int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2), x)
```

3.101 $\int f^{a+bx+cx^2} \sin^2(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 268

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)dx = \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{2ie-b\log(f)+2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}} - \frac{e^{2id+\frac{(2e-ib\log(f))^2}{8if+4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{2ie+b\log(f)+2x(2if+c\log(f))}{2\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}$$

```
[Out] 1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)-1/8*exp(-2*I*d-(2*e+I*b*ln(f))^2/(8*I*f-4*c*ln(f)))*f^a*erf(1
/2*(2*I*e-b*ln(f)+2*x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*I
*f-c*ln(f))^(1/2)-1/8*exp(2*I*d+(2*e-I*b*ln(f))^2/(8*I*f+4*c*ln(f)))*f^a*er
fi(1/2*(2*I*e+b*ln(f)+2*x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/
(2*I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used

$= \{4560, 2266, 2235, 2325, 2236\}$

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a\exp\left(-\frac{(2e+ib\log(f))^2}{-4c\log(f)+8if}-2id\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+2if)+2ie}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}f^a\exp\left(\frac{(2e-ib\log(f))^2}{4c\log(f)+8if}+2id\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)+2ie}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d - (2*e + I*b*Log[f])^2/((8*I)*f - 4*
c*Log[f]))*f^a*Sqrt[Pi]*Erf[((2*I)*e - b*Log[f] + 2*x*((2*I)*f - c*Log[f]))
/(2*Sqrt[(2*I)*f - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*I)*d
+ (2*e - I*b*Log[f])^2/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e +
 b*Log[f] + 2*x*((2*I)*f + c*Log[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqr
t[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}f^{a+bx+cx^2} - \frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2} - \frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx+cx^2}\right)dx \\
&= -\left(\frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2}\,dx\right) - \frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+bx+cx^2}\,dx + \frac{1}{2}\int f^{a+bx+cx^2}\,dx \\
&= -\left(\frac{1}{4}\int \exp\left(-2id + a\log(f) - x(2ie - b\log(f)) - x^2(2if - c\log(f))\right)dx\right) \\
&\quad - \frac{1}{4}\int \exp\left(2id + a\log(f) + x(2ie + b\log(f)) + x^2(2if + c\log(f))\right)dx \\
&\quad + \frac{1}{2}f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}\,dx \\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{1}{4}\left(\exp\left(-2id\right.\right. \\
&\quad \left.\left. - \frac{(2e+ib\log(f))^2}{8if-4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(-2ie+b\log(f)+2x(-2if+c\log(f)))^2}{4(-2if+c\log(f))}\right)dx \\
&\quad - \frac{1}{4}\left(\exp\left(2id\right.\right. \\
&\qquad \left.\left. + \frac{(2e-ib\log(f))^2}{8if+4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(2ie+b\log(f)+2x(2if+c\log(f)))^2}{4(2if+c\log(f))}\right)dx \\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} \\
&\quad - \frac{\exp\left(-2id - \frac{(2e+ib\log(f))^2}{8if-4c\log(f)}\right)f^a\sqrt{\pi}\,\text{erf}\left(\frac{2ie-b\log(f)+2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}} \\
&\quad - \frac{\exp\left(2id + \frac{(2e-ib\log(f))^2}{8if+4c\log(f)}\right)f^a\sqrt{\pi}\,\text{erfi}\left(\frac{2ie+b\log(f)+2x(2if+c\log(f))}{2\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1120 vs. 2(268) = 536.

Time = 6.53 (sec) , antiderivative size = 1120, normalized size of antiderivative = 4.18

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx$$

$$= \frac{f^a\sqrt{\pi}\left(8\sqrt{c}f^{2-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}+2c^{5/2}f^{-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\log^{\frac{5}{2}}(f)+2\sqrt[4]{-1}ce^{\frac{i\left(-4e^2+4ibe\log\cdots\right.}{4(2f-\cdots}}\cdots\right.}{\cdots}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(8*Sqrt[c]*f^(2 - b^2/(4*c))*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/
(2*Sqrt[c])]*Sqrt[Log[f]] + (2*c^(5/2)*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*S
qrt[c])]*Log[f]^(5/2))/f^(b^2/(4*c)) + 2*(-1)^(1/4)*c*E^(((I/4)*(-4*e^2 + (
4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Cos[2*d]*Erf[((-1)^(
3/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f
]])]*Log[f]*Sqrt[2*f - I*c*Log[f]] + (-1)^(3/4)*c^2*E^(((I/4)*(-4*e^2 + (4*
I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Cos[2*d]*Erf[((-1)^(3/4)
*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]
*Log[f]^2*Sqrt[2*f - I*c*Log[f]] + (2*(-1)^(3/4)*c*f*Cos[2*d]*Erf[((-1)^(1/
4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]]
)]*Log[f]*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] + b^
2*Log[f]^2))/(2*f + I*c*Log[f])) + ((-1)^(1/4)*c^2*Cos[2*d]*Erf[((-1)^(1/4)
*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]
*Log[f]^2*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] + b^
2*Log[f]^2))/(2*f + I*c*Log[f])) + 2*(-1)^(3/4)*c*E^(((I/4)*(-4*e^2 + (4*I)
*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Erf[((-1)^(3/4)*(2*e + 4
*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Log[f]*S
qrt[2*f - I*c*Log[f]]*Sin[2*d] - (-1)^(1/4)*c^2*E^(((I/4)*(-4*e^2 + (4*I)*b
*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Erf[((-1)^(3/4)*(2*e + 4*f*x
 - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Log[f]^2*Sqr
t[2*f - I*c*Log[f]]*Sin[2*d] + (2*(-1)^(1/4)*c*f*Erf[((-1)^(1/4)*(2*e + 4*f
*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]*Log[f]*Sqr
t[2*f + I*c*Log[f]]*Sin[2*d])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] + b^2*Lo
g[f]^2))/(2*f + I*c*Log[f])) - ((-1)^(3/4)*c^2*Erf[((-1)^(1/4)*(2*e + 4*f*x
 + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]*Log[f]^2*Sqr
t[2*f + I*c*Log[f]]*Sin[2*d])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] + b^2*Lo
g[f]^2))/(2*f + I*c*Log[f]))))/(8*c*Log[f]*(2*f - I*c*Log[f])*(2*f + I*c*Lo
g[f]))
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.98

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-4 i \ln(f) b e+8 i d \ln(f) c+16 d f-4 e^{2}}{4(c \ln(f)-2 i f)}} \operatorname{erf}\left(-x \sqrt{2 i f-c \ln(f)}+\frac{b \ln(f)-2 i e}{2 \sqrt{2 i f-c \ln(f)}}\right)}{8 \sqrt{2 i f-c \ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i \ln(f) b e-8 i d \ln(f) c+\cdots}{4(2 i f+c \ln(f))}}}{8 \cdots}$

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*b*e+8*I*d*ln(f)*c+16*d*f-4
*e^2)/(c*ln(f)-2*I*f))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)+1
/2*(b*ln(f)-2*I*e)/(2*I*f-c*ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^
2*b^2+4*I*ln(f)*b*e-8*I*d*ln(f)*c+16*d*f-4*e^2)/(2*I*f+c*ln(f)))/(-c*ln(f)-
2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-c*ln(f)-2*
I*f)^(1/2))-1/4*Pi^(1/2)*f^(-1/4*b^2/c)*f^a/(-c*ln(f))^(1/2)*erf(-(-c*ln(f)
)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(199) = 398.

Time = 0.26 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.75

$$\int f^{a+bx+cx^2} \sin^2\left(d+ex+fx^2\right)\, dx$$

$$= \frac{\sqrt{\pi}\left(c^2 \log(f)^2 - 2i\, cf \log(f)\right)\sqrt{-c\log(f) - 2i\, f}\, \operatorname{erf}\left(\frac{\left(8\, f^2 x + (2\, c^2 x + bc) \log(f)^2 + 4\, ef - 2\,(-i\, ce + i\, bf)\log(f)\right)\sqrt{-c\log(f) \cdots}}{2\left(c^2\log(f)^2 + 4\, f^2\right)} \cdots\right.}{\cdots}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(1
/2*(8*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*e*f - 2*(-I*c*e + I*b*f)*log(f))
*sqrt(-c*log(f) - 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)
*log(f)^3 + 8*I*e^2*f - 32*I*d*f^2 + 2*(-4*I*c^2*d + 2*I*b*c*e - I*b^2*f)*l
og(f)^2 - 4*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) + s
qrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(1/2*(8*
f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*e*f - 2*(I*c*e - I*b*f)*log(f))*sqrt(-
c*log(f) + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)
^3 - 8*I*e^2*f + 32*I*d*f^2 + 2*(4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2
- 4*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) - 2*sqrt(pi
)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f)
)/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + e*x + f*x**2)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 1487, normalized size of antiderivative = 5.55

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*cos(-1/2*(4*e^2*f - 16*
d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))*e^(c*
e^2*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c) + f^a*e^(c*e^2*log(f)
/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*sin(-1/2*(4*e^2*f - 16*d*f^2 -
(4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(
c*log(f) - 2*I*f)*x + b*log(f) - 2*I*e)*sqrt(-c*log(f) + 2*I*f)/(c*log(f) -
 2*I*f)) + (-I*f^a*cos(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*
f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2)
 + 1/4*b^2*log(f)/c) + f^a*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2
*log(f)/c)*sin(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f
)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) + 2*I*f)*x + b*log(f) +
2*I*e)*sqrt(-c*log(f) - 2*I*f)/(c*log(f) + 2*I*f)))*sqrt(c*log(f) + sqrt(c^
2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2
)*((f^a*cos(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2
)/(c^2*log(f)^2 + 4*f^2))*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*
log(f)/c) - I*f^a*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c
)*sin(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2
*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f) - 2*I*e)*sq
rt(-c*log(f) + 2*I*f)/(c*log(f) - 2*I*f)) + (f^a*cos(-1/2*(4*e^2*f - 16*d*f
^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))*e^(c*e^2
*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c) + I*f^a*e^(c*e^2*log(f)/
(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*sin(-1/2*(4*e^2*f - 16*d*f^2 - (
4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c
*log(f) + 2*I*f)*x + b*log(f) + 2*I*e)*sqrt(-c*log(f) - 2*I*f)/(c*log(f) +
2*I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) + 2*s
```

```
qrt(pi)*((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*b*e*f*lo
g(f)/(c^2*log(f)^2 + 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(
c^2*log(f)^2 + 4*f^2) + 2*b*e*f*log(f)/(c^2*log(f)^2 + 4*f^2)))*erf(-1/2*b*
conjugate(1/sqrt(-c*log(f)))*log(f) + x*conjugate(sqrt(-c*log(f)))) - (c^2*
f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*b*e*f*log(f)/(c^2*log(
f)^2 + 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 +
 4*f^2) + 2*b*e*f*log(f)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*c*x*log(f) + b
*log(f))/sqrt(-c*log(f)))))/((c^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f
^2) + 2*b*e*f*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*log(f)^2 +
4*f^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*b*e*f*log(f)/(c^2*lo
g(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c))*sqrt(-c*log(f)))
```

Giac [F]

$$\int f^{a+bx+cx^2} \sin^2\left(d+ex+fx^2\right)\,dx = \int f^{cx^2+bx+a} \sin\left(fx^2+ex+d\right)^2\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \sin^2\left(d+ex+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\,\sin\left(f\,x^2+e\,x+d\right)^2\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2)^2, x)
```

3.102 $\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 430

$$\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx=\frac{3ie^{-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id-\frac{(3e+ib\log(f))^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{3ie-b\log(f)+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}}-\frac{3ie^{id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}}+\frac{ie^{3id-\frac{(3ie+b\log(f))^2}{4(3if+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie+b\log(f)+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

```
[Out] 3/16*I*exp(-I*d-(e+I*b*ln(f))^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e-b*ln(f)
+2*x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/16*
I*exp(-3*I*d-1/4*(3*e+I*b*ln(f))^2/(3*I*f-c*ln(f)))*f^a*erf(1/2*(3*I*e-b*ln
(f)+2*x*(3*I*f-c*ln(f)))/(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1
/2)-3/16*I*exp(I*d+(e-I*b*ln(f))^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*e+b*l
n(f)+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)+1
/16*I*exp(3*I*d-1/4*(3*I*e+b*ln(f))^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(3*I*e+
b*ln(f)+2*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f)
)^(1/2)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4560, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx = \frac{3i\sqrt{\pi}f^a\exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}} - \frac{i\sqrt{\pi}f^a\exp\left(-\frac{(3e+ib\log(f))^2}{4(-c\log(f)+3if)}-3id\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}} - \frac{3i\sqrt{\pi}f^a\exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}} + \frac{i\sqrt{\pi}f^a\exp\left(3id-\frac{(b\log(f)+3ie)^2}{4(c\log(f)+3if)}\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3if)+3ie}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] (((3*I)/16)*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt
[Pi]*Erf[(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])
/Sqrt[I*f - c*Log[f]] - ((I/16)*E^((-3*I)*d - (3*e + I*b*Log[f])^2/(4*((3*I
)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e - b*Log[f] + 2*x*((3*I)*f - c*L
og[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/Sqrt[(3*I)*f - c*Log[f]] - (((3*I)/1
6)*E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I
*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f +
 c*Log[f]] + ((I/16)*E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*((3*I)*f + c*Lo
g[f])))*f^a*Sqrt[Pi]*Erfi[((3*I)*e + b*Log[f] + 2*x*((3*I)*f + c*Log[f]))/(
2*Sqrt[(3*I)*f + c*Log[f]])])/Sqrt[(3*I)*f + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{1}{8}ie^{-3i(d+ex+fx^2)}f^{a+bx+cx^2} + \frac{3}{8}i\exp\left(2id+2iex+2ifx^2-3i(d+ex+fx^2)\right)f^{a+bx+cx^2} - \frac{3}{8}i\exp\left(4id+4iex+4ifx^2-3i(d+ex+fx^2)\right)f^{a+bx+cx^2} + \frac{1}{8}i\exp\left(6id+6iex+6ifx^2-3i(d+ex+fx^2)\right)f^{a+bx+cx^2}\right)dx$$

$$= -\left(\frac{1}{8}i\int e^{-3i(d+ex+fx^2)}f^{a+bx+cx^2}\,dx\right) + \frac{1}{8}i\int \exp\left(6id+6iex+6ifx^2-3i(d+ex+fx^2)\right)f^{a+bx+cx^2}\,dx + \frac{3}{8}i\int \exp\left(2id+2iex+2ifx^2-3i(d+ex+fx^2)\right)f^{a+bx+cx^2}\,dx - \frac{3}{8}i\int \exp\left(4id+4iex+4ifx^2-3i(d+ex+fx^2)\right)f^{a+bx+cx^2}\,dx$$

$$= -\left(\frac{1}{8}i\int \exp\left(-3id+a\log(f)-x(3ie-b\log(f))-x^2(3if-c\log(f))\right)dx\right) + \frac{1}{8}i\int \exp\left(3id+a\log(f)+x(3ie+b\log(f))+x^2(3if+c\log(f))\right)dx + \frac{3}{8}i\int \exp\left(-id+a\log(f)-x(ie-b\log(f))-x^2(if-c\log(f))\right)dx - \frac{3}{8}i\int \exp\left(id+a\log(f)+x(ie+b\log(f))+x^2(if+c\log(f))\right)dx$$

$$= \frac{1}{8}\left(3i\exp\left(-id - \frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(-ie+b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx - \frac{1}{8}\left(i\exp\left(-3id - \frac{(3e+ib\log(f))^2}{4(3if-c\log(f))}\right)f^a\right)\int \exp\left(\frac{(-3ie+b\log(f)+2x(-3if+c\log(f)))^2}{4(-3if+c\log(f))}\right)dx + \frac{1}{8}\left(i\exp\left(3id - \frac{(3ie+b\log(f))^2}{4(3if+c\log(f))}\right)f^a\right)\int \exp\left(\frac{(3ie+b\log(f)+2x(3if+c\log(f)))^2}{4(3if+c\log(f))}\right)dx - \frac{1}{8}\left(3i\exp\left(id + \frac{(e-ib\log(f))^2}{4if+4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2x(if+c\log(f)))^2}{4(if+c\log(f))}\right)dx$$

$$= \frac{3i\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} - \frac{i\exp\left(-3id-\frac{(3e+ib\log(f))^2}{4(3if-c\log(f))}\right)f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{3ie-b\log(f)+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}} - \frac{3i\exp\left(id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} + \frac{i\exp\left(3id-\frac{(3ie+b\log(f))^2}{4(3if+c\log(f))}\right)f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3ie+b\log(f)+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3835 vs. $2(430) = 860$.

Time = 7.20 (sec) , antiderivative size = 3835, normalized size of antiderivative = 8.92

$$\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx = \text{Result too large to show}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[
f]^2))/(f - I*c*Log[f]))*f^3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f
] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*
```

(-1)^(1/4)*c*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*L
og[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log
[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4)*
c^2*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f
*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((
I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*Cos[d]*Erf
i[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*
Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] + 3*(-1)^(3/4)*E^(((I/4)*(-9*e^2 +
(6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[((-
1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*L
og[f]])]*Sqrt[3*f - I*c*Log[f]] - (-1)^(1/4)*c*E^(((I/4)*(-9*e^2 + (6*I)*b*
e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^2*Cos[3*d]*Erfi[((-1)^(1/4)
*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]
*Log[f]*Sqrt[3*f - I*c*Log[f]] + 3*(-1)^(3/4)*c^2*E^(((I/4)*(-9*e^2 + (6*I)
*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(1/4
)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])
]*Log[f]^2*Sqrt[3*f - I*c*Log[f]] - (-1)^(1/4)*c^3*E^(((I/4)*(-9*e^2 + (6*I
)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)
*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]
*Log[f]^3*Sqrt[3*f - I*c*Log[f]] + (27*(-1)^(1/4)*f^3*Cos[d]*Erfi[((-1)^(3/
4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*S
qrt[f + I*c*Log[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f
 + I*c*Log[f])) - (27*(-1)^(3/4)*c*f^2*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x +
 I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f +
I*c*Log[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*L
og[f])) + (3*(-1)^(1/4)*c^2*f*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[
f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log
[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f]))
 - (3*(-1)^(3/4)*c^3*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I
)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]])/E^(
((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - (3*(-1
)^(1/4)*f^3*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x
*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*(-9
*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(3/4)*
c*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f
]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*(-
9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) - (3*(-1)^(1/
4)*c^2*f*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Lo
g[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]])/E^(((I/
4)*(-9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^
(3/4)*c^3*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*L
og[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[f]])/E^(((I
/4)*(-9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + 27*(-
1)^(1/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f

```
]))*f^3*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*Sin[d] + 27*(-1)^(3/4)*c*E^(((I/4
)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f^2*Erfi[((-1
)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]
])]*Log[f]*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*(-e^2 +
 (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(e
+ 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^
2*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(3/4)*c^3*E^(((I/4)*(-e^2 + (2*I)*b*
e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(e + 2*f*x - I
*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f -
I*c*Log[f]]*Sin[d] - (27*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*L
og[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*S
in[d])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])
) - (27*(-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c
*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d])/E
^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - (3*(
-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]
))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4
)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - (3*(-1)^(1/
4)*c^3*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqr
t[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*(-e^2 -
 (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - 3*(-1)^(1/4)*E^(((I/
4)*(-9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^3*Erfi
[((-1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I
*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - (-1)^(3/4)*c*E^(((I/4)*(-9*e
^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^2*Erfi[((-1)^(
1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f
]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - 3*(-1)^(1/4)*c^2*E^(((I/4)*(-
9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f*Erfi[((-1)^
(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[
f]])]*Log[f]^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - (-1)^(3/4)*c^3*E^(((I/4)*(
-9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*Erfi[((-1)^(
1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f
]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (3*(-1)^(3/4)*f^3*Erfi[((-1
)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Lo
g[f]])]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b*e*Log[
f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4
)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])
]*Log[f]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b*e*Log
[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(
3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f
]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b*
e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c^3*Erfi[((-1)^
(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[
f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b
```

```
*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f]))))/(16*(I*f - c*Log[f])*(f -
I*c*Log[f])*(3*f - I*c*Log[f])*(3*f + I*c*Log[f]))
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+6 i \ln(f) b e-12 i d \ln(f) c+36 d f-9 e^{2}}{4(3 i f+c \ln(f))}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-3 i f}\, x+\frac{3 i e+b \ln(f)}{2 \sqrt{-c \ln(f)-3 i f}}\right)}{16 \sqrt{-c \ln(f)-3 i f}}+\frac{i\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-6 i \ln(f) b e+12 i d}{4(c \ln(f)-3}}}{}$

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*I*ln(f)*b*e-12*I*d*ln(f)*c+36*
d*f-9*e^2)/(3*I*f+c*ln(f)))/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1
/2)*x+1/2*(3*I*e+b*ln(f))/(-c*ln(f)-3*I*f)^(1/2))+1/16*I*Pi^(1/2)*f^a*exp(-
1/4*(ln(f)^2*b^2-6*I*ln(f)*b*e+12*I*d*ln(f)*c+36*d*f-9*e^2)/(c*ln(f)-3*I*f)
)/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-3*I*e)/(3
*I*f-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*b*
e+4*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-I*f))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c
*ln(f))^(1/2)+1/2*(b*ln(f)-I*e)/(I*f-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*ex
p(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c+4*d*f-e^2)/(I*f+c*ln(f)))/(
-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f)
-I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(312) = 624.

Time = 0.29 (sec) , antiderivative size = 865, normalized size of antiderivative = 2.01

$$\int f^{a+bx+cx^2} \sin^3\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*
e*f - 3*(-I*c*e + I*b*f)*log(f))*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*
f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 27*I*e^2*f - 108*I*d*f^2 + 3*(-
4*I*c^2*d + 2*I*b*c*e - I*b^2*f)*log(f)^2 - 9*(c*e^2 - 2*b*e*f + 4*a*f^2)*l
og(f))/(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*(-I*c^3*log(f)^3 - c^2*f*log(f)
^2 - 9*I*c*f^2*log(f) - 9*f^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x + (2*
c^2*x + b*c)*log(f)^2 + e*f + (I*c*e - I*b*f)*log(f))*sqrt(-c*log(f) - I*f)
```

```
/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*e^2*f - 4*I*
d*f^2 - (4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f + 4*a
*f^2)*log(f))/(c^2*log(f)^2 + f^2)) - 3*sqrt(pi)*(I*c^3*log(f)^3 - c^2*f*lo
g(f)^2 + 9*I*c*f^2*log(f) - 9*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x +
 (2*c^2*x + b*c)*log(f)^2 + e*f + (-I*c*e + I*b*f)*log(f))*sqrt(-c*log(f) +
 I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 - I*e^2*f +
 4*I*d*f^2 - (-4*I*c^2*d + 2*I*b*c*e - I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f
 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(I*c^3*log(f)^3 - 3*c^
2*f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(18*
f^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*e*f - 3*(I*c*e - I*b*f)*log(f))*sqrt(-
c*log(f) + 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)
^3 - 27*I*e^2*f + 108*I*d*f^2 + 3*(4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^
2 - 9*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^2)))/(c^4*log
(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \sin^3\left(d+ex+fx^2\right)\,dx = \text{Timed out}$$

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4343 vs. $2(312) = 624$.

Time = 0.32 (sec) , antiderivative size = 4343, normalized size of antiderivative = 10.10

$$\int f^{a+bx+cx^2} \sin^3\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f
)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*
e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3
/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f
*log(f)/(c^2*log(f)^2 + f^2)))*cos(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^2*d - 2*
b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + (-I*c^2*f^a*e^(1/4*b^2*c
*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) +
1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 - I*f^(a + 2)*e^(1/4*b^2*c*
log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1
```

```
/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*sin(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^
2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log
(f) - 3*I*f)*x + b*log(f) - 3*I*e)*sqrt(-c*log(f) + 3*I*f)/(c*log(f) - 3*I*
f)) + ((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(
f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2
 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/
(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*cos(-3/4*(
9*e^2*f - 36*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 +
9*f^2)) + (I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2
*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log
(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*
log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*sin
(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(
f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + b*log(f) + 3*I*e)*sqrt(-c
*log(f) - 3*I*f)/(c*log(f) + 3*I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 9
*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((c^2*f^a*e^(1/4*b^2*c*lo
g(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2) + 9/2
*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*l
og(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2) + 9/
2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*
d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (-I*c^2*f^a*e^(1/4*b
^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2
) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - 9*I*f^(a + 2)*e^(1/
4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 +
f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*sin(-1/4*(e^2*f - 4*d*f^2
- (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(
c*log(f) - I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I*f)/(c*log(f) - I*f))
 + ((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f
)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2
+ 9*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(
f)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*cos(-1/
4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 +
f^2)) + (I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2
*log(f)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log
(f)^2 + 9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*
e^2*log(f)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))
*sin(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log
(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(
f) - I*f)/(c*log(f) + I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) + sq
rt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^
2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log
(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2
*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(
f)/(c^2*log(f)^2 + f^2)))*cos(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^2*d - 2*b*c*e
 + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^
```

3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*
f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(
c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*l
og(f)/(c^2*log(f)^2 + f^2)))*sin(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^2*d - 2*b*
c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*
f)*x + b*log(f) - 3*I*e)*sqrt(-c*log(f) + 3*I*f)/(c*log(f) - 3*I*f)) + ((-I
*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2
*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 - I*f^
(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*
log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*cos(-3/4*(9*e^2
*f - 36*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2
)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)
/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 +
f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c
^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*sin(-3/4*(9*
e^2*f - 36*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*
f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + b*log(f) + 3*I*e)*sqrt(-c*log(f) -
3*I*f)/(c*log(f) + 3*I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) -
3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((I*c^2*f^a*e^(1/4*b^2*c*log(f)^3
/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f
*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*I*f^(a + 2)*e^(1/4*b^2*c*log(f
)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2) + 9/2*b*
e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d -
2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (c^2*f^a*e^(1/4*b^2*c*lo
g(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2) + 9/2
*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*l
og(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2) + 9/
2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*sin(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*
d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f)
- I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I*f)/(c*log(f) - I*f)) + ((-I*c
^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2
*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - 9*I*
f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(
c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*cos(-1/4*(e
^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)
) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f
)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2
+ 9*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(
f)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*sin(-1/
4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 +
f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f) - I*f
)/(c*log(f) + I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*e^(1/
4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f)^2
+ f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*lo
g(f)^2 + f^2))*log(f)^4 + 10*c^2*f^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 +

```
9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*lo
g(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^4*e
^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f
)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^
2*log(f)^2 + f^2)))
```

Giac [F]

$$\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+bx+a}\sin\left(fx^2+ex+d\right)^3\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\,\sin\left(f\,x^2+e\,x+d\right)^3\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2)^3,x)
[Out] int(f^(a + b*x + c*x^2)*sin(d + e*x + f*x^2)^3, x)
```

3.103 $\int f^{a+bx+cx^2} \sin(a+bx+ex^2)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 213

$$\int f^{a+bx+cx^2}\sin(a+bx+ex^2)\,dx = \frac{ie^{-\left((i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)}\sqrt{\pi}\,\mathrm{erf}\left(\frac{b(i-\log(f))+2x(ie-c\log(f))}{2\sqrt{ie-c\log(f)}}\right)}{4\sqrt{ie-c\log(f)}} - \frac{ie^{(i+\log(f))\left(a-\frac{b^2(i+\log(f))}{4ie+4c\log(f)}\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b(i+\log(f))+2x(ie+c\log(f))}{2\sqrt{ie+c\log(f)}}\right)}{4\sqrt{ie+c\log(f)}}$$

```
[Out] -1/4*I*erf(1/2*(-b*(I-ln(f))-2*x*(I*e-c*ln(f)))/(I*e-c*ln(f))^(1/2))*Pi^(1/
2)/exp((I-ln(f))*(a-b^2*(I-ln(f))/(4*I*e-4*c*ln(f))))/(I*e-c*ln(f))^(1/2)-1
/4*I*exp((I+ln(f))*(a-b^2*(I+ln(f))/(4*I*e+4*c*ln(f))))*erfi(1/2*(b*(I+ln(f
))+2*x*(I*e+c*ln(f)))/(I*e+c*ln(f))^(1/2))*Pi^(1/2)/(I*e+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4560, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\sin(a+bx+ex^2)\,dx = \frac{i\sqrt{\pi}\exp\left(-\left((-\log(f)+i)\left(a-\frac{b^2(-\log(f)+i)}{-4c\log(f)+4ie}\right)\right)\right)\mathrm{erf}\left(\frac{b(-\log(f)+i)+2x(-c\log(f)+ie)}{2\sqrt{-c\log(f)+ie}}\right)}{4\sqrt{-c\log(f)+ie}} - \frac{i\sqrt{\pi}\exp\left((\log(f)+i)\left(a-\frac{b^2(\log(f)+i)}{4c\log(f)+4ie}\right)\right)\mathrm{erfi}\left(\frac{b(\log(f)+i)+2x(c\log(f)+ie)}{2\sqrt{c\log(f)+ie}}\right)}{4\sqrt{c\log(f)+ie}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sin[a + b*x + e*x^2],x]
```

```
[Out] ((I/4)*Sqrt[Pi]*Erf[(b*(I - Log[f]) + 2*x*(I*e - c*Log[f]))/(2*Sqrt[I*e - c
*Log[f]])])/(E^((I - Log[f])*(a - (b^2*(I - Log[f]))/((4*I)*e - 4*c*Log[f])
))*Sqrt[I*e - c*Log[f]]) - ((I/4)*E^((I + Log[f])*(a - (b^2*(I + Log[f]))/(
(4*I)*e + 4*c*Log[f])))*Sqrt[Pi]*Erfi[(b*(I + Log[f]) + 2*x*(I*e + c*Log[f]
))/(2*Sqrt[I*e + c*Log[f]])])/Sqrt[I*e + c*Log[f]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2}ie^{-ia-ibx-iex^2}f^{a+bx+cx^2}-\frac{1}{2}ie^{ia+ibx+iex^2}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}i\int e^{-ia-ibx-iex^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}i\int e^{ia+ibx+iex^2}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}i\int \exp\left(-a(i-\log(f))-bx(i-\log(f))-x^2(ie-c\log(f))\right)dx\\ &\quad -\frac{1}{2}i\int \exp\left(a(i+\log(f))+bx(i+\log(f))+x^2(ie+c\log(f))\right)dx\end{aligned}$$

$$= \frac{1}{2}\left(i\exp\left(-\left((i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)\right)\right)\int\exp\left(\frac{(-b(i-\log(f))+2x(-ie+c\log(f$$
$$-\frac{1}{2}\left(i\exp\left((i+\log(f))\left(a-\frac{b^2(i+\log(f))}{4ie+4c\log(f)}\right)\right)\right)\int\exp\left(\frac{(b(i+\log(f))+2x(ie+c\log(f)))^2}{4(ie+c\log(f))}\right)$$
$$= \frac{i\exp\left(-\left((i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)\right)\sqrt{\pi}\operatorname{erf}\left(\frac{b(i-\log(f))+2x(ie-c\log(f))}{2\sqrt{ie-c\log(f)}}\right)}{4\sqrt{ie-c\log(f)}}$$
$$-\frac{i\exp\left((i+\log(f))\left(a-\frac{b^2(i+\log(f))}{4ie+4c\log(f)}\right)\right)\sqrt{\pi}\operatorname{erfi}\left(\frac{b(i+\log(f))+2x(ie+c\log(f))}{2\sqrt{ie+c\log(f)}}\right)}{4\sqrt{ie+c\log(f)}}$$

Mathematica [A] (warning: unable to verify)

Time = 1.61 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.52

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx$$
$$= \frac{e^{-\frac{b^2c\log^3(f)}{2(e^2+c^2\log^2(f))}}f^{a-\frac{b^2}{2(e-ic\log(f))}}\sqrt{\pi}\left(-e^{\frac{1}{4}b^2\left(\frac{1}{-ie+c\log(f)}+\frac{\log^2(f)}{ie+c\log(f)}\right)}f^{\frac{ib^2c\log(f)}{e^2+c^2\log^2(f)}}\operatorname{erfi}\left(\frac{-i(b+2ex)+(b+2cx)\log(f)}{2\sqrt{-ie+c\log(f)}}\right)(e-ic\right.}{}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[a + b*x + e*x^2],x]
```

```
[Out] (f^(a - b^2/(2*(e - I*c*Log[f])))*Sqrt[Pi]*(-(E^((b^2*(((-I)*e + c*Log[f])^
(-1) + Log[f]^2/(I*e + c*Log[f])))/4)*f^((I*b^2*c*Log[f])/(e^2 + c^2*Log[f]
^2))*Erfi[((-I)*(b + 2*e*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[(-I)*e + c*Log[f]
])]*(e - I*c*Log[f])*Sqrt[(-I)*e + c*Log[f]]*(Cos[a] - I*Sin[a])) + E^((b^2
*(Log[f]^2/((-I)*e + c*Log[f]) + (I*e + c*Log[f])^(-1)))/4)*Erfi[((-I)*(b +
 2*e*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[I*e + c*Log[f]])]*(e + I*c*Log[f])*Sq
rt[I*e + c*Log[f]]*(Cos[a] + I*Sin[a])))/(4*E^((b^2*c*Log[f]^3)/(2*(e^2 + c
^2*Log[f]^2)))*(e^2 + c^2*Log[f]^2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

method	result
risch	$\frac{i\sqrt{\pi}\,f^a\mathrm{e}^{\frac{-\ln(f)^2b^2+4i\ln(f)ac-2i\ln(f)b^2-4ae+b^2}{4ie+4c\ln(f)}}\operatorname{erf}\left(-\sqrt{-c\ln(f)-ie}\,x+\frac{b\ln(f)+ib}{2\sqrt{-c\ln(f)-ie}}\right)}{4\sqrt{-c\ln(f)-ie}}-\frac{i\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2+4i\ln(f)ac-2i\ln(f)b^2}{4(c\ln(f)-ie)}}}{4\sqrt{}}$

```
[In] int(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*(-ln(f)^2*b^2+4*I*ln(f)*a*c-2*I*ln(f)*b^2-4*a*e+
b^2)/(I*e+c*ln(f)))/(-c*ln(f)-I*e)^(1/2)*erf(-(-c*ln(f)-I*e)^(1/2)*x+1/2*(b
*ln(f)+I*b)/(-c*ln(f)-I*e)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+
4*I*ln(f)*a*c-2*I*ln(f)*b^2+4*a*e-b^2)/(c*ln(f)-I*e))/(I*e-c*ln(f))^(1/2)*e
rf(-(I*e-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*b)/(I*e-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(153) = 306$.

Time = 0.26 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.78

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx$$

$$= \frac{\sqrt{\pi}(i\,c\log\left(f\right)+e)\sqrt{-c\log\left(f\right)-i\,e}\,\text{erf}\left(\frac{\left(2\,e^2x+\left(2\,c^2x+bc\right)\log(f)^2+be+\left(i\,bc-i\,be\right)\log(f)\right)\sqrt{-c\log(f)-i\,e}}{2\left(c^2\log(f)^2+e^2\right)}\right)e^{\left(-\frac{\left(b^2c-4\,ac^2\right)\log}{}\right.}}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + e)*sqrt(-c*log(f) - I*e)*erf(1/2*(2*e^2*x + (2*
c^2*x + b*c)*log(f)^2 + b*e + (I*b*c - I*b*e)*log(f))*sqrt(-c*log(f) - I*e)
/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*b^2*e - 4*I*
a*e^2 - (-2*I*b^2*c + 4*I*a*c^2 + I*b^2*e)*log(f)^2 - (b^2*c - 2*b^2*e + 4*
a*e^2)*log(f))/(c^2*log(f)^2 + e^2)) + sqrt(pi)*(-I*c*log(f) + e)*sqrt(-c*l
og(f) + I*e)*erf(1/2*(2*e^2*x + (2*c^2*x + b*c)*log(f)^2 + b*e + (-I*b*c +
I*b*e)*log(f))*sqrt(-c*log(f) + I*e)/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c
- 4*a*c^2)*log(f)^3 - I*b^2*e + 4*I*a*e^2 - (2*I*b^2*c - 4*I*a*c^2 - I*b^2*
e)*log(f)^2 - (b^2*c - 2*b^2*e + 4*a*e^2)*log(f))/(c^2*log(f)^2 + e^2)))/(c
^2*log(f)^2 + e^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx = \int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sin(e*x**2+b*x+a),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(a + b*x + e*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 1054, normalized size of antiderivative = 4.95

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*((f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(-I*cos(1/2*arctan2(e
, -c*log(f))) + sin(1/2*arctan2(e, -c*log(f))))*cos(-1/4*(b^2*e - 4*a*e^2 +
 (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)) + f^(1/4*b^2*c
/(c^2*log(f)^2 + e^2))*f^a*(cos(1/2*arctan2(e, -c*log(f))) + I*sin(1/2*arct
an2(e, -c*log(f))))*sin(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)
*log(f)^2)/(c^2*log(f)^2 + e^2)))*erf(x*conjugate(sqrt(-c*log(f) + I*e)) -
1/2*(b*log(f) + I*b)*conjugate(1/sqrt(-c*log(f) + I*e))) + (f^(1/4*b^2*c/(c
^2*log(f)^2 + e^2))*f^a*(I*cos(1/2*arctan2(e, -c*log(f))) + sin(1/2*arctan2
(e, -c*log(f))))*cos(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)*lo
g(f)^2)/(c^2*log(f)^2 + e^2)) + f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(cos
(1/2*arctan2(e, -c*log(f))) - I*sin(1/2*arctan2(e, -c*log(f))))*sin(-1/4*(b
^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)
))*erf(x*conjugate(sqrt(-c*log(f) - I*e)) - 1/2*(b*log(f) - I*b)*conjugate(
1/sqrt(-c*log(f) - I*e))) + (f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(I*cos(
1/2*arctan2(e, -c*log(f))) + sin(1/2*arctan2(e, -c*log(f))))*cos(-1/4*(b^2*
e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)) +
 f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(cos(1/2*arctan2(e, -c*log(f))) - I
*sin(1/2*arctan2(e, -c*log(f))))*sin(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a
*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)))*erf(1/2*(2*(c*log(f) - I*e)*
x + b*log(f) - I*b)*sqrt(-c*log(f) + I*e)/(c*log(f) - I*e)) + (f^(1/4*b^2*c
/(c^2*log(f)^2 + e^2))*f^a*(-I*cos(1/2*arctan2(e, -c*log(f))) + sin(1/2*arc
tan2(e, -c*log(f))))*cos(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e
)*log(f)^2)/(c^2*log(f)^2 + e^2)) + f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*
(cos(1/2*arctan2(e, -c*log(f))) + I*sin(1/2*arctan2(e, -c*log(f))))*sin(-1/
4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 +
e^2)))*erf(1/2*(2*(c*log(f) + I*e)*x + b*log(f) + I*b)*sqrt(-c*log(f) - I*e
)/(c*log(f) + I*e)))*e^(-1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + e^2) - 1/2*b^2*
e*log(f)/(c^2*log(f)^2 + e^2))/(c^2*log(f)^2 + e^2)^(1/4)
```

Giac [F]

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx=\int f^{cx^2+bx+a}\sin\left(ex^2+bx+a\right)\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*sin(e*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\,\sin\left(e\,x^2+b\,x+a\right)\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sin(a + b*x + e*x^2),x)
[Out] int(f^(a + b*x + c*x^2)*sin(a + b*x + e*x^2), x)
```

3.104 $\int e^x \cos(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int e^x \cos(a+bx)\,dx = \frac{e^x \cos(a+bx)}{1+b^2} + \frac{be^x \sin(a+bx)}{1+b^2}$$

```
[Out] exp(x)*cos(b*x+a)/(b^2+1)+b*exp(x)*sin(b*x+a)/(b^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{4518\}$

$$\int e^x \cos(a+bx)\,dx = \frac{be^x \sin(a+bx)}{b^2+1} + \frac{e^x \cos(a+bx)}{b^2+1}$$

```
[In] Int[E^x*Cos[a + b*x],x]
```

```
[Out] (E^x*Cos[a + b*x])/(1 + b^2) + (b*E^x*Sin[a + b*x])/(1 + b^2)
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = \frac{e^x \cos(a+bx)}{1+b^2} + \frac{be^x \sin(a+bx)}{1+b^2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int e^x \cos(a+bx)\,dx = \frac{e^x(\cos(a+bx)+b\sin(a+bx))}{1+b^2}$$

```
[In] Integrate[E^x*Cos[a + b*x],x]
[Out] (E^x*(Cos[a + b*x] + b*Sin[a + b*x]))/(1 + b^2)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{e^x(b\sin(xb+a)+\cos(xb+a))}{b^2+1}$	26
default	$\frac{e^x\cos(xb+a)}{b^2+1}+\frac{b\,e^x\sin(xb+a)}{b^2+1}$	35
risch	$\frac{ie^x(2i\cos(xb+a)+2ib\sin(xb+a))}{2(-b+i)(i+b)}$	40
norman	$\frac{\frac{e^x}{b^2+1}-\frac{e^x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{b^2+1}+\frac{2b\,e^x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b^2+1}}{1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}$	71

```
[In] int(exp(x)*cos(b*x+a),x,method=_RETURNVERBOSE)
[Out] exp(x)/(b^2+1)*(b*sin(b*x+a)+cos(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int e^x \cos(a+bx)\,dx = \frac{be^x\sin(bx+a)+\cos(bx+a)\,e^x}{b^2+1}$$

```
[In] integrate(exp(x)*cos(b*x+a),x, algorithm="fricas")
[Out] (b*e^x*sin(b*x + a) + cos(b*x + a)*e^x)/(b^2 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.17

$$\int e^x \cos(a+bx)\,dx = \begin{cases} -\frac{ixe^x \sin(a-ix)}{2} + \frac{xe^x \cos(a-ix)}{2} + \frac{ie^x \sin(a-ix)}{2} & \text{for } b=-i \\ \frac{ixe^x \sin(a+ix)}{2} + \frac{xe^x \cos(a+ix)}{2} + \frac{e^x \cos(a+ix)}{2} & \text{for } b=i \\ \frac{be^x \sin(a+bx)}{b^2+1} + \frac{e^x \cos(a+bx)}{b^2+1} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(x)*cos(b*x+a),x)
```

```
[Out] Piecewise((-I*x*exp(x)*sin(a - I*x)/2 + x*exp(x)*cos(a - I*x)/2 + I*exp(x)*
sin(a - I*x)/2, Eq(b, -I)), (I*x*exp(x)*sin(a + I*x)/2 + x*exp(x)*cos(a + I
*x)/2 + exp(x)*cos(a + I*x)/2, Eq(b, I)), (b*exp(x)*sin(a + b*x)/(b**2 + 1)
 + exp(x)*cos(a + b*x)/(b**2 + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int e^x \cos(a+bx)\,dx = \frac{(b\sin(bx+a) + \cos(bx+a))e^x}{b^2+1}$$

```
[In] integrate(exp(x)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (b*sin(b*x + a) + cos(b*x + a))*e^x/(b^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int e^x \cos(a+bx)\,dx = \left(\frac{b\sin(bx+a)}{b^2+1} + \frac{\cos(bx+a)}{b^2+1}\right)e^x$$

```
[In] integrate(exp(x)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] (b*sin(b*x + a)/(b^2 + 1) + cos(b*x + a)/(b^2 + 1))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int e^x \cos(a+bx)\,dx = \frac{e^x\,(\cos(a+b\,x)+b\,\sin(a+b\,x))}{b^2+1}$$

```
[In] int(cos(a + b*x)*exp(x),x)
[Out] (exp(x)*(cos(a + b*x) + b*sin(a + b*x)))/(b^2 + 1)
```

3.105 $\int e^x \cos\left(a+cx^2\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 115

$$\int e^x \cos\left(a+cx^2\right)\,dx = -\frac{\sqrt[4]{-1}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{\sqrt[4]{-1}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] -1/4*(-1)^(1/4)*exp(1/4*I*(4*a+1/c))*erf(1/2*(-1)^(1/4)*(1+2*I*c*x)/c^(1/2)
)*Pi^(1/2)/c^(1/2)+1/4*(-1)^(1/4)*erfi(1/2*(-1)^(1/4)*(1-2*I*c*x)/c^(1/2))*
Pi^(1/2)/exp(1/4*I*(4*a+1/c))/c^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4561, 2266, 2235, 2236}

$$\int e^x \cos\left(a+cx^2\right)\,dx = \frac{\sqrt[4]{-1}\sqrt{\pi}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} - \frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\mathrm{erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[In] Int[E^x*Cos[a + c*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((I/4)*(4*a + c^(-1)))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + (2*
I)*c*x))/(2*Sqrt[c])])/Sqrt[c] + ((-1)^(1/4)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 -
 (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]*E^((I/4)*(4*a + c^(-1))))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-ia+x-icx^2}+\frac{1}{2}e^{ia+x+icx^2}\right) dx \\
&= \frac{1}{2}\int e^{-ia+x-icx^2}\,dx+\frac{1}{2}\int e^{ia+x+icx^2}\,dx \\
&= \frac{1}{2}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\int e^{\frac{i(1-2icx)^2}{4c}}\,dx+\frac{1}{2}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\int e^{-\frac{i(1+2icx)^2}{4c}}\,dx \\
&= -\frac{\sqrt[4]{-1}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt[4]{-1}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int e^x \cos\left(a+cx^2\right)\,dx$$
$$=\frac{\sqrt[4]{-1}e^{-\frac{i}{4}/c}\sqrt{\pi}\left(-\text{erfi}\left(\frac{(-1)^{3/4}(i+2cx)}{2\sqrt{c}}\right)(\cos(a)-i\sin(a))+e^{\frac{i}{2}/c}\text{erfi}\left(\frac{\sqrt[4]{-1}(-i+2cx)}{2\sqrt{c}}\right)(-i\cos(a)+\sin(a))\right)}{4\sqrt{c}}$$

```
[In] Integrate[E^x*Cos[a + c*x^2],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[Pi]*(-(Erfi[((-1)^(3/4)*(I + 2*c*x))/(2*Sqrt[c])]*(Cos[a]
- I*Sin[a])) + E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + 2*c*x))/(2*Sqrt[c])]*((-I
)*Cos[a] + Sin[a])))/(4*Sqrt[c]*E^((I/4)/c))
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{\sqrt{\pi}\,e^{-\frac{i(4ac+1)}{4c}}\operatorname{erf}\left(\sqrt{ic}\,x-\frac{1}{2\sqrt{ic}}\right)}{4\sqrt{ic}}+\frac{\sqrt{\pi}\,e^{\frac{i(4ac+1)}{4c}}\operatorname{erf}\left(\sqrt{-ic}\,x-\frac{1}{2\sqrt{-ic}}\right)}{4\sqrt{-ic}}$	86

```
[In] int(exp(x)*cos(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*I*(4*a*c+1)/c)/(I*c)^(1/2)*erf((I*c)^(1/2)*x-1/2/(I*c
)^(1/2))+1/4*Pi^(1/2)*exp(1/4*I*(4*a*c+1)/c)/(-I*c)^(1/2)*erf((-I*c)^(1/2)*
x-1/2/(-I*c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(73) = 146.

Time = 0.26 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.26

$$\int e^x \cos\left(a+cx^2\right)\,dx$$
$$=\frac{\sqrt{2}\left(\pi\cos\left(\frac{4\,ac+1}{4\,c}\right)-i\,\pi\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\sqrt{\frac{c}{\pi}}\,C\left(\frac{\sqrt{2}(2\,cx+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-\sqrt{2}\left(\pi\cos\left(\frac{4\,ac+1}{4\,c}\right)+i\,\pi\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\sqrt{\frac{c}{\pi}}\,C\left(-\right.}{}$$

```
[In] integrate(exp(x)*cos(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(pi*cos(1/4*(4*a*c + 1)/c) - I*pi*sin(1/4*(4*a*c + 1)/c))*sqrt
(c/pi)*fresnel_cos(1/2*sqrt(2)*(2*c*x + I)*sqrt(c/pi)/c) - sqrt(2)*(pi*cos(
1/4*(4*a*c + 1)/c) + I*pi*sin(1/4*(4*a*c + 1)/c))*sqrt(c/pi)*fresnel_cos(-1
/2*sqrt(2)*(2*c*x - I)*sqrt(c/pi)/c) + sqrt(2)*(-I*pi*cos(1/4*(4*a*c + 1)/c
) - pi*sin(1/4*(4*a*c + 1)/c))*sqrt(c/pi)*fresnel_sin(1/2*sqrt(2)*(2*c*x +
I)*sqrt(c/pi)/c) + sqrt(2)*(-I*pi*cos(1/4*(4*a*c + 1)/c) + pi*sin(1/4*(4*a*
c + 1)/c))*sqrt(c/pi)*fresnel_sin(-1/2*sqrt(2)*(2*c*x - I)*sqrt(c/pi)/c))/c
```

Sympy [F]

$$\int e^x \cos \left(a+cx^2\right)\,dx=\int e^x \cos \left(a+cx^2\right)dx$$

```
[In] integrate(exp(x)*cos(c*x**2+a),x)
```

```
[Out] Integral(exp(x)*cos(a + c*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int e^x \cos \left(a+cx^2\right)\,dx=$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left((i-1)\,\cos\left(\frac{4\,ac+1}{4\,c}\right)+(i+1)\,\sin\left(\frac{4\,ac+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2i\,cx-1}{2\,\sqrt{i\,c}}\right)+\left((i+1)\,\cos\left(\frac{4\,ac+1}{4\,c}\right)+(i-1)\,\sin\left(\frac{4\,ac}{4\,c}\right.\right.}{8\,\sqrt{c}}$$

```
[In] integrate(exp(x)*cos(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*(((I - 1)*cos(1/4*(4*a*c + 1)/c) + (I + 1)*sin(1/4*(4
*a*c + 1)/c))*erf(1/2*(2*I*c*x - 1)/sqrt(I*c)) + ((I + 1)*cos(1/4*(4*a*c +
1)/c) + (I - 1)*sin(1/4*(4*a*c + 1)/c))*erf(1/2*(2*I*c*x + 1)/sqrt(-I*c)))/
sqrt(c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\int e^{x}\cos\left(a+cx^{2}\right)\,dx=-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{4}i\sqrt{2}\left(2x+\frac{i}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{4iac+i}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}+\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}i\sqrt{2}\left(2x-\frac{i}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-4iac-i}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

```
[In] integrate(exp(x)*cos(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(2)*sqrt(pi)*erf(1/4*I*sqrt(2)*(2*x + I/c)*(-I*c/abs(c) + 1)*sqr
t(abs(c)))*e^(-1/4*(4*I*a*c + I)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c))) + 1/4*
I*sqrt(2)*sqrt(pi)*erf(-1/4*I*sqrt(2)*(2*x - I/c)*(I*c/abs(c) + 1)*sqrt(abs
(c)))*e^(-1/4*(-4*I*a*c - I)/c)/((I*c/abs(c) + 1)*sqrt(abs(c)))
```

Mupad [F(-1)]

Timed out.

$$\int e^{x}\cos\left(a+cx^{2}\right)\,dx=\int \mathrm{e}^{x}\,\cos\left(c\,x^{2}+a\right)\,dx$$

```
[In] int(exp(x)*cos(a + c*x^2),x)
```

```
[Out] int(exp(x)*cos(a + c*x^2), x)
```

3.106 $\int e^x \cos(a+bx+cx^2)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 144

$$\int e^x \cos(a+bx+cx^2)\,dx = -\frac{\sqrt[4]{-1}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(1+ib+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{\sqrt[4]{-1}e^{-ia+\frac{i(i+b)^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(1-ib-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] -1/4*(-1)^(1/4)*exp(1/4*I*(4*a+(1+I*b)^2/c))*erf(1/2*(-1)^(1/4)*(1+I*b+2*I*
c*x)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/4*(-1)^(1/4)*exp(-I*a+1/4*I*(I+b)^2/c)*erf
i(1/2*(-1)^(1/4)*(1-I*b-2*I*c*x)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4561, 2266, 2235, 2236}

$$\int e^x \cos(a+bx+cx^2)\,dx = \frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{i(b+i)^2}{4c}-ia}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(-ib-2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}} - \frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(ib+2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[In] Int[E^x*Cos[a + b*x + c*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((I/4)*(4*a + (1 + I*b)^2/c))*Sqrt[Pi]*Erf[((-1)^(1/4)*(
1 + I*b + (2*I)*c*x))/(2*Sqrt[c])])/Sqrt[c] + ((-1)^(1/4)*E^((-I)*a + ((I/4
)*(I + b)^2)/c)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 - I*b - (2*I)*c*x))/(2*Sqrt[c]
)])/(4*Sqrt[c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-ia+(1-ib)x-icx^2}+\frac{1}{2}e^{ia+(1+ib)x+icx^2}\right)\,dx\\
&= \frac{1}{2}\int e^{-ia+(1-ib)x-icx^2}\,dx+\frac{1}{2}\int e^{ia+(1+ib)x+icx^2}\,dx\\
&= \frac{1}{2}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\int e^{-\frac{i(1+ib+2icx)^2}{4c}}\,dx+\frac{1}{2}e^{-\frac{i\left(1-2ib-b^2+4ac\right)}{4c}}\int e^{\frac{i(1-ib-2icx)^2}{4c}}\,dx\\
&= -\frac{\sqrt[4]{-1}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(1+ib+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}\\
&\quad+\frac{\sqrt[4]{-1}e^{-\frac{i\left(1-2ib-b^2+4ac\right)}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(1-ib-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94

$$\int e^x \cos\left(a+bx+cx^2\right)\,dx = \frac{\sqrt[4]{-1}e^{-\frac{i\left(1-2ib+b^2\right)}{4c}}\sqrt{\pi}\left(-e^{\frac{ib^2}{2c}}\operatorname{erfi}\left(\frac{(-1)^{3/4}(i+b+2cx)}{2\sqrt{c}}\right)(\cos(a)-i\sin(a))+e^{\frac{i}{2}/c}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(-i+b+2cx)}{2\sqrt{c}}\right)(-i\cos(a)\right.}{4\sqrt{c}}$$

```
[In] Integrate[E^x*Cos[a + b*x + c*x^2],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[Pi]*(-(E^(((I/2)*b^2)/c)*Erfi[((-1)^(3/4)*(I + b + 2*c*x))
/(2*Sqrt[c])]*(Cos[a] - I*Sin[a])) + E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + b +
 2*c*x))/(2*Sqrt[c])]*((-I)*Cos[a] + Sin[a])))/(4*Sqrt[c]*E^(((I/4)*(1 - (2
*I)*b + b^2))/c))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{i\left(4ac-b^2-2ib+1\right)}{4c}}\operatorname{erf}\left(\sqrt{ic}\,x-\frac{-ib+1}{2\sqrt{ic}}\right)}{4\sqrt{ic}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{i\left(4ac-b^2+2ib+1\right)}{4c}}\operatorname{erf}\left(-\sqrt{-ic}\,x+\frac{ib+1}{2\sqrt{-ic}}\right)}{4\sqrt{-ic}}$	117

```
[In] int(exp(x)*cos(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*I*(-b^2-2*I*b+4*a*c+1)/c)/(I*c)^(1/2)*erf((I*c)^(1/2)
*x-1/2*(-I*b+1)/(I*c)^(1/2))-1/4*Pi^(1/2)*exp(1/4*I*(-b^2+2*I*b+4*a*c+1)/c)
/(-I*c)^(1/2)*erf(-(-I*c)^(1/2)*x+1/2*(1+I*b)/(-I*c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(91) = 182.

Time = 0.26 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.59

$$\int e^x \cos\left(a+bx+cx^2\right)\,dx = \frac{\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{i\,b^2-4i\,ac-2\,b-i}{4\,c}\right)}\operatorname{C}\left(\frac{\sqrt{2}(2\,cx+b+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{-i\,b^2+4i\,ac-2\,b+i}{4\,c}\right)}\operatorname{C}\left(-\frac{\sqrt{2}(2\,cx+b-i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e\right.}{4\,c}$$

```
[In] integrate(exp(x)*cos(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/c)*fresnel_co
s(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) - sqrt(2)*pi*sqrt(c/pi)*e^(1/4*
(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_cos(-1/2*sqrt(2)*(2*c*x + b - I)*sq
rt(c/pi)/c) - I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/c)
*fresnel_sin(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) - I*sqrt(2)*pi*sqrt(
c/pi)*e^(1/4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_sin(-1/2*sqrt(2)*(2*c*
x + b - I)*sqrt(c/pi)/c))/c
```

Sympy [F]

$$\int e^x \cos\left(a+bx+cx^2\right)\,dx=\int e^x \cos\left(a+bx+cx^2\right)dx$$

```
[In] integrate(exp(x)*cos(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x)*cos(a + b*x + c*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91

$$\int e^x \cos\left(a+bx+cx^2\right)\,dx=$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i-1)\cos\left(-\frac{b^2-4\,ac-1}{4\,c}\right)-(i+1)\sin\left(-\frac{b^2-4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{i\,(2i\,cx+i\,b-1)\sqrt{i\,c}}{2\,c}\right)+\left((i+1)\cos\left(-\right.\right.\right.}{8\,\sqrt{c}}$$

```
[In] integrate(exp(x)*cos(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I - 1)*cos(-1/4*(b^2 - 4*a*c - 1)/c) - (I + 1)*si
n(-1/4*(b^2 - 4*a*c - 1)/c))*erf(1/2*I*(2*I*c*x + I*b - 1)*sqrt(I*c)/c) + (
(I + 1)*cos(-1/4*(b^2 - 4*a*c - 1)/c) + (I - 1)*sin(-1/4*(b^2 - 4*a*c - 1)/
c))*erf(1/2*I*(2*I*c*x + I*b + 1)*sqrt(-I*c)/c))*e^(-1/2*b/c)/sqrt(c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

$$\int e^x \cos\left(a+bx+cx^2\right) dx = \frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}i\sqrt{2}\left(2x+\frac{b-i}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2-4iac+2b-i}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}} - \frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{4}i\sqrt{2}\left(2x+\frac{b+i}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-ib^2+4iac+2b+i}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

```
[In] integrate(exp(x)*cos(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/4*I*sqrt(2)*(2*x + (b - I)/c)*(I*c/abs(c) + 1
)*sqrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c + 2*b - I)/c)/((I*c/abs(c) + 1)*sq
rt(abs(c))) - 1/4*I*sqrt(2)*sqrt(pi)*erf(1/4*I*sqrt(2)*(2*x + (b + I)/c)*(-
I*c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c + 2*b + I)/c)/((-I*
c/abs(c) + 1)*sqrt(abs(c)))
```

Mupad [F(-1)]

Timed out.

$$\int e^x \cos\left(a+bx+cx^2\right) dx = \int \mathrm{e}^x \cos\left(c\,x^2+b\,x+a\right) dx$$

```
[In] int(exp(x)*cos(a + b*x + c*x^2),x)
```

```
[Out] int(exp(x)*cos(a + b*x + c*x^2), x)
```

3.107 $\int e^{x^2} \cos(a + bx) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int e^{x^2} \cos(a + bx) \, dx = \frac{1}{4} e^{-ia+\frac{b^2}{4}} \sqrt{\pi} \operatorname{erfi}\left(\frac{1}{2}(-ib + 2x)\right) + \frac{1}{4} e^{ia+\frac{b^2}{4}} \sqrt{\pi} \operatorname{erfi}\left(\frac{1}{2}(ib + 2x)\right)$$

```
[Out] -1/4*exp(-I*a+1/4*b^2)*erfi(1/2*I*b-x)*Pi^(1/2)+1/4*exp(I*a+1/4*b^2)*erfi(1
/2*I*b+x)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4561, 2266, 2235}

$$\int e^{x^2} \cos(a + bx) \, dx = \frac{1}{4} \sqrt{\pi} e^{\frac{b^2}{4}-ia} \operatorname{erfi}\left(\frac{1}{2}(2x - ib)\right) + \frac{1}{4} \sqrt{\pi} e^{\frac{b^2}{4}+ia} \operatorname{erfi}\left(\frac{1}{2}(2x + ib)\right)$$

```
[In] Int[E^x^2*Cos[a + b*x],x]
```

```
[Out] (E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(I*a + b^2/4)*S
qrt[Pi]*Erfi[(I*b + 2*x)/2])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-ia-ibx+x^2}+\frac{1}{2}e^{ia+ibx+x^2}\right)dx \\
&= \frac{1}{2}\int e^{-ia-ibx+x^2}\,dx+\frac{1}{2}\int e^{ia+ibx+x^2}\,dx \\
&= \frac{1}{2}e^{-ia+\frac{b^2}{4}}\int e^{\frac{1}{4}(-ib+2x)^2}\,dx+\frac{1}{2}e^{ia+\frac{b^2}{4}}\int e^{\frac{1}{4}(ib+2x)^2}\,dx \\
&= \frac{1}{4}e^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int e^{x^2}\cos(a+bx)\,dx = \frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\left(\cos(a)\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\cos(a)\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)\right.$$
$$\left.-\left(\mathrm{erf}\left(\frac{b}{2}-ix\right)+\mathrm{erf}\left(\frac{b}{2}+ix\right)\right)\sin(a)\right)$$

```
[In] Integrate[E^x^2*Cos[a + b*x],x]
[Out] (E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erfi[((-I)*b + 2*x)/2] + Cos[a]*Erfi[(I*b + 2*x
)/2] - (Erf[b/2 - I*x] + Erf[b/2 + I*x])*Sin[a]))/4
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(ix+\frac{b}{2}\right)}{4}+\frac{i\sqrt{\pi}\,\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(-ix+\frac{b}{2}\right)}{4}$	54

```
[In] int(exp(x^2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(1/4*b^2)*exp(-I*a)*erf(I*x+1/2*b)+1/4*I*Pi^(1/2)*exp(1/
4*b^2)*exp(I*a)*erf(-I*x+1/2*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60

$$\int e^{x^2}\cos(a+bx)\,dx=\frac{1}{4}\sqrt{\pi}\left(-i\operatorname{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2+ia\right)}-i\operatorname{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2-ia\right)}\right)$$

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(-I*erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - I*erf(1/2*b + I*x)*e
^(1/4*b^2 - I*a))
```

Sympy [F]

$$\int e^{x^2}\cos(a+bx)\,dx=\int e^{x^2}\cos(a+bx)\,dx$$

```
[In] integrate(exp(x**2)*cos(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cos(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int e^{x^2}\cos(a+bx)\,dx=$$
$$-\frac{1}{4}\sqrt{\pi}\left((i\cos(a)+\sin(a))\operatorname{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}+(i\cos(a)-\sin(a))\operatorname{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}\right)$$

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*((I*cos(a) + sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) + (I*cos(a)
 - sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Giac [F]

$$\int e^{x^2} \cos(a+bx)\,dx = \int \cos\left(bx+a\right)e^{\left(x^2\right)}\,dx$$

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*e^(x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \cos(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\mathrm{e}^{x^2}\,dx$$

```
[In] int(cos(a + b*x)*exp(x^2),x)
[Out] int(cos(a + b*x)*exp(x^2), x)
```

3.108 $\int e^{x^2} \cos(a + cx^2) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int e^{x^2} \cos(a + cx^2) \, dx = \frac{e^{-ia}\sqrt{\pi}\text{erfi}\left(\sqrt{1-ic}x\right)}{4\sqrt{1-ic}} + \frac{e^{ia}\sqrt{\pi}\text{erfi}\left(\sqrt{1+ic}x\right)}{4\sqrt{1+ic}}$$

```
[Out] 1/4*erfi(x*(1-I*c)^(1/2))*Pi^(1/2)/exp(I*a)/(1-I*c)^(1/2)+1/4*exp(I*a)*erfi
(x*(1+I*c)^(1/2))*Pi^(1/2)/(1+I*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4561, 2235}

$$\int e^{x^2} \cos(a + cx^2) \, dx = \frac{\sqrt{\pi}e^{-ia}\text{erfi}\left(\sqrt{1-ic}x\right)}{4\sqrt{1-ic}} + \frac{\sqrt{\pi}e^{ia}\text{erfi}\left(\sqrt{1+ic}x\right)}{4\sqrt{1+ic}}$$

```
[In] Int[E^x^2*Cos[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erfi[Sqrt[1 - I*c]*x])/(4*Sqrt[1 - I*c]*E^(I*a)) + (E^(I*a)*Sqrt[
Pi]*Erfi[Sqrt[1 + I*c]*x])/(4*Sqrt[1 + I*c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2}e^{-ia+(1-ic)x^2}+\frac{1}{2}e^{ia+(1+ic)x^2}\right)dx\\ &= \frac{1}{2}\int e^{-ia+(1-ic)x^2}\,dx+\frac{1}{2}\int e^{ia+(1+ic)x^2}\,dx\\ &= \frac{e^{-ia}\sqrt{\pi}\,\text{erfi}\left(\sqrt{1-ic}\,x\right)}{4\sqrt{1-ic}}+\frac{e^{ia}\sqrt{\pi}\,\text{erfi}\left(\sqrt{1+ic}\,x\right)}{4\sqrt{1+ic}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.29

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx$$
$$= \frac{\sqrt[4]{-1}\sqrt{\pi}\left(-\left((-i+c)\sqrt{i+c}\,\text{erfi}\left((-1)^{3/4}\sqrt{i+c}\,x\right)(\cos(a)-i\sin(a))\right)+(1-ic)\sqrt{-i+c}\,\text{erfi}\left(\sqrt[4]{-1}\sqrt{-i+c}\right.\right.}{4\left(1+c^2\right)}$$

```
[In] Integrate[E^x^2*Cos[a + c*x^2],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[Pi]*(-((-I + c)*Sqrt[I + c]*Erfi[(-1)^(3/4)*Sqrt[I + c]*x]
*(Cos[a] - I*Sin[a])) + (1 - I*c)*Sqrt[-I + c]*Erfi[(-1)^(1/4)*Sqrt[-I + c]
*x]*(Cos[a] + I*Sin[a])))/(4*(1 + c^2))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-ia}\,\mathrm{erf}\left(\sqrt{ic-1}\,x\right)}{4\sqrt{ic-1}}+\frac{\sqrt{\pi}\,\mathrm{e}^{ia}\,\mathrm{erf}\left(\sqrt{-ic-1}\,x\right)}{4\sqrt{-ic-1}}$	60

```
[In] int(exp(x^2)*cos(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(-I*a)/(I*c-1)^(1/2)*erf((I*c-1)^(1/2)*x)+1/4*Pi^(1/2)*exp(
I*a)/(-I*c-1)^(1/2)*erf((-I*c-1)^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx = \frac{\sqrt{\pi}((-i\,c-1)\cos(a)-(c-i)\sin(a))\sqrt{i\,c-1}\,\mathrm{erf}\left(\sqrt{i\,c-1}x\right)+\sqrt{\pi}((i\,c-1)\cos(a)-(c+i)\sin(a))\sqrt{-}}{4\left(c^2+1\right)}$$

```
[In] integrate(exp(x^2)*cos(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*((-I*c - 1)*cos(a) - (c - I)*sin(a))*sqrt(I*c - 1)*erf(sqrt(I
*c - 1)*x) + sqrt(pi)*((I*c - 1)*cos(a) - (c + I)*sin(a))*sqrt(-I*c - 1)*er
f(sqrt(-I*c - 1)*x))/(c^2 + 1)
```

Sympy [F]

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx=\int e^{x^2}\cos\left(a+cx^2\right)dx$$

```
[In] integrate(exp(x**2)*cos(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*cos(a + c*x**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(53) = 106.

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.60

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx = -\frac{\sqrt{\pi}\sqrt{2\,c^2+2}\left((i\,\cos(a)+\sin(a))\,\mathrm{erf}\left(\sqrt{i\,c-1}x\right)+(-i\,\cos(a)+\sin(a))\,\mathrm{erf}\left(\sqrt{-i\,c-1}x\right)\right)\sqrt{\sqrt{c^2+1}}}{8}$$

```
[In] integrate(exp(x^2)*cos(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(pi)*sqrt(2*c^2 + 2)*((I*cos(a) + sin(a))*erf(sqrt(I*c - 1)*x) +
(-I*cos(a) + sin(a))*erf(sqrt(-I*c - 1)*x))*sqrt(sqrt(c^2 + 1) + 1) - sqrt(
pi)*sqrt(2*c^2 + 2)*((cos(a) - I*sin(a))*erf(sqrt(I*c - 1)*x) + (cos(a) + I
*sin(a))*erf(sqrt(-I*c - 1)*x))*sqrt(sqrt(c^2 + 1) - 1))/(c^2 + 1)
```

Giac [F]

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx=\int\cos\left(cx^2+a\right)e^{\left(x^2\right)}\,dx$$

```
[In] integrate(exp(x^2)*cos(c*x^2+a),x, algorithm="giac")
[Out] integrate(cos(c*x^2 + a)*e^(x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\cos\left(a+cx^2\right)\,dx=\int e^{x^2}\cos\left(c\,x^2+a\right)\,dx$$

```
[In] int(exp(x^2)*cos(a + c*x^2),x)
[Out] int(exp(x^2)*cos(a + c*x^2), x)
```

3.109 $\int e^{x^2} \cos(a+bx+cx^2)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 151

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)\,dx=-\frac{e^{-i\left(a-\frac{b^2}{4i+4c}\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}+\frac{e^{ia+\frac{b^2}{4(1+ic)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}$$

```
[Out] -1/4*erfi(1/2*(I*b-2*(1-I*c)*x)/(1-I*c)^(1/2))*Pi^(1/2)/exp(I*(a-b^2/(4*I+4
*c)))/(1-I*c)^(1/2)+1/4*exp(I*a+1/4*b^2/(1+I*c))*erfi(1/2*(I*b+2*(1+I*c)*x)
/(1+I*c)^(1/2))*Pi^(1/2)/(1+I*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4561, 2266, 2235}

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)\,dx=\frac{\sqrt{\pi}e^{ia+\frac{b^2}{4(1+ic)}}\,\mathrm{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}-\frac{\sqrt{\pi}e^{-i\left(a-\frac{b^2}{4c+4i}\right)}\,\mathrm{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}$$

```
[In] Int[E^x^2*Cos[a + b*x + c*x^2],x]
```

```
[Out] -1/4*(Sqrt[Pi]*Erfi[(I*b - 2*(1 - I*c)*x)/(2*Sqrt[1 - I*c])])/(Sqrt[1 - I*c
]*E^(I*(a - b^2/(4*I + 4*c)))) + (E^(I*a + b^2/(4*(1 + I*c)))*Sqrt[Pi]*Erfi
[(I*b + 2*(1 + I*c)*x)/(2*Sqrt[1 + I*c])])/(4*Sqrt[1 + I*c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-ia-ibx+(1-ic)x^2}+\frac{1}{2}e^{ia+ibx+(1+ic)x^2}\right)\,dx\\
&=\frac{1}{2}\int e^{-ia-ibx+(1-ic)x^2}\,dx+\frac{1}{2}\int e^{ia+ibx+(1+ic)x^2}\,dx\\
&=\frac{1}{2}e^{ia+\frac{b^2}{4(1+ic)}}\int \exp\left(\frac{(ib+2(1+ic)x)^2}{4(1+ic)}\right)\,dx+\frac{1}{2}e^{-i\left(a-\frac{b^2}{4i+4c}\right)}\int \exp\left(\frac{(-ib+2(1-ic)x)^2}{4(1-ic)}\right)\,dx\\
&=-\frac{e^{-i\left(a-\frac{b^2}{4i+4c}\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}+\frac{e^{ia+\frac{b^2}{4(1+ic)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.10

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)\,dx$$

$$=\frac{\sqrt[4]{-1}e^{\frac{ib^2}{4i-4c}}\sqrt{\pi}\left(-\left((-i+c)\sqrt{i+c}e^{\frac{ib^2c}{2+2c^2}}\mathrm{erfi}\left(\frac{(-1)^{3/4}(b+2(i+c)x)}{2\sqrt{i+c}}\right)(\cos(a)-i\sin(a))\right)+\sqrt{-i+c}(i+c)\mathrm{erfi}\left(\cdots\right.\right.}{4\left(1+c^2\right)}$$

```
[In] Integrate[E^x^2*Cos[a + b*x + c*x^2],x]
```

```
[Out] ((-1)^(1/4)*E^((I*b^2)/(4*I - 4*c))*Sqrt[Pi]*(-((-I + c)*Sqrt[I + c]*E^((I*
b^2*c)/(2 + 2*c^2))*Erfi[((-1)^(3/4)*(b + 2*(I + c)*x))/(2*Sqrt[I + c])]*(C
os[a] - I*Sin[a])) + Sqrt[-I + c]*(I + c)*Erfi[((-1)^(1/4)*(b + 2*(-I + c)*
x))/(2*Sqrt[-I + c])]*((-I)*Cos[a] + Sin[a])))/(4*(1 + c^2))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac+4ia-b^2}{4ic-4}}\,\mathrm{erf}\left(\sqrt{ic-1}\,x+\frac{ib}{2\sqrt{ic-1}}\right)}{4\sqrt{ic-1}}-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-4ia-b^2}{4(ic+1)}}\,\mathrm{erf}\left(-\sqrt{-ic-1}\,x+\frac{ib}{2\sqrt{-ic-1}}\right)}{4\sqrt{-ic-1}}$	127

```
[In] int(exp(x^2)*cos(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(1/4*(4*a*c+4*I*a-b^2)/(I*c-1))/(I*c-1)^(1/2)*erf((I*c-1)^(
1/2)*x+1/2*I*b/(I*c-1)^(1/2))-1/4*Pi^(1/2)*exp(-1/4*(4*a*c-4*I*a-b^2)/(1+I*
c))/(-I*c-1)^(1/2)*erf(-(-I*c-1)^(1/2)*x+1/2*I*b/(-I*c-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.09

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}(i\,c+1)\sqrt{i\,c-1}\,\mathrm{erf}\left(-\frac{(bc+2\,(c^2+1)x-i\,b)\sqrt{i\,c-1}}{2\,(c^2+1)}\right)e^{\left(\frac{i\,b^2c-4i\,ac^2+b^2-4i\,a}{4\,(c^2+1)}\right)}+\sqrt{\pi}(i\,c-1)\sqrt{-i\,c-1}\,\mathrm{erf}\left(\frac{(bc+2\,(c^2\cdots}{\cdots}\right.}{4\,(c^2+1)}$$

```
[In] integrate(exp(x^2)*cos(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c + 1)*sqrt(I*c - 1)*erf(-1/2*(b*c + 2*(c^2 + 1)*x - I*b)*
sqrt(I*c - 1)/(c^2 + 1))*e^(1/4*(I*b^2*c - 4*I*a*c^2 + b^2 - 4*I*a)/(c^2 +
1)) + sqrt(pi)*(I*c - 1)*sqrt(-I*c - 1)*erf(1/2*(b*c + 2*(c^2 + 1)*x + I*b)
*sqrt(-I*c - 1)/(c^2 + 1))*e^(1/4*(-I*b^2*c + 4*I*a*c^2 + b^2 + 4*I*a)/(c^2
 + 1)))/(c^2 + 1)
```

Sympy [F]

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)\,dx=\int e^{x^2}\cos\left(a+bx+cx^2\right)dx$$

```
[In] integrate(exp(x**2)*cos(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cos(a + b*x + c*x**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(101) = 202.

Time = 0.22 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.14

$$\int e^{x^2} \cos \left(a+bx+cx^2\right) \, dx$$
$$= \frac{\sqrt{\pi}\sqrt{2\,c^2+2}\left(\left(\left(-i\,\cos\left(-\frac{b^2c-4\,ac^2-4\,a}{4\,(c^2+1)}\right)e^{\left(\frac{b^2}{4\,(c^2+1)}\right)} - e^{\left(\frac{b^2}{4\,(c^2+1)}\right)}\sin\left(-\frac{b^2c-4\,ac^2-4\,a}{4\,(c^2+1)}\right)\right)\operatorname{erf}\left(-\frac{2\,(-i\,c+1)x-i\,b}{2\,\sqrt{i\,c-1}}\right)\right.\right.}{}$$

```
[In] integrate(exp(x^2)*cos(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2 + 2)*((-I*cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 +
1))*e^(1/4*b^2/(c^2 + 1)) - e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2
 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c + 1)*x - I*b)/sqrt(I*c - 1)) + (-I*co
s(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)) + e^(1/4*b^
2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c
 - 1)*x - I*b)/sqrt(-I*c - 1)))*sqrt(sqrt(c^2 + 1) + 1) + sqrt(pi)*sqrt(2*c
^2 + 2)*((cos(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1))
 - I*e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf
(-1/2*(2*(-I*c + 1)*x - I*b)/sqrt(I*c - 1)) - (cos(-1/4*(b^2*c - 4*a*c^2 -
4*a)/(c^2 + 1))*e^(1/4*b^2/(c^2 + 1)) + I*e^(1/4*b^2/(c^2 + 1))*sin(-1/4*(b
^2*c - 4*a*c^2 - 4*a)/(c^2 + 1)))*erf(-1/2*(2*(-I*c - 1)*x - I*b)/sqrt(-I*c
 - 1)))*sqrt(sqrt(c^2 + 1) - 1))/(c^2 + 1)
```

Giac [F]

$$\int e^{x^2} \cos \left(a+bx+cx^2\right) \, dx = \int \cos \left(cx^2+bx+a\right) e^{\left(x^2\right)} \, dx$$

```
[In] integrate(exp(x^2)*cos(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cos(c*x^2 + b*x + a)*e^(x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \cos \left(a+bx+cx^2\right) \, dx=\int \mathrm{e}^{x^2}\,\cos \left(c\,x^2+b\,x+a\right) \,d x$$

```
[In] int(exp(x^2)*cos(a + b*x + c*x^2),x)
[Out] int(exp(x^2)*cos(a + b*x + c*x^2), x)
```

3.110 $\int f^{a+bx} \cos \left(d+f x^2\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 142

$$\begin{aligned}\int f^{a+bx} \cos \left(d+f x^2\right) \, dx &= -\frac{1}{4}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right)\\&\quad-\frac{1}{4}\sqrt[4]{-1}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt[4]{-1}(2ifx-b\log(f))}{2\sqrt{f}}\right)\end{aligned}$$

```
[Out] -1/4*(-1)^(1/4)*exp(1/4*I*(4*d+b^2*ln(f)^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4
)*(2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-1/4*(-1)^(1/4)*f^(-1/2+a)*erfi(1/2*(-
1)^(1/4)*(2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)/exp(1/4*I*(4*d+b^2*ln(f)^2/f))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4561, 2325, 2266, 2235, 2236}

$$\begin{aligned}\int f^{a+bx} \cos \left(d+f x^2\right) \, dx &= -\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)\\&\quad-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+2ifx)}{2\sqrt{f}}\right)\end{aligned}$$

```
[In] Int[f^(a + b*x)*Cos[d + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*E
rf[((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])]) - ((-1)^(1/4)*f^(-1/2
+ a)*Sqrt[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(4*E^(
(I/4)*(4*d + (b^2*Log[f]^2)/f)))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-id-ifx^2}f^{a+bx}+\frac{1}{2}e^{id+ifx^2}f^{a+bx}\right)dx\\
&= \frac{1}{2}\int e^{-id-ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int e^{id+ifx^2}f^{a+bx}\,dx\\
&= \frac{1}{2}\int e^{-id-ifx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{2}\int e^{id+ifx^2+a\log(f)+bx\log(f)}\,dx\\
&= \frac{1}{2}\left(e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-2ifx+b\log(f))^2}{4f}}\,dx+\frac{1}{2}\left(e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{-\frac{i(2ifx+b\log(f))^2}{4f}}\,dx
\end{aligned}$$

$$= -\frac{1}{4}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right) - \frac{1}{4}\sqrt[4]{-1}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt[4]{-1}(2ifx-b\log(f))}{2\sqrt{f}}\right)$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int f^{a+bx}\cos\left(d+fx^2\right)\,dx = \frac{1}{4}\sqrt[4]{-1}e^{-\frac{ib^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(-\text{erfi}\left(\frac{(-1)^{3/4}(2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d)) + e^{\frac{ib^2\log^2(f)}{2f}}\text{erfi}\left(\frac{\sqrt[4]{-1}(2fx-ib\log(f))}{2\sqrt{f}}\right)(-i\cos(d)+\sin(d))\right)$$

```
[In] Integrate[f^(a + b*x)*Cos[d + f*x^2],x]
```

```
[Out] ((-1)^(1/4)*f^(-1/2 + a)*Sqrt[Pi]*(-(Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f]))
/(2*Sqrt[f])]*(Cos[d] - I*Sin[d])) + E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[((-1)^
(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*((-I)*Cos[d] + Sin[d])))/(4*E^(((I
/4)*b^2*Log[f]^2)/f))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{i\left(\ln(f)^2b^2+4df\right)}{4f}}\operatorname{erf}\left(-\sqrt{if}\,x+\frac{\ln(f)b}{2\sqrt{if}}\right)}{4\sqrt{if}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{i\left(\ln(f)^2b^2+4df\right)}{4f}}\operatorname{erf}\left(-\sqrt{-if}\,x+\frac{\ln(f)b}{2\sqrt{-if}}\right)}{4\sqrt{-if}}$	114

```
[In] int(f^(b*x+a)*cos(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^
(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*
d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(98) = 196.

Time = 0.26 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.87

$$\int f^{a+bx}\cos\left(d+fx^2\right)\,dx$$

$$=\frac{\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-ib^2\log(f)^2+4\,af\log(f)-4i\,df}{4\,f}\right)}\operatorname{C}\left(\frac{\sqrt{2}(2\,fx+ib\log(f))\sqrt{\frac{f}{\pi}}}{2\,f}\right)-\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{ib^2\log(f)^2+4\,af\log(f)+4i\,df}{4\,f}\right)}\operatorname{C}\left(-\frac{\sqrt{2}(2\,fx\cdots}{}\right.}{}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f
)/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - sqrt(2)*p
i*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_co
s(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)
*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqrt(
2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b
^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x -
I*b*log(f))*sqrt(f/pi)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\cos\left(d+fx^2\right)\,dx=\int f^{a+bx}\cos\left(d+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*cos(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + f*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04

$$\int f^{a+bx}\cos\left(d+fx^2\right)\,dx=$$

$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left((i-1)\,f^a\cos\left(\frac{b^2\log(f)^2+4\,df}{4\,f}\right)+(i+1)\,f^a\sin\left(\frac{b^2\log(f)^2+4\,df}{4\,f}\right)\right)\operatorname{erf}\left(\frac{2i\,fx-b\log(f)}{2\,\sqrt{i\,f}}\right)+\left((i+1)\,f^a\cdots\right.\right.}{8\,\sqrt{f}}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*(((I - 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f)/f) + (I
+ 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*f*x - b*log(f))/sq
rt(I*f)) + ((I + 1)*f^a*cos(1/4*(b^2*log(f)^2 + 4*d*f)/f) + (I - 1)*f^a*sin
(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*f*x + b*log(f))/sqrt(-I*f)))/s
qrt(f)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(98) = 196$.

Time = 0.32 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.11

$$\int f^{a+bx}\cos\left(d+fx^2\right)\,dx$$
$$= \frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}i\sqrt{2}\left(4x-\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)}{f}\right)\left(\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}-\frac{i\pi^2b^2}{8f}-\frac{\pi b^2\log(|f|)}{4f}+\right.}}{4\left(\frac{if}{|f|}+1\right)\sqrt{|f|}}$$
$$-\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{8}i\sqrt{2}\left(4x+\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}-\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}+\frac{i\pi^2b^2}{8f}+\frac{\pi b^2\log(|\right.}}{4\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*
b*log(abs(f)))/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/f
 + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs(
f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(
abs(f)) + I*d)/((I*f/abs(f) + 1)*sqrt(abs(f))) - 1/4*I*sqrt(2)*sqrt(pi)*erf
(1/8*I*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(-I*f/abs
(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))
*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs
(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) - I*d)/((-I*f/abs
(f) + 1)*sqrt(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cos \left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,\cos \left(f\,x^2+d\right)\,dx$$

```
[In] int(f^(a + b*x)*cos(d + f*x^2),x)
[Out] int(f^(a + b*x)*cos(d + f*x^2), x)
```

3.111 $\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 157

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx = \left(-\frac{1}{16}-\frac{i}{16}\right)e^{2id+\frac{ib^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx+b\log(f))}{\sqrt{f}}\right) - \left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx-b\log(f))}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

```
[Out] 1/2*f^(b*x+a)/b/ln(f)-(1/16+1/16*I)*exp(2*I*d+1/8*I*b^2*ln(f)^2/f)*f^(-1/2+
a)*erf((1/4+1/4*I)*(4*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-(1/16+1/16*I)*f^(-1/
2+a)*erfi((1/4+1/4*I)*(4*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)/exp(1/8*I*(16*d+b
^2*ln(f)^2/f))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4561, 2225, 2325, 2266, 2235, 2236}

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx = \left(-\frac{1}{16}-\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{8f}+2id}\mathrm{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+4ifx)}{\sqrt{f}}\right) - \left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{8}i\left(\frac{b^2\log^2(f)}{f}+16d\right)}\mathrm{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\log(f)+4ifx)}{\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

```
[In] Int[f^(a + b*x)*Cos[d + f*x^2]^2,x]
```

```
[Out] (-1/16 - I/16)*E^((2*I)*d + ((I/8)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*E
rf[((1/4 + I/4)*((4*I)*f*x + b*Log[f]))/Sqrt[f]] - ((1/16 + I/16)*f^(-1/2 +
 a)*Sqrt[Pi]*Erfi[((1/4 + I/4)*((4*I)*f*x - b*Log[f]))/Sqrt[f]])/E^((I/8)*(
16*d + (b^2*Log[f]^2)/f)) + f^(a + b*x)/(2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{2}f^{a+bx} + \frac{1}{4}e^{-2id-2ifx^2}f^{a+bx} + \frac{1}{4}e^{2id+2ifx^2}f^{a+bx}\right)dx$$

$$= \frac{1}{4}\int e^{-2id-2ifx^2}f^{a+bx}\,dx + \frac{1}{4}\int e^{2id+2ifx^2}f^{a+bx}\,dx + \frac{1}{2}\int f^{a+bx}\,dx$$

$$= \frac{f^{a+bx}}{2b\log(f)} + \frac{1}{4}\int e^{-2id-2ifx^2+a\log(f)+bx\log(f)}\,dx + \frac{1}{4}\int e^{2id+2ifx^2+a\log(f)+bx\log(f)}\,dx$$

$$= \frac{f^{a+bx}}{2b\log(f)} + \frac{1}{4}\left(e^{2id+\frac{ib^2\log^2(f)}{8f}}f^a\right)\int e^{-\frac{i(4ifx+b\log(f))^2}{8f}}\,dx + \frac{1}{4}\left(e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-4ifx+b\log(f))^2}{8f}}\,dx$$

$$= \left(-\frac{1}{16}-\frac{i}{16}\right)e^{2id+\frac{ib^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx+b\log(f))}{\sqrt{f}}\right) - \left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx-b\log(f))}{\sqrt{f}}\right) + \frac{f^{a+bx}}{2b\log(f)}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx = \frac{1}{16}f^a\left(\frac{8f^{bx}}{b\log(f)} + \frac{(1-i)e^{-\frac{ib^2\log^2(f)}{8f}}\sqrt{\pi}\operatorname{erf}\left(\frac{(4+4i)fx-(1-i)b\log(f)}{4\sqrt{f}}\right)(\cos(d)-i\sin(d))^2}{\sqrt{f}} + \frac{(1+i)e^{\frac{ib^2\log^2(f)}{8f}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(4+4i)fx+(1-i)b\log(f)}{4\sqrt{f}}\right)(-i\cos(2d)+\sin(2d))}{\sqrt{f}}\right)$$

```
[In] Integrate[f^(a + b*x)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^a*((8*f^(b*x))/(b*Log[f]) + ((1 - I)*Sqrt[Pi]*Erf[((4 + 4*I)*f*x - (1 -
I)*b*Log[f])/(4*Sqrt[f])]*(Cos[d] - I*Sin[d])^2)/(E^(((I/8)*b^2*Log[f]^2)/f
)*Sqrt[f]) + ((1 + I)*E^(((I/8)*b^2*Log[f]^2)/f)*Sqrt[Pi]*Erfi[((4 + 4*I)*f
*x + (1 - I)*b*Log[f])/(4*Sqrt[f])]*((-I)*Cos[2*d] + Sin[2*d]))/Sqrt[f]))/1
6
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+16 d f\right)}{8 f}} \sqrt{2}\, \operatorname{erf}\left(-\sqrt{2}\, \sqrt{i f}\, x+\frac{b \ln(f) \sqrt{2}}{4 \sqrt{i f}}\right)}{16 \sqrt{i f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+16 d f\right)}{8 f}} \operatorname{erf}\left(-\sqrt{-2 i f}\, x+\frac{b \ln(f)}{2 \sqrt{-2 i f}}\right)}{8 \sqrt{-2 i f}}+\frac{f^{x b+a}}{2 b \ln(f)}$

```
[In] int(f^(b*x+a)*cos(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/8*I*(ln(f)^2*b^2+16*d*f)/f)*2^(1/2)/(I*f)^(1/2)*e
rf(-2^(1/2)*(I*f)^(1/2)*x+1/4*b*ln(f)*2^(1/2)/(I*f)^(1/2))-1/8*Pi^(1/2)*f^a
*exp(1/8*I*(ln(f)^2*b^2+16*d*f)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2
*b*ln(f)/(-2*I*f)^(1/2))+1/2*f^(b*x+a)/b/ln(f)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(103) = 206.

Time = 0.24 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.72

$$\int f^{a+bx} \cos^2\left(d+fx^2\right)\,dx$$
$$=\frac{\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i b^2 \log(f)^2+8 a f \log(f)-16 i d f}{8 f}\right)}\operatorname{C}\left(\frac{(4 f x+i b \log(f))\sqrt{\frac{f}{\pi}}}{2 f}\right)\log(f)-\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{i b^2 \log(f)^2+8 a f \log(f)+16 i d f}{8 f}\right)}\operatorname{C}\left(-\frac{(4 fx-$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)*
fresnel_cos(1/2*(4*f*x + I*b*log(f))*sqrt(f/pi)/f)*log(f) - pi*b*sqrt(f/pi)
*e^(1/8*(I*b^2*log(f)^2 + 8*a*f*log(f) + 16*I*d*f)/f)*fresnel_cos(-1/2*(4*f
*x - I*b*log(f))*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*lo
g(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)*fresnel_sin(1/2*(4*f*x + I*b*log(f))*s
qrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 + 8*a*f*log(
f) + 16*I*d*f)/f)*fresnel_sin(-1/2*(4*f*x - I*b*log(f))*sqrt(f/pi)/f)*log(f
) + 4*f*f^(b*x + a))/(b*f*log(f))
```

Sympy [F]

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx=\int f^{a+bx}\cos^2\left(d+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*cos(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.18

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx=$$

$$-\frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left((i-1)\,bf^a\cos\left(\frac{b^2\log(f)^2+16\,df}{8\,f}\right)\log(f)+(i+1)\,bf^a\log(f)\sin\left(\frac{b^2\log(f)^2+16\,df}{8\,f}\right)\right)\operatorname{erf}\left(\frac{4i\,fx-b\log}{2\sqrt{2i\,f}}\right.\right.}{}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*b*f^a*cos(1/8*(b^2*log(f)^2 + 16*
d*f)/f)*log(f) + (I + 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*d*f)/f))*e
rf(1/2*(4*I*f*x - b*log(f))/sqrt(2*I*f)) + ((I + 1)*b*f^a*cos(1/8*(b^2*log(
f)^2 + 16*d*f)/f)*log(f) + (I - 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 + 16*
d*f)/f))*erf(1/2*(4*I*f*x + b*log(f))/sqrt(-2*I*f)))*f^(3/2) - 16*f^(b*x)*f
^(a + 2))/(b*f^2*log(f))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(103) = 206.

Time = 0.34 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.32

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(
abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - pi*
b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2
*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(abs(f)
```

```
)) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I
*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I*e^(-1/2*I*pi*b*x*
sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) +
 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f))) + 1/8*I*s
qrt(pi)*erf(-1/8*I*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/
f)*(I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*log(abs(f))*s
gn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*b^2*log(abs(
f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 2*I*d)/(sqrt(f)*
(I*f/abs(f) + 1)) - 1/8*I*sqrt(pi)*erf(1/8*I*sqrt(f)*(8*x + (pi*b*sgn(f) -
pi*b + 2*I*b*log(abs(f)))/f)*(-I*f/abs(f) + 1))*e^(-1/16*I*pi^2*b^2*sgn(f)/
f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f + 1/8*pi*b^2*log(ab
s(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*lo
g(abs(f)) - 2*I*d)/(sqrt(f)*(-I*f/abs(f) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,\cos\left(f\,x^2+d\right)^2\,dx$$

```
[In] int(f^(a + b*x)*cos(d + f*x^2)^2,x)
[Out] int(f^(a + b*x)*cos(d + f*x^2)^2, x)
```

3.112 $\int f^{a+bx} \cos^3(d+fx^2)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 298

$$\begin{aligned}\int f^{a+bx}\cos^3(d+fx^2)\,dx &= -\frac{3}{16}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right)\\ &\quad-\left(\frac{1}{16}+\frac{i}{16}\right)e^{3id+\frac{ib^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(6ifx+b\log(f))}{\sqrt{6}\sqrt{f}}\right)\\ &\quad-\frac{3}{16}\sqrt[4]{-1}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(2ifx-b\log(f))}{2\sqrt{f}}\right)\\ &\quad-\left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{12}i\left(36d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(6ifx-b\log(f))}{\sqrt{6}\sqrt{f}}\right)\end{aligned}$$

```
[Out] (-1/96-1/96*I)*exp(3*I*d+1/12*I*b^2*ln(f)^2/f)*f^(-1/2+a)*erf((1/12+1/12*I)
*(6*I*f*x+b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)-(1/96+1/96*I)*f^(-1/2+
a)*erfi((1/12+1/12*I)*(6*I*f*x-b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)/e
xp(1/12*I*(36*d+b^2*ln(f)^2/f))-3/16*(-1)^(1/4)*exp(1/4*I*(4*d+b^2*ln(f)^2/
f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)*(2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*
(-1)^(1/4)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/
2)/exp(1/4*I*(4*d+b^2*ln(f)^2/f))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4561, 2325, 2266, 2235, 2236}

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx$$
$$= -\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)$$
$$-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{12f}+3id}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$
$$-\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+2ifx)}{2\sqrt{f}}\right)$$
$$-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{-\frac{1}{12}i\left(\frac{b^2\log^2(f)}{f}+36d\right)}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-b\log(f)+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$

```
[In] Int[f^(a + b*x)*Cos[d + f*x^2]^3,x]
```

```
[Out] (-3*(-1)^(1/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf
[((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 + I/16)*E^((3
*I)*d + ((I/12)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]*Erf[((1/2 + I/2)*(
(6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3*(-1)^(1/4)*f^(-1/2 + a)*Sqrt
[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(16*E^((I/4)*(4
*d + (b^2*Log[f]^2)/f))) - ((1/16 + I/16)*f^(-1/2 + a)*Sqrt[Pi/6]*Erfi[((1/
2 + I/2)*((6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])])/E^((I/12)*(36*d + (b^2
*Log[f]^2)/f))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{3}{8}e^{-id-ifx^2}f^{a+bx}+\frac{3}{8}e^{id+ifx^2}f^{a+bx}+\frac{1}{8}e^{-3id-3ifx^2}f^{a+bx}+\frac{1}{8}e^{3id+3ifx^2}f^{a+bx}\right)dx\\
&= \frac{1}{8}\int e^{-3id-3ifx^2}f^{a+bx}\,dx+\frac{1}{8}\int e^{3id+3ifx^2}f^{a+bx}\,dx\\
&\quad+\frac{3}{8}\int e^{-id-ifx^2}f^{a+bx}\,dx+\frac{3}{8}\int e^{id+ifx^2}f^{a+bx}\,dx\\
&= \frac{1}{8}\int e^{-3id-3ifx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{8}\int e^{3id+3ifx^2+a\log(f)+bx\log(f)}\,dx\\
&\quad+\frac{3}{8}\int e^{-id-ifx^2+a\log(f)+bx\log(f)}\,dx+\frac{3}{8}\int e^{id+ifx^2+a\log(f)+bx\log(f)}\,dx\\
&= \frac{1}{8}\left(e^{3id+\frac{ib^2\log^2(f)}{12f}}f^a\right)\int e^{-\frac{i(6ifx+b\log(f))^2}{12f}}\,dx\\
&\quad+\frac{1}{8}\left(3e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-2ifx+b\log(f))^2}{4f}}\,dx\\
&\quad+\frac{1}{8}\left(3e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{-\frac{i(2ifx+b\log(f))^2}{4f}}\,dx\\
&\quad+\frac{1}{8}\left(e^{-\frac{1}{12}i\left(36d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-6ifx+b\log(f))^2}{12f}}\,dx\\
&= -\frac{3}{16}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right)\\
&\quad-\left(\frac{1}{16}+\frac{i}{16}\right)e^{3id+\frac{ib^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(6ifx+b\log(f))}{\sqrt{6}\sqrt{f}}\right)\\
&\quad-\frac{3}{16}\sqrt[4]{-1}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt[4]{-1}(2ifx-b\log(f))}{2\sqrt{f}}\right)\\
&\quad-\left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{12}i\left(36d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\,\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(6ifx-b\log(f))}{\sqrt{6}\sqrt{f}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.90

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx$$
$$=\frac{1}{48}\sqrt[4]{-1}e^{-\frac{ib^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(-9\mathrm{erfi}\left(\frac{(-1)^{3/4}(2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))\right.$$
$$+9e^{\frac{ib^2\log^2(f)}{2f}}\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(2fx-ib\log(f))}{2\sqrt{f}}\right)(-i\cos(d)+\sin(d))$$
$$+\sqrt{3}e^{\frac{ib^2\log^2(f)}{6f}}\left(-\mathrm{erfi}\left(\frac{(-1)^{3/4}(6fx+ib\log(f))}{2\sqrt{3}\sqrt{f}}\right)(\cos(3d)-i\sin(3d))+e^{\frac{ib^2\log^2(f)}{6f}}\mathrm{erfi}\left(\frac{(6+6i)fx+(1-}{2\sqrt{6}\sqrt{f}}\right.\right.$$

```
[In] Integrate[f^(a + b*x)*Cos[d + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^(-1/2 + a)*Sqrt[Pi]*(-9*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f])
)/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) + 9*E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[((-1
)^(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*((-I)*Cos[d] + Sin[d]) + Sqrt[3]
*E^(((I/6)*b^2*Log[f]^2)/f)*(-(Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f]))/(2*Sq
rt[3]*Sqrt[f])]*(Cos[3*d] - I*Sin[3*d])) + E^(((I/6)*b^2*Log[f]^2)/f)*Erfi[
((6 + 6*I)*f*x + (1 - I)*b*Log[f])/(2*Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Si
n[3*d]))))/(48*E^(((I/4)*b^2*Log[f]^2)/f))
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{i\left(\ln(f)^2b^2+36df\right)}{12f}}\sqrt{3}\,\mathrm{erf}\left(-\sqrt{3}\,\sqrt{if}\,x+\frac{\ln(f)b\sqrt{3}}{6\sqrt{if}}\right)}{48\sqrt{if}}-\frac{3\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{i\left(\ln(f)^2b^2+4df\right)}{4f}}\mathrm{erf}\left(-\sqrt{if}\,x+\frac{\ln(f)b}{2\sqrt{if}}\right)}{16\sqrt{if}}-\frac{3\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{i(\ln(f}{}}}{}$

```
[In] int(f^(b*x+a)*cos(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*Pi^(1/2)*f^a*exp(-1/12*I*(ln(f)^2*b^2+36*d*f)/f)*3^(1/2)/(I*f)^(1/2)*
erf(-3^(1/2)*(I*f)^(1/2)*x+1/6*ln(f)*b*3^(1/2)/(I*f)^(1/2))-3/16*Pi^(1/2)*f
^a*exp(-1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*ln(
f)*b/(I*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(-I*f)
^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*exp(
1/12*I*(ln(f)^2*b^2+36*d*f)/f)/(-3*I*f)^(1/2)*erf(-(-3*I*f)^(1/2)*x+1/2*ln(
f)*b/(-3*I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(196) = 392$.

Time = 0.27 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.76

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx = \frac{\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+12\,af\log(f)-36i\,df}{12\,f}\right)}\,\mathrm{C}\left(\frac{\sqrt{6}(6\,fx+i\,b\log(f))\sqrt{\frac{f}{\pi}}}{6\,f}\right)-\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2+12\,af\log(f)+36i\,df}{12\,f}\right)}\,\mathrm{C}\left(-\frac{\sqrt{6}(6\,f}{}\right.}{}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 12*a*f*log(f) - 36*I
*d*f)/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f))*sqrt(f/pi)/f) - sqrt(
6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 + 12*a*f*log(f) + 36*I*d*f)/f)*fre
snel_cos(-1/6*sqrt(6)*(6*f*x - I*b*log(f))*sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqr
t(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_cos(1/
2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1
/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_cos(-1/2*sqrt(2)*(2
*f*x - I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*
log(f)^2 + 12*a*f*log(f) - 36*I*d*f)/f)*fresnel_sin(1/6*sqrt(6)*(6*f*x + I*
b*log(f))*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 +
 12*a*f*log(f) + 36*I*d*f)/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*log(f))
*sqrt(f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*
log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi
)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*
I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx = \int f^{a+bx}\cos^3\left(d+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*cos(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx =$$
$$-\frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left((i-1)\,f^a\cos\left(\frac{b^2\log(f)^2+36\,df}{12\,f}\right)+(i+1)\,f^a\sin\left(\frac{b^2\log(f)^2+36\,df}{12\,f}\right)\right)\operatorname{erf}\left(\frac{6i\,fx-b\log(f)}{2\,\sqrt{3i\,f}}\right)+\left((i+1\right.\right.}{}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*d
*f)/f) + (I + 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f)/f))*erf(1/2*(6*I*f*x
- b*log(f))/sqrt(3*I*f)) + ((I + 1)*f^a*cos(1/12*(b^2*log(f)^2 + 36*d*f)/f)
 + (I - 1)*f^a*sin(1/12*(b^2*log(f)^2 + 36*d*f)/f))*erf(1/2*(6*I*f*x + b*lo
g(f))/sqrt(-3*I*f)))*f^(3/2) - 9*sqrt(2)*sqrt(pi)*((-(I - 1)*f^a*cos(1/4*(b
^2*log(f)^2 + 4*d*f)/f) - (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*er
f(1/2*(2*I*f*x - b*log(f))/sqrt(I*f)) + (-(I + 1)*f^a*cos(1/4*(b^2*log(f)^2
 + 4*d*f)/f) - (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 + 4*d*f)/f))*erf(1/2*(2*I*
f*x + b*log(f))/sqrt(-I*f)))*f^(3/2))/f^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(196) = 392.

Time = 0.38 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.00

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I
*b*log(abs(f)))/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/
f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs
(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log
(abs(f)) + I*d)/((I*f/abs(f) + 1)*sqrt(abs(f))) + 1/48*I*sqrt(6)*sqrt(pi)*e
rf(-1/24*I*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))
/f)*(I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*b^2*log(abs(f))
*sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f + 1/12*I*b^2*log(
abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 3*I*d)/(sqrt
(f)*(I*f/abs(f) + 1)) - 1/48*I*sqrt(6)*sqrt(pi)*erf(1/24*I*sqrt(6)*sqrt(f)*
(12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(-I*f/abs(f) + 1))*e^(-
```

```
1/24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*sgn(f)/f + 1/24*I*pi^2*b
^2/f + 1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*
sgn(f) + 1/2*I*pi*a + a*log(abs(f)) - 3*I*d)/(sqrt(f)*(-I*f/abs(f) + 1)) -
3/16*I*sqrt(2)*sqrt(pi)*erf(1/8*I*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*
b*log(abs(f)))/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)
/f - 1/4*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(ab
s(f))/f - 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*lo
g(abs(f)) - I*d)/((-I*f/abs(f) + 1)*sqrt(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\cos\left(f\,x^2+d\right)^3\,dx$$

```
[In] int(f^(a + b*x)*cos(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*cos(d + f*x^2)^3, x)
```

3.113 $\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 162

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx = -\frac{1}{4}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right) - \frac{1}{4}\sqrt[4]{-1}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt[4]{-1}(ie+2ifx-b\log(f))}{2\sqrt{f}}\right)$$

```
[Out] -1/4*(-1)^(1/4)*exp(1/4*I*(4*d+(I*e+b*ln(f))^2/f))*f^(-1/2+a)*erf(1/2*(-1)^
(1/4)*(I*e+2*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-1/4*(-1)^(1/4)*exp(-I*d+1/4*I
*(e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(I*e+2*I*f*x-b*ln(f))/f^
(1/2))*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4561, 2325, 2266, 2235, 2236}

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx = -\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right) - \frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\text{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)$$

```
[In] Int[f^(a + b*x)*Cos[d + e*x + f*x^2],x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[P
i]*Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])]) - ((-1)^(1/4
)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Erfi[((-1
)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-id-iex-ifx^2}f^{a+bx}+\frac{1}{2}e^{id+iex+ifx^2}f^{a+bx}\right)dx\\
&= \frac{1}{2}\int e^{-id-iex-ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int e^{id+iex+ifx^2}f^{a+bx}\,dx\\
&= \frac{1}{2}\int \exp\left(-id-ifx^2+a\log(f)-x(ie-b\log(f))\right)dx\\
&\quad+\frac{1}{2}\int \exp\left(id+ifx^2+a\log(f)+x(ie+b\log(f))\right)dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}\left(e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^a\right)\int e^{\frac{i(-ie-2ifx+b\log(f))^2}{4f}}\,dx \\
&\quad + \frac{1}{2}\left(e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^a\right)\int e^{-\frac{i(ie+2ifx+b\log(f))^2}{4f}}\,dx \\
&= -\frac{1}{4}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right) \\
&\quad - \frac{1}{4}\sqrt[4]{-1}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(ie+2ifx-b\log(f))}{2\sqrt{f}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

$$
\begin{aligned}
&\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx \\
&= \frac{1}{4}\sqrt[4]{-1}e^{-\frac{i\left(e^2+b^2\log^2(f)\right)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(-e^{\frac{ie^2}{2f}}\mathrm{erfi}\left(\frac{(-1)^{3/4}(e+2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d))\right. \\
&\qquad \left. + e^{\frac{ib^2\log^2(f)}{2f}}\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-ib\log(f))}{2\sqrt{f}}\right)(-i\cos(d)+\sin(d))\right)
\end{aligned}
$$

```
[In] Integrate[f^(a + b*x)*Cos[d + e*x + f*x^2],x]
```

```
[Out] ((-1)^(1/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(-(E^(((I/2)*e^2)/f)*Erfi[((-1
)^(3/4)*(e + 2*f*x + I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] - I*Sin[d])) + E^(((
I/2)*b^2*Log[f]^2)/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f]
)]*((-I)*Cos[d] + Sin[d])))/(4*E^(((I/4)*(e^2 + b^2*Log[f]^2))/f))
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+4df-e^{2}\right)}{4f}} \mathrm{erf}\left(-\sqrt{if}\, x+\frac{b\ln(f)-ie}{2\sqrt{if}}\right)}{4\sqrt{if}}-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+4df-e^{2}\right)}{4f}} \mathrm{erf}\left(-\sqrt{-if}\, x+\frac{ie+b\ln(f)}{2\sqrt{-if}}\right)}{4\sqrt{-if}}$

```
[In] int(f^(b*x+a)*cos(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/4*I*(ln(f)^2*b^2+4*d*f-e^2)/f)/(I*f
)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))-1/4*Pi^(1/2)*f^a*
f^(-1/2/f*b*e)*exp(1/4*I*(ln(f)^2*b^2+4*d*f-e^2)/f)/(-I*f)^(1/2)*erf(-(-I*f
)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(109) = 218$.

Time = 0.25 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.93

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+i\,e^2-4i\,df-2\,(be-2\,af)\log(f)}{4\,f}\right)}\mathrm{C}\left(\frac{\sqrt{2}(2\,fx+i\,b\log(f)+e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)-\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2-i\,e^2+4i\,df-2\,(be-2\,af)\log\cdots}{4\,f}\right.}\cdots}{\cdots}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*e^2 - 4*I*d*f - 2*(b
*e - 2*a*f)*log(f))/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f) + e)*sqr
t(f/pi)/f) - sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 - I*e^2 + 4*I*d*f
 - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f)
+ e)*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*e^
2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I
*b*log(f) + e)*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)
^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*sqrt(2)*
(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx = \int f^{a+bx}\cos\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*cos(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + e*x + f*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.17

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx =$$
$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(-(i-1)\,f^a\cos\left(\frac{b^2\log(f)^2-e^2+4\,df}{4\,f}\right)-(i+1)\,f^a\sin\left(\frac{b^2\log(f)^2-e^2+4\,df}{4\,f}\right)\right)\mathrm{erf}\left(\frac{i\,(2i\,fx-b\log(f)+i\,e)\sqrt{i\,f}}{2\,f}\right.\cdots}{8\sqrt{f}\,f\cdots}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*((-(I - 1)*f^a*cos(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f
) - (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))*erf(1/2*I*(2*I*f*x
 - b*log(f) + I*e)*sqrt(I*f)/f) + ((I + 1)*f^a*cos(1/4*(b^2*log(f)^2 - e^2
+ 4*d*f)/f) + (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))*erf(1/2*
I*(2*I*f*x + b*log(f) + I*e)*sqrt(-I*f)/f))/(sqrt(f)*f^(1/2*b*e/f))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(109) = 218.

Time = 0.34 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.33

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx$$
$$= \frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}i\sqrt{2}\left(4x-\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)-2e}{f}\right)\left(\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}-\frac{i\pi^2b^2}{8f}-\frac{\pi b^2\log}{4f}\right.}}{4\left(\frac{if}{|f|}+1\right)\sqrt{|f|}}$$
$$-\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{8}i\sqrt{2}\left(4x+\frac{\pi b\mathrm{sgn}(f)-\pi b+2i\,b\log(|f|)+2e}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(-\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}-\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}+\frac{i\pi^2b^2}{8f}+\right.}}{4\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*
b*log(abs(f)) - 2*e)/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sg
n(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*lo
g(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*
sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f)
)/f + I*d - 1/4*I*e^2/f)/((I*f/abs(f) + 1)*sqrt(abs(f))) - 1/4*I*sqrt(2)*sq
rt(pi)*erf(1/8*I*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 2
*e)/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi
*b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1
/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*
I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - I*d + 1/4
*I*e^2/f)/((-I*f/abs(f) + 1)*sqrt(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\,\cos\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int(f^(a + b*x)*cos(d + e*x + f*x^2),x)
[Out] int(f^(a + b*x)*cos(d + e*x + f*x^2), x)
```

3.114 $\int f^{a+bx} \cos^2\left(d+ex+fx^2\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)dx = \left(-\frac{1}{16}-\frac{i}{16}\right)e^{2id+\frac{i(2ie+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx+b\log(f))}{\sqrt{f}}\right) - \left(\frac{1}{16}+\frac{i}{16}\right)e^{-2id+\frac{i(2e+ib\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx-b\log(f))}{\sqrt{f}}\right) + \frac{f^{a+bx}}{2b\log(f)}$$

```
[Out] 1/2*f^(b*x+a)/b/ln(f)-(1/16+1/16*I)*exp(2*I*d+1/8*I*(2*I*e+b*ln(f))^2/f)*f^
(-1/2+a)*erf((1/4+1/4*I)*(2*I*e+4*I*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)-(1/16+1/
16*I)*exp(-2*I*d+1/8*I*(2*e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi((1/4+1/4*I)*(2*
I*e+4*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{4561, 2225, 2325, 2266, 2235, 2236\}$

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx$$
$$=\left(-\frac{1}{16}-\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+2ie)^2}{8f}+2id}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)$$
$$-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(2e+ib\log(f))^2}{8f}-2id}\operatorname{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)$$
$$+\frac{f^{a+bx}}{2b\log(f)}$$

```
[In] Int[f^(a + b*x)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (-1/16 - I/16)*E^((2*I)*d + ((I/8)*((2*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*
Sqrt[Pi]*Erf[((1/4 + I/4)*((2*I)*e + (4*I)*f*x + b*Log[f]))/Sqrt[f]] - (1/1
6 + I/16)*E^((-2*I)*d + ((I/8)*(2*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[P
i]*Erfi[((1/4 + I/4)*((2*I)*e + (4*I)*f*x - b*Log[f]))/Sqrt[f]] + f^(a + b*
x)/(2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx}+\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx}\right)dx\\
&= \frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx\\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int \exp\left(-2id-2ifx^2+a\log(f)-x(2ie-b\log(f))\right)dx\\
&\quad+\frac{1}{4}\int \exp\left(2id+2ifx^2+a\log(f)+x(2ie+b\log(f))\right)dx\\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\exp\left(-2id+a\log(f)-\frac{i(-2ie+b\log(f))^2}{8f}\right)\int e^{\frac{i(-2ie-4ifx+b\log(f))^2}{8f}}\,dx\\
&\quad+\frac{1}{4}\left(e^{2id+\frac{i(2ie+b\log(f))^2}{8f}}f^a\right)\int e^{-\frac{i(2ie+4ifx+b\log(f))^2}{8f}}\,dx\\
&= \left(-\frac{1}{16}-\frac{i}{16}\right)e^{2id+\frac{i(2ie+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx+b\log(f))}{\sqrt{f}}\right)\\
&\quad-\left(\frac{1}{16}+\frac{i}{16}\right)\exp\left(-\frac{1}{8}i\left(16d\right.\right.\\
&\qquad\left.\left.+\frac{(2ie-b\log(f))^2}{f}\right)\right)f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx-b\log(f))}{\sqrt{f}}\right)\\
&\quad+\frac{f^{a+bx}}{2b\log(f)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.37

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx = \frac{e^{-\frac{i\left(4e^2+b^2\log^2(f)\right)}{8f}}f^{a-\frac{be+f}{2f}}\left(8e^{\frac{i\left(4e^2+b^2\log^2(f)\right)}{8f}}f^{\frac{1}{2}+b\left(\frac{e}{2f}+x\right)}+\sqrt[4]{-1}be^{\frac{ib^2\log^2(f)}{4f}}\sqrt{2\pi}\,\text{erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2e+4fx-ib\log(f))}{\sqrt{f}}\right)\log(f)\right.}{16b\log(f)}$$

```
[In] Integrate[f^(a + b*x)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*(8*E^(((I/8)*(4*e^2 + b^2*Log[f]^2))/f)*f^(1/2 + b
*(e/(2*f) + x)) + (-1)^(1/4)*b*E^(((I/4)*b^2*Log[f]^2)/f)*Sqrt[2*Pi]*Erfi[(
(1/4 + I/4)*(2*e + 4*f*x - I*b*Log[f]))/Sqrt[f]]*Log[f]*((-I)*Cos[2*d] + Si
n[2*d]) - (-1)^(1/4)*b*E^((I*e^2)/f)*Sqrt[2*Pi]*Erf[((1/4 + I/4)*(2*e + 4*f
*x + I*b*Log[f]))/Sqrt[f]]*Log[f]*(I*Cos[2*d] + Sin[2*d])))/(16*b*E^(((I/8)
*(4*e^2 + b^2*Log[f]^2))/f)*Log[f])
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{\sqrt{\pi}\,f^{a} f^{-\frac{be}{2f}}\,\mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+16 d f-4 e^{2}\right)}{8 f}}\sqrt{2}\,\mathrm{erf}\left(-\sqrt{2}\,\sqrt{i f}\,x+\frac{(b\ln(f)-2 i e)\sqrt{2}}{4\sqrt{i f}}\right)}{16\sqrt{i f}}-\frac{\sqrt{\pi}\,f^{a} f^{-\frac{be}{2f}}\,\mathrm{e}^{\frac{i\left(\ln(f)^{2} b^{2}+16 d f-4 e^{2}\right)}{8 f}}\,\mathrm{erf}\left(-\sqrt{-2 i f}\,x+\right.}{8\sqrt{-2 i f}}$

```
[In] int(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/8*I*(ln(f)^2*b^2+16*d*f-4*e^2)/f)*
2^(1/2)/(I*f)^(1/2)*erf(-2^(1/2)*(I*f)^(1/2)*x+1/4*(b*ln(f)-2*I*e)*2^(1/2)/
(I*f)^(1/2))-1/8*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(1/8*I*(ln(f)^2*b^2+16*d*f-
4*e^2)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-2*I*f)
^(1/2))+1/2*f^(b*x+a)/b/ln(f)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(116) = 232.

Time = 0.26 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.82

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx$$

$$=\frac{\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+4i\,e^2-16i\,df-4\,(be-2\,af)\log(f)}{8\,f}\right)}\operatorname{C}\left(\frac{(4\,fx+i\,b\log(f)+2\,e)\sqrt{\frac{f}{\pi}}}{2\,f}\right)\log(f)-\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2-4i\,e^2+16i\,df-4\,(be}{8\,f}\right.}}{}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 4*I*e^2 - 16*I*d*f - 4*(b*e
- 2*a*f)*log(f))/f)*fresnel_cos(1/2*(4*f*x + I*b*log(f) + 2*e)*sqrt(f/pi)/f
)*log(f) - pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 - 4*I*e^2 + 16*I*d*f - 4*
(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/2*(4*f*x - I*b*log(f) + 2*e)*sqrt(f
/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 4*I*e^2 - 16*I
*d*f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*(4*f*x + I*b*log(f) + 2*e
)*sqrt(f/pi)/f)*log(f) - I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 - 4*I*e^2
 + 16*I*d*f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*(4*f*x - I*b*log(
f) + 2*e)*sqrt(f/pi)/f)*log(f) + 4*f*f^(b*x + a))/(b*f*log(f))
```

Sympy [F]

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*cos(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + e*x + f*x**2)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(116) = 232.

Time = 0.32 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.34

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx=$$

$$-\frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-(i-1)\,bf^a\cos\left(\frac{b^2\log(f)^2-4\,e^2+16\,df}{8\,f}\right)\log(f)-(i+1)\,bf^a\log(f)\sin\left(\frac{b^2\log(f)^2-4\,e^2+16\,df}{8\,f}\right)\right)\mathrm{e}}{}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*b*f^a*cos(1/8*(b^2*log(f)^2 - 4*
e^2 + 16*d*f)/f)*log(f) - (I + 1)*b*f^a*log(f)*sin(1/8*(b^2*log(f)^2 - 4*e^
2 + 16*d*f)/f))*erf(1/4*I*(4*I*f*x - b*log(f) + 2*I*e)*sqrt(2*I*f)/f) + ((I
 + 1)*b*f^a*cos(1/8*(b^2*log(f)^2 - 4*e^2 + 16*d*f)/f)*log(f) + (I - 1)*b*f
^a*log(f)*sin(1/8*(b^2*log(f)^2 - 4*e^2 + 16*d*f)/f))*erf(1/4*I*(4*I*f*x +
b*log(f) + 2*I*e)*sqrt(-2*I*f)/f))*f^(3/2) - 16*f^(a + 2)*e^(b*x*log(f) + 1
/2*b*e*log(f)/f))/(b*f^2*f^(1/2*b*e/f)*log(f))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(116) = 232$.

Time = 0.37 (sec) , antiderivative size = 599, normalized size of antiderivative = 3.35

$$\int f^{a+bx} \cos^2\left(d+ex+fx^2\right) dx = \text{Too large to display}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(
abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - pi*
b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2
*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(abs(f)
)) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I
*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I*e^(-1/2*I*pi*b*x*
sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) +
 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f))) + 1/8*I*s
qrt(pi)*erf(-1/8*I*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) -
 4*e)/f)*(I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*log(abs
(f))*sgn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*b^2*lo
g(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/
4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 2*I*d - 1/2*I*e^2/f)
/(sqrt(f)*(I*f/abs(f) + 1)) - 1/8*I*sqrt(pi)*erf(1/8*I*sqrt(f)*(8*x + (pi*b
*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 4*e)/f)*(-I*f/abs(f) + 1))*e^(-1/16*I*
pi^2*b^2*sgn(f)/f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f + 1
/8*pi*b^2*log(abs(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1
/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*
e*log(abs(f))/f - 2*I*d + 1/2*I*e^2/f)/(sqrt(f)*(-I*f/abs(f) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\cos\left(f\,x^2+e\,x+d\right)^2\,dx$$

```
[In] int(f^(a + b*x)*cos(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + b*x)*cos(d + e*x + f*x^2)^2, x)
```

3.115 $\int f^{a+bx} \cos^3\left(d+ex+fx^2\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 340

$$\begin{aligned}
&\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)dx \\
&= -\frac{3}{16}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right) \\
&\quad-\left(\frac{1}{16}+\frac{i}{16}\right)e^{3id+\frac{i(3ie+b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3ie+6ifx+b\log(f))}{\sqrt{6}\sqrt{f}}\right) \\
&\quad-\frac{3}{16}\sqrt[4]{-1}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(ie+2ifx-b\log(f))}{2\sqrt{f}}\right) \\
&\quad-\left(\frac{1}{16}+\frac{i}{16}\right)e^{-3id+\frac{i(3e+ib\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3ie+6ifx-b\log(f))}{\sqrt{6}\sqrt{f}}\right)
\end{aligned}$$

```
[Out] (-1/96-1/96*I)*exp(3*I*d+1/12*I*(3*I*e+b*ln(f))^2/f)*f^(-1/2+a)*erf((1/12+1
/12*I)*(3*I*e+6*I*f*x+b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)-(1/96+1/96
*I)*exp(-3*I*d+1/12*I*(3*e+I*b*ln(f))^2/f)*f^(-1/2+a)*erfi((1/12+1/12*I)*(3
*I*e+6*I*f*x-b*ln(f))*6^(1/2)/f^(1/2))*6^(1/2)*Pi^(1/2)-3/16*(-1)^(1/4)*exp
(1/4*I*(4*d+(I*e+b*ln(f))^2/f))*f^(-1/2+a)*erf(1/2*(-1)^(1/4)*(I*e+2*I*f*x+
b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*(-1)^(1/4)*exp(-I*d+1/4*I*(e+I*b*ln(f))^2/f
)*f^(-1/2+a)*erfi(1/2*(-1)^(1/4)*(I*e+2*I*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4561, 2325, 2266, 2235, 2236}

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx$$

$$= -\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right) - \left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+3ie)^2}{12f}+3id}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+3ie+6ifx)}{\sqrt{6}\sqrt{f}}\right) - \frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\text{erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+ie+2ifx)}{2\sqrt{f}}\right) - \left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(3e+ib\log(f))^2}{12f}-3id}\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-b\log(f)+3ie+6ifx)}{\sqrt{6}\sqrt{f}}\right)$$

```
[In] Int[f^(a + b*x)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (-3*(-1)^(1/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]
*Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 + I
/16)*E^((3*I)*d + ((I/12)*((3*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6
]*Erf[((1/2 + I/2)*((3*I)*e + (6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (
3*(-1)^(1/4)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi
]*Erfi[((-1)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 +
 I/16)*E^((-3*I)*d + ((I/12)*(3*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/
6]*Erfi[((1/2 + I/2)*((3*I)*e + (6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} = \int \Bigg(& \frac{1}{8}e^{-3i(d+ex+fx^2)}f^{a+bx} + \frac{3}{8}\exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx} \\
& + \frac{3}{8}\exp\left(4id+4iex+4ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx} \\
& + \frac{1}{8}\exp\left(6id+6iex+6ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx}\Bigg)\,dx \\
= & \frac{1}{8}\int e^{-3i(d+ex+fx^2)}f^{a+bx}\,dx \\
& + \frac{1}{8}\int \exp\left(6id+6iex+6ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx}\,dx \\
& + \frac{3}{8}\int \exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx}\,dx \\
& + \frac{3}{8}\int \exp\left(4id+4iex+4ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx}\,dx \\
= & \frac{1}{8}\int \exp\left(-3id-3ifx^2+a\log(f)-x(3ie-b\log(f))\right)\,dx \\
& + \frac{1}{8}\int \exp\left(3id+3ifx^2+a\log(f)+x(3ie+b\log(f))\right)\,dx \\
& + \frac{3}{8}\int \exp\left(-id-ifx^2+a\log(f)-x(ie-b\log(f))\right)\,dx \\
& + \frac{3}{8}\int \exp\left(id+ifx^2+a\log(f)+x(ie+b\log(f))\right)\,dx
\end{aligned}$$

$$= \frac{1}{8}\exp\left(-3id + a\log(f) - \frac{i(-3ie + b\log(f))^2}{12f}\right)\int e^{\frac{i(-3ie-6ifx+b\log(f))^2}{12f}}\,dx$$
$$+ \frac{1}{8}\left(3e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^a\right)\int e^{\frac{i(-ie-2ifx+b\log(f))^2}{4f}}\,dx$$
$$+ \frac{1}{8}\left(3e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^a\right)\int e^{-\frac{i(ie+2ifx+b\log(f))^2}{4f}}\,dx$$
$$+ \frac{1}{8}\left(e^{3id+\frac{i(3ie+b\log(f))^2}{12f}}f^a\right)\int e^{-\frac{i(3ie+6ifx+b\log(f))^2}{12f}}\,dx$$
$$= -\frac{3}{16}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)$$
$$- \left(\frac{1}{16}+\frac{i}{16}\right)e^{3id+\frac{i(3ie+b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3ie+6ifx+b\log(f))}{\sqrt{6}\sqrt{f}}\right)$$
$$- \frac{3}{16}\sqrt[4]{-1}e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(ie+2ifx-b\log(f))}{2\sqrt{f}}\right)$$
$$- \left(\frac{1}{16}+\frac{i}{16}\right)\exp\left(-\frac{1}{12}i\left(36d + \frac{(3ie-b\log(f))^2}{f}\right)\right)f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{6}}\operatorname{erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(3ie+6ifx-b\log(f))}{\sqrt{6}\sqrt{f}}\right)$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.95

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx$$
$$= \frac{1}{48}\sqrt[4]{-1}e^{-\frac{i\left(3e^2+b^2\log^2(f)\right)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(9e^{\frac{i\left(e^2+b^2\log^2(f)\right)}{2f}}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-ib\log(f))}{2\sqrt{f}}\right)(-i\cos(d) + \sin(d))\right.$$
$$\left. + e^{\frac{ie^2}{f}}\left(-9\operatorname{erfi}\left(\frac{(-1)^{3/4}(e+2fx+ib\log(f))}{2\sqrt{f}}\right)(\cos(d)-i\sin(d)) - \sqrt{3}e^{\frac{i\left(3e^2+b^2\log^2(f)\right)}{6f}}\operatorname{erfi}\left(\frac{(-1)^{3/4}(3e+6f}{2\sqrt{3}}\right.\right.\right.$$

```
[In] Integrate[f^(a + b*x)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(9*E^(((I/2)*(e^2 + b^2*Log[f]
^2))/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*((-I)*Cos[d
] + Sin[d]) + E^((I*e^2)/f)*(-9*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f]))/
(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) - Sqrt[3]*E^(((I/6)*(3*e^2 + b^2*Log[f]^2)
)/f)*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f]))/(2*Sqrt[3]*Sqrt[f])]*(Cos
[3*d] - I*Sin[3*d])) + Sqrt[3]*E^(((I/3)*b^2*Log[f]^2)/f)*Erfi[((1/2 + I/2)
*(3*e + 6*f*x - I*b*Log[f]))/(Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Sin[3*d]))
)/(48*E^(((I/4)*(3*e^2 + b^2*Log[f]^2))/f))
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+36 d f-9 e^{2}\right)}{12 f}} \sqrt{3}\, \mathrm{erf}\left(-\sqrt{3}\, \sqrt{i f}\, x+\frac{(b\ln(f)-3 i e)\sqrt{3}}{6\sqrt{i f}}\right)}{48\sqrt{i f}}-\frac{3\sqrt{\pi}\, f^{a} f^{-\frac{be}{2f}} \mathrm{e}^{-\frac{i\left(\ln(f)^{2} b^{2}+4 d f-e^{2}\right)}{4 f}} \mathrm{erf}\left(-\sqrt{i f}\, x+\frac{b}{}\right.}{16\sqrt{i f}}$

```
[In] int(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/12*I*(ln(f)^2*b^2+36*d*f-9*e^2)/f)
*3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*x+1/6*(b*ln(f)-3*I*e)*3^(1/2)
/(I*f)^(1/2))-3/16*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(-1/4*I*(ln(f)^2*b^2+4*d*
f-e^2)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))-3/1
6*Pi^(1/2)*f^a*f^(-1/2/f*b*e)*exp(1/4*I*(ln(f)^2*b^2+4*d*f-e^2)/f)/(-I*f)^(
1/2)*erf(-(-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*
f^(-1/2/f*b*e)*exp(1/12*I*(ln(f)^2*b^2+36*d*f-9*e^2)/f)/(-3*I*f)^(1/2)*erf(
-(-3*I*f)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-3*I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(220) = 440.

Time = 0.27 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.85

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log(f)^2+9i\,e^2-36i\,df-6\,(be-2\,af)\log(f)}{12\,f}\right)}\mathrm{C}\left(\frac{\sqrt{6}(6\,fx+i\,b\log(f)+3\,e)\sqrt{\frac{f}{\pi}}}{6\,f}\right)-\sqrt{6}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log(f)^2-9i\,e^2+36i\,df-6\,(be-2}{12\,f}\right.}}{}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 9*I*e^2 - 36*I*d*f -
 6*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f) + 3
*e)*sqrt(f/pi)/f) - sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 - 9*I*e^2
 + 36*I*d*f - 6*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/6*sqrt(6)*(6*f*x -
I*b*log(f) + 3*e)*sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*lo
g(f)^2 + I*e^2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/2*sqrt(
2)*(2*f*x + I*b*log(f) + e)*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*
(I*b^2*log(f)^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(
-1/2*sqrt(2)*(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/p
i)*e^(1/12*(-I*b^2*log(f)^2 + 9*I*e^2 - 36*I*d*f - 6*(b*e - 2*a*f)*log(f))/
f)*fresnel_sin(1/6*sqrt(6)*(6*f*x + I*b*log(f) + 3*e)*sqrt(f/pi)/f) - I*sqr
```

```
t(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 - 9*I*e^2 + 36*I*d*f - 6*(b*e -
2*a*f)*log(f))/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*log(f) + 3*e)*sqrt(
f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*e^2 - 4*I*
d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*log(f
) + e)*sqrt(f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 - I
*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x
 - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*cos(f*x**2+e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + e*x + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.11

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx=$$

$$-\frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-(i-1)\ f^a\cos\left(\frac{b^2\log(f)^2-9\,e^2+36\,df}{12\,f}\right)-(i+1)\ f^a\sin\left(\frac{b^2\log(f)^2-9\,e^2+36\,df}{12\,f}\right)\right)\mathrm{erf}\left(\frac{i\,(6i\,fx-b\log(f)}{6\,f}\right.\right.}{}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*f^a*cos(1/12*(b^2*log(f)^2 - 9*e
^2 + 36*d*f)/f) - (I + 1)*f^a*sin(1/12*(b^2*log(f)^2 - 9*e^2 + 36*d*f)/f))*
erf(1/6*I*(6*I*f*x - b*log(f) + 3*I*e)*sqrt(3*I*f)/f) + ((I + 1)*f^a*cos(1/
12*(b^2*log(f)^2 - 9*e^2 + 36*d*f)/f) + (I - 1)*f^a*sin(1/12*(b^2*log(f)^2
- 9*e^2 + 36*d*f)/f))*erf(1/6*I*(6*I*f*x + b*log(f) + 3*I*e)*sqrt(-3*I*f)/f
))*f^(3/2) - 9*sqrt(2)*sqrt(pi)*(((I - 1)*f^a*cos(1/4*(b^2*log(f)^2 - e^2 +
 4*d*f)/f) + (I + 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f))*erf(1/2*I
*(2*I*f*x - b*log(f) + I*e)*sqrt(I*f)/f) + (-(I + 1)*f^a*cos(1/4*(b^2*log(f
)^2 - e^2 + 4*d*f)/f) - (I - 1)*f^a*sin(1/4*(b^2*log(f)^2 - e^2 + 4*d*f)/f)
)*erf(1/2*I*(2*I*f*x + b*log(f) + I*e)*sqrt(-I*f)/f))*f^(3/2))/(f^2*f^(1/2*
b*e/f))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(220) = 440$.

Time = 0.49 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.21

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/8*I*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I
*b*log(abs(f)) - 2*e)/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*s
gn(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*l
og(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e
*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f
))/f + I*d - 1/4*I*e^2/f)/((I*f/abs(f) + 1)*sqrt(abs(f))) + 1/48*I*sqrt(6)*
sqrt(pi)*erf(-1/24*I*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*lo
g(abs(f)) - 6*e)/f)*(I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi
*b^2*log(abs(f))*sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f +
 1/12*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1
/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 3*I*d
- 3/4*I*e^2/f)/(sqrt(f)*(I*f/abs(f) + 1)) - 1/48*I*sqrt(6)*sqrt(pi)*erf(1/2
4*I*sqrt(6)*sqrt(f)*(12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 6*e)/
f)*(-I*f/abs(f) + 1))*e^(-1/24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f)
)*sgn(f)/f + 1/24*I*pi^2*b^2/f + 1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log
(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4
*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - 3*I*d + 3/4*I*e^2/f)/
(sqrt(f)*(-I*f/abs(f) + 1)) - 3/16*I*sqrt(2)*sqrt(pi)*erf(1/8*I*sqrt(2)*(4*
x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 2*e)/f)*(-I*f/abs(f) + 1)*sqr
t(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*sgn(f)/f +
1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f))^2/f - 1
/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*
log(abs(f)) - 1/2*b*e*log(abs(f))/f - I*d + 1/4*I*e^2/f)/((-I*f/abs(f) + 1)
*sqrt(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx = \int f^{a+b\,x}\cos\left(f\,x^2+e\,x+d\right)^3\,dx$$

```
[In] int(f^(a + b*x)*cos(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*cos(d + e*x + f*x^2)^3, x)
```

3.116 $\int f^{a+cx^2}\cos(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 147

$$\int f^{a+cx^2}\cos(d+ex)\,dx=-\frac{e^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 1/4*exp(-I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-I*e+2*c*x*ln(f))/c^(1/2)/ln(f)
^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(I*d+1/4*e^2/c/ln(f))*f^a*erfi(
1/2*(I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4561, 2325, 2266, 2235}

$$\int f^{a+cx^2}\cos(d+ex)\,dx=\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\mathrm{erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\mathrm{erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x],x]
```

```
[Out] -1/4*(E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[f])/
(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + (E^(I*d + e^2/(4*c*Log[
```

```
f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*S
qrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-id-iex}f^{a+cx^2}+\frac{1}{2}e^{id+iex}f^{a+cx^2}\right)\,dx\\
&= \frac{1}{2}\int e^{-id-iex}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{id+iex}f^{a+cx^2}\,dx\\
&= \frac{1}{2}\int e^{-id-iex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{2}\int e^{id+iex+a\log(f)+cx^2\log(f)}\,dx\\
&= \frac{1}{2}\left(e^{-id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-ie+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{2}\left(e^{id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(ie+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&= -\frac{e^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

$$\int f^{a+cx^2}\cos(d+ex)\,dx$$
$$=\frac{e^{\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(\operatorname{erfi}\left(\frac{-ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)-i\sin(d))+\operatorname{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x],x]
```

```
[Out] (E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*(Erfi[((-I)*e + 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])]*(Cos[d] - I*Sin[d]) + Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])]*(Cos[d] + I*Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{4id\ln(f)c-e^{2}}{4\ln(f)c}}\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{ie}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{4id\ln(f)c+e^{2}}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{ie}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}$	121

```
[In] int(f^(c*x^2+a)*cos(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf
((-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*(4*I
*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*I*e/(
-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int f^{a+cx^2}\cos(d+ex)\,dx=$$
$$-\frac{\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{(2\,cx\log(f)+i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(\frac{4\,ac\log(f)^2+4i\,cd\log(f)+e^2}{4\,c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{(2\,cx\log(f)-i\,e)}{2\,c\log(}\right.}{4\,c\log\left(f\right)}$$

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*log(f))
/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))) + s
qrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f))/(c*log
(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))))/(c*log(f)
)
```

Sympy [F]

$$\int f^{a+cx^2}\cos(d+ex)\,dx=\int f^{a+cx^2}\cos(d+ex)\,dx$$

```
[In] integrate(f**(c*x**2+a)*cos(e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.39

$$\int f^{a+cx^2}\cos(d+ex)\,dx=$$
$$-\frac{\sqrt{\pi}\left(f^a(\cos(d)-i\sin(d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}+\frac{1}{2}ie\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^2}{4c\log(f)}\right)}+f^a(\cos(d)+i\sin(d))\operatorname{erf}\left(x\right.\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(f^a*(cos(d) - I*sin(d))*erf(x*conjugate(sqrt(-c*log(f))) + 1
/2*I*e*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(cos(d) +
 I*sin(d))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*I*e*conjugate(1/sqrt(-c*l
og(f))))*e^(1/4*e^2/(c*log(f))) - f^a*(cos(d) + I*sin(d))*erf(1/2*(2*c*x*lo
g(f) + I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))) - f^a*(cos(d) - I*sin(d
))*erf(1/2*(2*c*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))))*sq
rt(-c*log(f))/(c*log(f))
```

Giac [F]

$$\int f^{a+cx^2}\cos(d+ex)\,dx=\int f^{cx^2+a}\cos\left(ex+d\right)\,dx$$

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d),x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*cos(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos(d+ex)\,dx=\int f^{c\,x^2+a}\cos\left(d+e\,x\right)\,dx$$

```
[In] int(f^(a + c*x^2)*cos(d + e*x),x)
[Out] int(f^(a + c*x^2)*cos(d + e*x), x)
```

3.117 $\int f^{a+cx^2} \cos^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 171

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx=\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 1/8*exp(-2*I*d+e^2/c/ln(f))*f^a*erfi((-I*e+c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*
Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*I*d+e^2/c/ln(f))*f^a*erfi((I*e+c*x*l
n(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^a*erfi(x*c^(1
/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4561, 2235, 2325, 2266}

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx=-\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)}-2id}\,\mathrm{erfi}\left(\frac{-cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)}+2id}\,\mathrm{erfi}\left(\frac{cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(
(-2*I)*d + e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - c*x*Log[f])/(Sqrt[c]*Sq
rt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^((2*I)*d + e^2/(c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[L
og[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2} f^{a+cx^2} + \frac{1}{4} e^{-2id-2iex} f^{a+cx^2} + \frac{1}{4} e^{2id+2iex} f^{a+cx^2}\right) dx \\
&= \frac{1}{4} \int e^{-2id-2iex} f^{a+cx^2}\, dx + \frac{1}{4} \int e^{2id+2iex} f^{a+cx^2}\, dx + \frac{1}{2} \int f^{a+cx^2}\, dx \\
&= \frac{f^a \sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{1}{4} \int e^{-2id-2iex+a\log(f)+cx^2\log(f)}\, dx \\
&\quad + \frac{1}{4} \int e^{2id+2iex+a\log(f)+cx^2\log(f)}\, dx \\
&= \frac{f^a \sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{1}{4}\left(e^{-2id+\frac{e^2}{c\log(f)}} f^a\right) \int e^{\frac{(-2ie+2cx\log(f))^2}{4c\log(f)}}\, dx \\
&\quad + \frac{1}{4}\left(e^{2id+\frac{e^2}{c\log(f)}} f^a\right) \int e^{\frac{(2ie+2cx\log(f))^2}{4c\log(f)}}\, dx
\end{aligned}
$$

$$= \frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} + \frac{e^{2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.77

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx$$

$$= \frac{f^a\sqrt{\pi}\left(2\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right) + e^{\frac{e^2}{c\log(f)}}\left(\text{erfi}\left(\frac{-ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)-i\sin(2d)) + \text{erfi}\left(\frac{ie+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d) + \right.\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + E^(e^2/(c*Log[f]))*(Erfi[((
-I)*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + Erfi[
(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] + I*Sin[2*d]))))/(8*Sq
rt[c]*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

method	result
risch	$\frac{\sqrt{\pi}\, f^a \mathrm{e}^{-\frac{2id\ln(f)c-e^2}{\ln(f)c}}\,\text{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{ie}{\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}} - \frac{\sqrt{\pi}\, f^a \mathrm{e}^{\frac{2id\ln(f)c+e^2}{\ln(f)c}}\,\text{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{ie}{\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}} + \frac{f^a\sqrt{\pi}\,\text{erf}\left(\sqrt{-c\ln\right.}{4\sqrt{-c\ln(f}}$

```
[In] int(f^(c*x^2+a)*cos(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c
*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp((2*I*d*ln(f)*c+e
^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2))
+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.93

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx =$$

$$-\frac{2\sqrt{\pi}\sqrt{-c\log(f)}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)+\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(cx\log(f)+ie)\sqrt{-c\log(f)}}{c\log(f)}\right)e^{\left(\frac{ac\log(f)^2+2icd\log(f}{c\log(f)}\right.}}{8c\log(f)}$$

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log(f))*x) + sqrt(pi)*sqrt
(-c*log(f))*erf((c*x*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(
f)^2 + 2*I*c*d*log(f) + e^2)/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf((c*
x*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(f)^2 - 2*I*c*d*log(
f) + e^2)/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx = \int f^{a+cx^2}\cos^2(d+ex)\,dx$$

```
[In] integrate(f**(c*x**2+a)*cos(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.38

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx$$

$$=\frac{\sqrt{\pi}\left(f^a(\cos(2d)-i\sin(2d))\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}+ie\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^2}{c\log(f)}\right)}+f^a(\cos(2d)+i\sin(2d))\operatorname{erf}\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*(f^a*(cos(2*d) - I*sin(2*d))*erf(x*conjugate(sqrt(-c*log(f)))
 + I*e*conjugate(1/sqrt(-c*log(f))))*e^(e^2/(c*log(f))) + f^a*(cos(2*d) + I
```

```
*sin(2*d))*erf(x*conjugate(sqrt(-c*log(f))) - I*e*conjugate(1/sqrt(-c*log(f
))))*e^(e^2/(c*log(f))) - f^a*(cos(2*d) + I*sin(2*d))*erf((c*x*log(f) + I*e
)/sqrt(-c*log(f)))*e^(e^2/(c*log(f))) - f^a*(cos(2*d) - I*sin(2*d))*erf((c*
x*log(f) - I*e)/sqrt(-c*log(f)))*e^(e^2/(c*log(f))) + 2*f^a*erf(x*conjugate
(sqrt(-c*log(f)))) + 2*f^a*erf(sqrt(-c*log(f))*x))/sqrt(-c*log(f))
```

Giac [F]

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx = \int f^{cx^2+a}\cos\left(ex+d\right)^2\,dx$$

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx = \int f^{c\,x^2+a}\cos\left(d+e\,x\right)^2\,dx$$

```
[In] int(f^(a + c*x^2)*cos(d + e*x)^2,x)
```

```
[Out] int(f^(a + c*x^2)*cos(d + e*x)^2, x)
```

3.118 $\int f^{a+cx^2} \cos^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 293

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx=-\frac{3e^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3e^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{e^{3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 3/16*exp(-I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-I*e+2*c*x*ln(f))/c^(1/2)/ln(f
)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*I*d+9/4*e^2/c/ln(f))*f^a*
erfi(1/2*(-3*I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(
1/2)+3/16*exp(I*d+1/4*e^2/c/ln(f))*f^a*erfi(1/2*(I*e+2*c*x*ln(f))/c^(1/2)/l
n(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*I*d+9/4*e^2/c/ln(f))*f^
a*erfi(1/2*(3*I*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^
(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4561, 2325, 2266, 2235}

$$\int f^{a+cx^2} \cos^3(d+ex)\,dx = -\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\operatorname{erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi}f^a e^{\frac{9e^2}{4c\log(f)}-3id}\operatorname{erfi}\left(\frac{-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\operatorname{erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi}f^a e^{\frac{9e^2}{4c\log(f)}+3id}\operatorname{erfi}\left(\frac{2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x]^3,x]
```

```
[Out] (-3*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[f])/(2
*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^((-3*I)*d + (9*e^2)
/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[L
og[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (3*E^(I*d + e^2/(4*c*Log[f]))*f^a*Sqr
t[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt
[Log[f]]) + (E^((3*I)*d + (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e
+ 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{8}e^{-id-iex}f^{a+cx^2}+\frac{3}{8}e^{id+iex}f^{a+cx^2}+\frac{1}{8}e^{-3id-3iex}f^{a+cx^2}+\frac{1}{8}e^{3id+3iex}f^{a+cx^2}\right)dx\\
&= \frac{1}{8}\int e^{-3id-3iex}f^{a+cx^2}\,dx+\frac{1}{8}\int e^{3id+3iex}f^{a+cx^2}\,dx\\
&\quad+\frac{3}{8}\int e^{-id-iex}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{id+iex}f^{a+cx^2}\,dx\\
&= \frac{1}{8}\int e^{-3id-3iex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{8}\int e^{3id+3iex+a\log(f)+cx^2\log(f)}\,dx\\
&\quad+\frac{3}{8}\int e^{-id-iex+a\log(f)+cx^2\log(f)}\,dx+\frac{3}{8}\int e^{id+iex+a\log(f)+cx^2\log(f)}\,dx\\
&= \frac{1}{8}\left(3e^{-id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-ie+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{8}\left(3e^{id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(ie+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&\quad+\frac{1}{8}\left(e^{-3id+\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-3ie+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{8}\left(e^{3id+\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(3ie+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&= -\frac{3e^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{3ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}\\
&\quad+\frac{3e^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{e^{3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{3ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.74

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx$$

$$=\frac{e^{\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(3\text{erfi}\left(\frac{-ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)-i\sin(d))+3\text{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d))+e^{\frac{2e^2}{c\log(f)}}\left(\text{er}\right.\right.}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x]^3,x]
```

```
[Out] (E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*(3*Erfi[((-I)*e + 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])]*(Cos[d] - I*Sin[d]) + 3*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt
[c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) + E^((2*e^2)/(c*Log[f]))*(Erfi[((-3*
```

```
I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] - I*Sin[3*d]) + Er
fi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] + I*Sin[3*d
]))))/(16*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 i d \ln(f) c-3 e^{2}\right)}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{3 i e}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}+\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 i d \ln(f) c-e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{i e}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}-\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{4i}}{}$

```
[In] int(f^(c*x^2+a)*cos(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3/4*(4*I*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*
erf((-c*ln(f))^(1/2)*x+3/2*I*e/(-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4
*(4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*I
*e/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c+e^2)/ln(f)/c)
/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))-1/16*Pi
^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c
*ln(f))^(1/2)*x+3/2*I*e/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.96

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx = -\frac{\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2\,cx\log(f)+3i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(\frac{4\,ac\log(f)^2+12i\,cd\log(f)+9\,e^2}{4\,c\log(f)}\right)}+3\,\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2\,cx\log(f)+}{2\,c}\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + 3*I*e)*sqrt(-c*log(
f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 12*I*c*d*log(f) + 9*e^2)/(c*log(f)
)) + 3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*log(f)
)/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))) +
3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f))/(c*
log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))) + sqrt(
pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f
)))*e^(1/4*(4*a*c*log(f)^2 - 12*I*c*d*log(f) + 9*e^2)/(c*log(f))))/(c*log(f
))
```

Sympy [F]

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx = \int f^{a+cx^2}\cos^3{(d+ex)}\,dx$$

```
[In] integrate(f**(c*x**2+a)*cos(e*x+d)**3,x)
[Out] Integral(f**(a + c*x**2)*cos(d + e*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.39

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx =$$

$$-\frac{\sqrt{\pi}\left(f^a(\cos{(3d)} - i\sin{(3d)})\operatorname{erf}\left(x\overline{\sqrt{-c\log{(f)}}} + \frac{3}{2}i\,e\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{9e^2}{4c\log(f)}\right)} + f^a(\cos{(3d)} + i\sin{(3d)}\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^3,x, algorithm="maxima")
[Out] -1/32*sqrt(pi)*(f^a*(cos(3*d) - I*sin(3*d))*erf(x*conjugate(sqrt(-c*log(f))
) + 3/2*I*e*conjugate(1/sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(cos
(3*d) + I*sin(3*d))*erf(x*conjugate(sqrt(-c*log(f))) - 3/2*I*e*conjugate(1/
sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) - f^a*(cos(3*d) + I*sin(3*d))*erf(
1/2*(2*c*x*log(f) + 3*I*e)/sqrt(-c*log(f)))*e^(9/4*e^2/(c*log(f))) - f^a*(c
os(3*d) - I*sin(3*d))*erf(1/2*(2*c*x*log(f) - 3*I*e)/sqrt(-c*log(f)))*e^(9/
4*e^2/(c*log(f))) + 3*f^a*(cos(d) - I*sin(d))*erf(x*conjugate(sqrt(-c*log(f
))) + 1/2*I*e*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + 3*f^a*
(cos(d) + I*sin(d))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*I*e*conjugate(1/
sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(cos(d) + I*sin(d))*erf(1/
2*(2*c*x*log(f) + I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c*log(f))) - 3*f^a*(cos
(d) - I*sin(d))*erf(1/2*(2*c*x*log(f) - I*e)/sqrt(-c*log(f)))*e^(1/4*e^2/(c
*log(f))))*sqrt(-c*log(f))/(c*log(f))
```

Giac [F]

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx=\int f^{cx^2+a}\cos(ex+d)^3\,dx$$

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*cos(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx=\int f^{c\,x^2+a}\cos(d+e\,x)^3\,dx$$

```
[In] int(f^(a + c*x^2)*cos(d + e*x)^3,x)
[Out] int(f^(a + c*x^2)*cos(d + e*x)^3, x)
```

3.119 $\int f^{a+cx^2} \cos(d+fx^2)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 103

$$\int f^{a+cx^2} \cos(d+fx^2)\, dx = \frac{e^{-id} f^a \sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{if - c\log(f)}\right)}{4\sqrt{if - c\log(f)}} + \frac{e^{id} f^a \sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{if + c\log(f)}\right)}{4\sqrt{if + c\log(f)}}$$

```
[Out] 1/4*f^a*erf(x*(I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(I*d)/(I*f-c*ln(f))^(1/2)+1/
4*exp(I*d)*f^a*erfi(x*(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4561, 2325, 2236, 2235}

$$\int f^{a+cx^2} \cos(d+fx^2)\, dx = \frac{\sqrt{\pi} e^{-id} f^a \mathrm{erf}\left(x\sqrt{-c\log(f)+if}\right)}{4\sqrt{-c\log(f)+if}} + \frac{\sqrt{\pi} e^{id} f^a \mathrm{erfi}\left(x\sqrt{c\log(f)+if}\right)}{4\sqrt{c\log(f)+if}}$$

```
[In] Int[f^(a + c*x^2)*Cos[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(4*E^(I*d)*Sqrt[I*f - c*Log[f]])
 + (E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])/(4*Sqrt[I*f + c*Log[
f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-id-ifx^2}f^{a+cx^2}+\frac{1}{2}e^{id+ifx^2}f^{a+cx^2}\right)dx\\
&= \frac{1}{2}\int e^{-id-ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{id+ifx^2}f^{a+cx^2}\,dx\\
&= \frac{1}{2}\int e^{-id+a\log(f)-x^2(if-c\log(f))}\,dx+\frac{1}{2}\int e^{id+a\log(f)+x^2(if+c\log(f))}\,dx\\
&= \frac{e^{-id}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{if-c\log(f)}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id}f^a\sqrt{\pi}\,\text{erfi}\left(x\sqrt{if+c\log(f)}\right)}{4\sqrt{if+c\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.65

$$\int f^{a+cx^2}\cos\left(d+fx^2\right)\,dx = -\frac{(-1)^{3/4}f^a\sqrt{\pi}\left(\text{erfi}\left(\sqrt[4]{-1}x\sqrt{f-ic\log(f)}\right)\sqrt{f-ic\log(f)}(f+ic\log(f))(\cos(d)+i\sin(d))+\sqrt{f+ic\log(f)}\cdots\right)}{4\left(f^2\cdots\right)}$$

```
[In] Integrate[f^(a + c*x^2)*Cos[d + f*x^2],x]
```

```
[Out] -1/4*((-1)^(3/4)*f^a*Sqrt[Pi]*(Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt
[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] + I*Sin[d]) + Sqrt[f + I*c*Log[f]
]*(f*Cos[d]*Erf[((1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]] - Erfi[(-1)^(3/4)
*x*Sqrt[f + I*c*Log[f]]]*(c*Cos[d]*Log[f] + (f - I*c*Log[f])*Sin[d]))))/(f^
2 + c^2*Log[f]^2)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-i d} \operatorname{erf}\left(x \sqrt{i f-c \ln \left(f\right)}\right)}{4 \sqrt{i f-c \ln \left(f\right)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{i d} \operatorname{erf}\left(\sqrt{-c \ln \left(f\right)-i f}\, x\right)}{4 \sqrt{-c \ln \left(f\right)-i f}}$	82

```
[In] int(f^(c*x^2+a)*cos(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2))+1
/4*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-c*ln(f)-I*f)^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06

$$\int f^{a+cx^2} \cos\left(d+fx^2\right)\,dx =$$
$$-\frac{\sqrt{\pi}(c\log\left(f\right)-i\,f)\sqrt{-c\log\left(f\right)-i\,f}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-i\,f}x\right)e^{(a\log(f)+i\,d)}+\sqrt{\pi}(c\log\left(f\right)+i\,f)\sqrt{-c}}{4\left(c^2\log\left(f\right)^2+f^2\right)}$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) -
I*f)*x)*e^(a*log(f) + I*d) + sqrt(pi)*(c*log(f) + I*f)*sqrt(-c*log(f) + I*f
)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d))/(c^2*log(f)^2 + f^2)
```

Sympy [F]

$$\int f^{a+cx^2}\cos\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\cos\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+d),x)
[Out] Integral(f**(a + c*x**2)*cos(d + f*x**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(73) = 146.

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.99

$$\int f^{a+cx^2}\cos\left(d+fx^2\right)\,dx=$$

$$-\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+2\,f^2}\left(f^a(i\,\cos\left(d\right)+\sin\left(d\right))\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+i\,fx}\right)+f^a(-i\,\cos\left(d\right)+\sin\left(d\right))\,\mathrm{erf}\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d),x, algorithm="maxima")
[Out] -1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(f^a*(I*cos(d) + sin(d))*erf(sq
rt(-c*log(f) + I*f)*x) + f^a*(-I*cos(d) + sin(d))*erf(sqrt(-c*log(f) - I*f)
*x))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^2*log(f)
^2 + 2*f^2)*(f^a*(cos(d) - I*sin(d))*erf(sqrt(-c*log(f) + I*f)*x) + f^a*(co
s(d) + I*sin(d))*erf(sqrt(-c*log(f) - I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*lo
g(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Giac [F]

$$\int f^{a+cx^2}\cos\left(d+fx^2\right)\,dx=\int f^{cx^2+a}\cos\left(fx^2+d\right)\,dx$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d),x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \cos\left(d+fx^2\right)\,dx = \int f^{c\,x^2+a}\,\cos\left(f\,x^2+d\right)\,dx$$

```
[In] int(f^(a + c*x^2)*cos(d + f*x^2),x)
[Out] int(f^(a + c*x^2)*cos(d + f*x^2), x)
```

3.120 $\int f^{a+cx^2} \cos^2(d+fx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx=\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{2if-c\log(f)}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{2if+c\log(f)}\right)}{8\sqrt{2if+c\log(f)}}$$

```
[Out] 1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*f^a*er
f(x*(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(2*I*d)/(2*I*f-c*ln(f))^(1/2)+1/8*ex
p(2*I*d)*f^a*erfi(x*(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4561, 2235, 2325, 2236}

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx=\frac{\sqrt{\pi}e^{-2id}f^a\operatorname{erf}\left(x\sqrt{-c\log(f)+2if}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}e^{2id}f^a\operatorname{erfi}\left(x\sqrt{c\log(f)+2if}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^a
*Sqrt[Pi]*Erf[x*Sqrt[(2*I)*f - c*Log[f]]])/(8*E^((2*I)*d)*Sqrt[(2*I)*f - c*
Log[f]]) + (E^((2*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(2*I)*f + c*Log[f]]])/(8*S
qrt[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2} f^{a+cx^2}+\frac{1}{4} e^{-2id-2ifx^2} f^{a+cx^2}+\frac{1}{4} e^{2id+2ifx^2} f^{a+cx^2}\right)\,dx\\
&= \frac{1}{4}\int e^{-2id-2ifx^2} f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2id+2ifx^2} f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int \exp\left(-2id+a\log(f)-x^2(2if-c\log(f))\right)\,dx\\
&\quad+\frac{1}{4}\int \exp\left(2id+a\log(f)+x^2(2if+c\log(f))\right)\,dx\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{2if-c\log(f)}\right)}{8\sqrt{2if-c\log(f)}}\\
&\quad+\frac{e^{2id}f^a\sqrt{\pi}\,\text{erfi}\left(x\sqrt{2if+c\log(f)}\right)}{8\sqrt{2if+c\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.35

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx=\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{\sqrt{c}\sqrt{\log(f)}}\right.$$
$$+\frac{\sqrt[4]{-1}\left(-\text{erfi}\left((-1)^{3/4}x\sqrt{2f+ic\log(f)}\right)(2f-ic\log(f))\sqrt{2f+ic\log(f)}(\cos(2d)-i\sin(2d))+\text{erfi}\left(\sqrt[4]{-}\right.\right.}{4f^2+c^2\log^2(f)}$$

```
[In] Integrate[f^(a + c*x^2)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + ((
-1)^(1/4)*(-(Erfi[(-1)^(3/4)*x*Sqrt[2*f + I*c*Log[f]]]*(2*f - I*c*Log[f])*S
qrt[2*f + I*c*Log[f]]*(Cos[2*d] - I*Sin[2*d])) + Erfi[(-1)^(1/4)*x*Sqrt[2*f
 - I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]]*((-2*I)*f + c*Log[f])*(Cos[2*d] + I*
Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-2 i d} \operatorname{erf}\left(x \sqrt{2 i f -c \ln \left(f \right)}\right)}{8 \sqrt{2 i f -c \ln \left(f \right)}}+\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{2 i d} \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)-2 i f}\, x \right)}{8 \sqrt{-c \ln \left(f \right)-2 i f}}+\frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)}\, x \right)}{4 \sqrt{-c \ln \left(f \right)}}$	107

```
[In] int(f^(c*x^2+a)*cos(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-2*I*d)/(2*I*f-c*ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(1
/2))+1/8*Pi^(1/2)*f^a*exp(2*I*d)/(-c*ln(f)-2*I*f)^(1/2)*erf((-c*ln(f)-2*I*f
)^(1/2)*x)+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.19

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx=$$
$$-\frac{2\sqrt{\pi}\left(c^2\log(f)^2+4f^2\right)\sqrt{-c\log(f)}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)+\sqrt{\pi}\left(c^2\log(f)^2-2icf\log(f)\right)\sqrt{-c\log(f}}{}$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log
(f))*x) + sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*
erf(sqrt(-c*log(f) - 2*I*f)*x)*e^(a*log(f) + 2*I*d) + sqrt(pi)*(c^2*log(f)^
2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(sqrt(-c*log(f) + 2*I*f)*x)*
e^(a*log(f) - 2*I*d))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\cos^2\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + f*x**2)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.25

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx=$$

$$-\frac{\sqrt{\pi}\sqrt{2\,c^2\log\left(f\right)^2+8\,f^2}\left(f^a(i\,\cos\left(2\,d\right)+\sin\left(2\,d\right))\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+2i\,f}x\right)+f^a(-i\,\cos\left(2\,d\right)+\sin\left(\right.\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(f^a*(I*cos(2*d) + sin(2*d))*e
rf(sqrt(-c*log(f) + 2*I*f)*x) + f^a*(-I*cos(2*d) + sin(2*d))*erf(sqrt(-c*lo
g(f) - 2*I*f)*x))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f
)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*(f^a*(cos(2*d) - I*sin(2*d))*erf
(sqrt(-c*log(f) + 2*I*f)*x) + f^a*(cos(2*d) + I*sin(2*d))*erf(sqrt(-c*log(f
) - 2*I*f)*x))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f))
 - 2*sqrt(pi)*((c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(x*conjugate(sqrt(-c*log
(f)))) + (c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(sqrt(-c*log(f))*x)))/((c^2*lo
g(f)^2 + 4*f^2)*sqrt(-c*log(f)))
```

Giac [F]

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx=\int f^{cx^2+a}\cos\left(fx^2+d\right)^2\,dx$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^2,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos^2\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\cos\left(f\,x^2+d\right)^2\,dx$$

```
[In] int(f^(a + c*x^2)*cos(d + f*x^2)^2,x)
[Out] int(f^(a + c*x^2)*cos(d + f*x^2)^2, x)
```

3.121 $\int f^{a+cx^2} \cos^3\left(d+fx^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 205

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)dx=\frac{3e^{-id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{if-c\log(f)}\right)}{16\sqrt{if-c\log(f)}}+\frac{e^{-3id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{3if-c\log(f)}\right)}{16\sqrt{3if-c\log(f)}}+\frac{3e^{id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{if+c\log(f)}\right)}{16\sqrt{if+c\log(f)}}+\frac{e^{3id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{3if+c\log(f)}\right)}{16\sqrt{3if+c\log(f)}}$$

```
[Out] 3/16*f^a*erf(x*(I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(I*d)/(I*f-c*ln(f))^(1/2)+1
/16*f^a*erf(x*(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(3*I*d)/(3*I*f-c*ln(f))^(1
/2)+3/16*exp(I*d)*f^a*erfi(x*(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1
/2)+1/16*exp(3*I*d)*f^a*erfi(x*(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(
f))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4561, 2325, 2236, 2235}

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)\,dx=\frac{3\sqrt{\pi}e^{-id}f^a\operatorname{erf}\left(x\sqrt{-c\log(f)+if}\right)}{16\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}e^{-3id}f^a\operatorname{erf}\left(x\sqrt{-c\log(f)+3if}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3\sqrt{\pi}e^{id}f^a\operatorname{erfi}\left(x\sqrt{c\log(f)+if}\right)}{16\sqrt{c\log(f)+if}}+\frac{\sqrt{\pi}e^{3id}f^a\operatorname{erfi}\left(x\sqrt{c\log(f)+3if}\right)}{16\sqrt{c\log(f)+3if}}$$

```
[In] Int[f^(a + c*x^2)*Cos[d + f*x^2]^3,x]
```

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(16*E^(I*d)*Sqrt[I*f - c*Log[f
]]) + (f^a*Sqrt[Pi]*Erf[x*Sqrt[(3*I)*f - c*Log[f]]])/(16*E^((3*I)*d)*Sqrt[(
3*I)*f - c*Log[f]]) + (3*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])
/(16*Sqrt[I*f + c*Log[f]]) + (E^((3*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(3*I)*f
+ c*Log[f]]])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{8}e^{-id-ifx^2}f^{a+cx^2}+\frac{3}{8}e^{id+ifx^2}f^{a+cx^2}+\frac{1}{8}e^{-3id-3ifx^2}f^{a+cx^2}+\frac{1}{8}e^{3id+3ifx^2}f^{a+cx^2}\right)\,dx\\
&= \frac{1}{8}\int e^{-3id-3ifx^2}f^{a+cx^2}\,dx+\frac{1}{8}\int e^{3id+3ifx^2}f^{a+cx^2}\,dx\\
&\quad+\frac{3}{8}\int e^{-id-ifx^2}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{id+ifx^2}f^{a+cx^2}\,dx\\
&= \frac{1}{8}\int \exp\left(-3id+a\log(f)-x^2(3if-c\log(f))\right)\,dx\\
&\quad+\frac{1}{8}\int \exp\left(3id+a\log(f)+x^2(3if+c\log(f))\right)\,dx\\
&\quad+\frac{3}{8}\int e^{-id+a\log(f)-x^2(if-c\log(f))}\,dx+\frac{3}{8}\int e^{id+a\log(f)+x^2(if+c\log(f))}\,dx\\
&= \frac{3e^{-id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{if-c\log(f)}\right)}{16\sqrt{if-c\log(f)}}+\frac{e^{-3id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{3if-c\log(f)}\right)}{16\sqrt{3if-c\log(f)}}\\
&\quad+\frac{3e^{id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{if+c\log(f)}\right)}{16\sqrt{if+c\log(f)}}+\frac{e^{3id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{3if+c\log(f)}\right)}{16\sqrt{3if+c\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.90

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt[4]{-1}f^a\sqrt{\pi}\left(3\operatorname{erfi}\left(\sqrt[4]{-1}x\sqrt{f-ic\log(f)}\right)\sqrt{f-ic\log(f)}\left(-9if^3+9cf^2\log(f)-ic^2f\log^2(f)+c^3\log^3(f\right.\right.}{}$$

```
[In] Integrate[f^(a + c*x^2)*Cos[d + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^a*Sqrt[Pi]*(3*Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt[f
- I*c*Log[f]]*((-9*I)*f^3 + 9*c*f^2*Log[f] - I*c^2*f*Log[f]^2 + c^3*Log[f]^
3)*(Cos[d] + I*Sin[d]) + (f - I*c*Log[f])*(-((3*f - I*c*Log[f])*(9*f*Erf[((
1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]]*Sqrt[f + I*c*Log[f]]*Sin[d] + 3*Erf
i[(-1)^(3/4)*x*Sqrt[f + I*c*Log[f]]]*Sqrt[f + I*c*Log[f]]*(Cos[d]*(3*f + I*
c*Log[f]) + c*Log[f]*Sin[d]) + Erfi[(-1)^(3/4)*x*Sqrt[3*f + I*c*Log[f]]]*(f
```

```
 + I*c*Log[f])*Sqrt[3*f + I*c*Log[f]]*(Cos[3*d] - I*Sin[3*d]))) + Erfi[(-1)
^(1/4)*x*Sqrt[3*f - I*c*Log[f]]]*Sqrt[3*f - I*c*Log[f]]*((-3*I)*f^2 + 4*c*f
*Log[f] + I*c^2*Log[f]^2)*(Cos[3*d] + I*Sin[3*d]))))/(16*(9*f^4 + 10*c^2*f^
2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-3 i d} \operatorname{erf}\left(x \sqrt{3 i f-c \ln(f)}\right)}{16 \sqrt{3 i f-c \ln(f)}}+\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{-i d} \operatorname{erf}\left(x \sqrt{i f-c \ln(f)}\right)}{16 \sqrt{i f-c \ln(f)}}+\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{i d} \operatorname{erf}\left(\sqrt{-c \ln(f)-i f}\, x\right)}{16 \sqrt{-c \ln(f)-i f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{3 i d} \operatorname{erf}\left(\sqrt{-c}\right.}{16 \sqrt{-c \ln(f}}$

```
[In] int(f^(c*x^2+a)*cos(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3*I*d)/(3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f-c*ln(f))^(
1/2))+3/16*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(
1/2))+3/16*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-c*ln(f)-I*f)^(1
/2)*x)+1/16*Pi^(1/2)*f^a*exp(3*I*d)/(-c*ln(f)-3*I*f)^(1/2)*erf((-c*ln(f)-3*
I*f)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(145) = 290.

Time = 0.26 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.52

$$\int f^{a+cx^2} \cos^3\left(d+fx^2\right)\,dx =$$
$$-\frac{\sqrt{\pi}\left(c^3 \log(f)^3 - 3i\,c^2 f \log(f)^2 + cf^2 \log(f) - 3i\,f^3\right)\sqrt{-c\log(f) - 3i\,f}\,\operatorname{erf}\left(\sqrt{-c\log(f) - 3i\,f}x\right) e^{(a\log}}{}$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(sqrt(-c*log(f) - 3*I*f)*x)*e^(a*log(f) + 3*I*
d) + 3*sqrt(pi)*(c^3*log(f)^3 - I*c^2*f*log(f)^2 + 9*c*f^2*log(f) - 9*I*f^3
)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) - I*f)*x)*e^(a*log(f) + I*d) + 3
*sqrt(pi)*(c^3*log(f)^3 + I*c^2*f*log(f)^2 + 9*c*f^2*log(f) + 9*I*f^3)*sqrt
(-c*log(f) + I*f)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d) + sqrt(pi
)*(c^3*log(f)^3 + 3*I*c^2*f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(
f) + 3*I*f)*erf(sqrt(-c*log(f) + 3*I*f)*x)*e^(a*log(f) - 3*I*d))/(c^4*log(f
)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F]

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\cos^3\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+d)**3,x)
[Out] Integral(f**(a + c*x**2)*cos(d + f*x**2)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 667 vs. 2(145) = 290.

Time = 0.25 (sec) , antiderivative size = 667, normalized size of antiderivative = 3.25

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)\,dx$$

$$=\frac{\sqrt{\pi}\sqrt{2\,c^2\log{(f)}^2+18\,f^2}\Big(\big((-i\,c^2\cos{(3\,d)}-c^2\sin{(3\,d)})f^a\log{(f)}^2+f^{a+2}(-i\,\cos{(3\,d)}-\sin{(3\,d)})\big)\,\mathrm{erf}\,\big(}{}$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^3,x, algorithm="maxima")
[Out] 1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((-I*c^2*cos(3*d) - c^2*sin(3
*d))*f^a*log(f)^2 + f^(a + 2)*(-I*cos(3*d) - sin(3*d)))*erf(sqrt(-c*log(f)
+ 3*I*f)*x) + ((I*c^2*cos(3*d) - c^2*sin(3*d))*f^a*log(f)^2 + f^(a + 2)*(I*
cos(3*d) - sin(3*d)))*erf(sqrt(-c*log(f) - 3*I*f)*x))*sqrt(c*log(f) + sqrt(
c^2*log(f)^2 + 9*f^2)) + 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((-I*c^2*
cos(d) - c^2*sin(d))*f^a*log(f)^2 + 9*f^(a + 2)*(-I*cos(d) - sin(d)))*erf(s
qrt(-c*log(f) + I*f)*x) + ((I*c^2*cos(d) - c^2*sin(d))*f^a*log(f)^2 + 9*f^(
a + 2)*(I*cos(d) - sin(d)))*erf(sqrt(-c*log(f) - I*f)*x))*sqrt(c*log(f) + s
qrt(c^2*log(f)^2 + f^2)) + sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*co
s(3*d) - I*c^2*sin(3*d))*f^a*log(f)^2 + f^(a + 2)*(cos(3*d) - I*sin(3*d)))*
erf(sqrt(-c*log(f) + 3*I*f)*x) + ((c^2*cos(3*d) + I*c^2*sin(3*d))*f^a*log(f
)^2 + f^(a + 2)*(cos(3*d) + I*sin(3*d)))*erf(sqrt(-c*log(f) - 3*I*f)*x))*sq
rt(-c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) + 3*sqrt(pi)*sqrt(2*c^2*log(f)^2
 + 2*f^2)*(((c^2*cos(d) - I*c^2*sin(d))*f^a*log(f)^2 + 9*f^(a + 2)*(cos(d)
- I*sin(d)))*erf(sqrt(-c*log(f) + I*f)*x) + ((c^2*cos(d) + I*c^2*sin(d))*f^
a*log(f)^2 + 9*f^(a + 2)*(cos(d) + I*sin(d)))*erf(sqrt(-c*log(f) - I*f)*x))
*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log
(f)^2 + 9*f^4)
```

Giac [F]

$$\int f^{a+cx^2} \cos^3\left(d+fx^2\right)\,dx = \int f^{cx^2+a} \cos\left(fx^2+d\right)^3\,dx$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \cos^3\left(d+fx^2\right)\,dx = \int f^{c\,x^2+a} \cos\left(f\,x^2+d\right)^3\,dx$$

```
[In] int(f^(a + c*x^2)*cos(d + f*x^2)^3,x)
[Out] int(f^(a + c*x^2)*cos(d + f*x^2)^3, x)
```

3.122 $\int f^{a+cx^2} \cos(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 183

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx=\frac{e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] 1/4*exp(-I*d-e^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e+2*x*(I*f-c*ln(f)))/(I*
f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/4*exp(I*d+e^2/(4*I*f+4*c*l
n(f)))*f^a*erfi(1/2*(I*e+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(
I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx=\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\,\mathrm{erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)+4if}+id}\,\mathrm{erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x + f*x^2],x]
```

```
[Out] (E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*(I*f -
 c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(4*Sqrt[I*f - c*Log[f]]) + (E^(I*d +
 e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*x*(I*f + c*Log[f]))
/(2*Sqrt[I*f + c*Log[f]])])/(4*Sqrt[I*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-id-iex-ifx^2}f^{a+cx^2}+\frac{1}{2}e^{id+iex+ifx^2}f^{a+cx^2}\right)\,dx\\
&= \frac{1}{2}\int e^{-id-iex-ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{id+iex+ifx^2}f^{a+cx^2}\,dx\\
&= \frac{1}{2}\int \exp\left(-id-iex+a\log(f)-x^2(if-c\log(f))\right)\,dx\\
&\quad+\frac{1}{2}\int \exp\left(id+iex+a\log(f)+x^2(if+c\log(f))\right)\,dx
\end{aligned}
$$

$$= \frac{1}{2}\left(e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(-ie+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx + \frac{1}{2}\left(e^{id+\frac{e^2}{4if+4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+2x(if+c\log(f)))^2}{4(if+c\log(f))}\right)dx$$

$$= \frac{e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}} + \frac{e^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.19

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx$$

$$= \frac{\sqrt[4]{-1}e^{\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\left(-e^{\frac{ie^2f}{2(f^2+c^2\log^2(f))}}\operatorname{erfi}\left(\frac{(-1)^{3/4}(e+2fx+2icx\log(f))}{2\sqrt{f+ic\log(f)}}\right)(f-ic\log(f))\sqrt{f+ic\log(f)}(\cos(d)\right.}{4\left(f^2+c^2\log^2(\right.}$$

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x + f*x^2],x]
```

```
[Out] ((-1)^(1/4)*E^(e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*(-(E^(((I/2)*e^2*f)
/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*
Sqrt[f + I*c*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(Cos[d] - I*Si
n[d])) + Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*L
og[f]])]*Sqrt[f - I*c*Log[f]]*((-I)*f + c*Log[f])*(Cos[d] + I*Sin[d])))/(4*
(f^2 + c^2*Log[f]^2))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\sqrt{\pi}\,f^a \mathrm{e}^{-\frac{4id\ln(f)c+4df-e^2}{4(c\ln(f)-if)}}\operatorname{erf}\left(x\sqrt{if-c\ln(f)}+\frac{ie}{2\sqrt{if-c\ln(f)}}\right)}{4\sqrt{if-c\ln(f)}} - \frac{\sqrt{\pi}\,f^a \mathrm{e}^{\frac{4id\ln(f)c-4df+e^2}{4if+4c\ln(f)}}\operatorname{erf}\left(-\sqrt{-c\ln(f)-if}\,x+\frac{ie}{2\sqrt{-c\ln(f)-if}}\right.}{4\sqrt{-c\ln(f)-if}}$

```
[In] int(f^(c*x^2+a)*cos(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-I*f))/(I*f-c*l
n(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(f))^(1/2))-1/4*Pi^(
1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1
/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(135) = 270.

Time = 0.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.64

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}(c\log(f)-i\,f)\sqrt{-c\log(f)-i\,f}\,\mathrm{erf}\left(\frac{\left(2\,c^2x\log(f)^2+2\,f^2x+i\,ce\log(f)+ef\right)\sqrt{-c\log(f)-i\,f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(\frac{4\,ac^2\log(f)^3+4i\,c^2d\log(\ldots}{4(\ldots}\right.}}{\ldots}$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(
f)^2 + 2*f^2*x + I*c*e*log(f) + e*f)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 +
f^2))*e^(1/4*(4*a*c^2*log(f)^3 + 4*I*c^2*d*log(f)^2 - I*e^2*f + 4*I*d*f^2 +
 (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c*log(f) + I*f
)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x - I*c*e*log(f)
+ e*f)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*log(f)^3
 - 4*I*c^2*d*log(f)^2 + I*e^2*f - 4*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^
2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Sympy [F]

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx = \int f^{a+cx^2}\cos\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x + f*x**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 761 vs. 2(135) = 270.

Time = 0.25 (sec) , antiderivative size = 761, normalized size of antiderivative = 4.16

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx$$

$$=\frac{\sqrt{\pi}\sqrt{2\,c^2\log(f)^2+2\,f^2}\left(\left(\left(i\,f^{\frac{ce^2}{4\left(c^2\log(f)^2+f^2\right)}}f^a\cos\left(\frac{4\,c^2d\log(f)^2-e^2f+4\,df^2}{4\left(c^2\log(f)^2+f^2\right)}\right)+f^{\frac{ce^2}{4\left(c^2\log(f)^2+f^2\right)}}f^a\sin\left(\frac{4\,c^2d\log(f)^2\ldots}{4\left(c^2\log(\ldots\right.}\right.\right.\right.\right.}{\ldots}$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*f^(1/4*c*e^2/(c^2*log(f)^2 +
 f^2))*f^a*cos(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2
)) + f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 - e^2
*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*e)/s
qrt(-c*log(f) + I*f)) + (-I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(1/4*
(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) + f^(1/4*c*e^2/(
c^2*log(f)^2 + f^2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*
log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f
)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^2*log(f)^
2 + 2*f^2)*((f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(1/4*(4*c^2*d*log(f)
^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) - I*f^(1/4*c*e^2/(c^2*log(f)^2
+ f^2))*f^a*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^
2)))*erf(1/2*(2*(c*log(f) - I*f)*x - I*e)/sqrt(-c*log(f) + I*f)) + (f^(1/4*
c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2
)/(c^2*log(f)^2 + f^2)) + I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(1/4*
(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*l
og(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*log
(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Giac [F]

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+a}\cos\left(fx^2+ex+d\right)\,dx$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\,\cos\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int(f^(a + c*x^2)*cos(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*cos(d + e*x + f*x^2), x)
```

3.123 $\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 211

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+x(2if-c\log(f))}{\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id+\frac{e^2}{2if+c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+x(2if+c\log(f))}{\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}$$

```
[Out] 1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(-2
*I*d-e^2/(2*I*f-c*ln(f)))*f^a*erf((I*e+x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(
1/2))*Pi^(1/2)/(2*I*f-c*ln(f))^(1/2)+1/8*exp(2*I*d+e^2/(2*I*f+c*ln(f)))*f^a
*erfi((I*e+x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f
))^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{4561, 2235, 2325, 2266, 2236\}$

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a e^{-\frac{e^2}{-c\log(f)+2if}-2id}\,\mathrm{erf}\left(\frac{x(-c\log(f)+2if)+ie}{\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}} + \frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)+2if}+2id}\,\mathrm{erfi}\left(\frac{x(c\log(f)+2if)+ie}{\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}} + \frac{\sqrt{\pi}f^a\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(
(-2*I)*d - e^2/((2*I)*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + x*((2*I)*f - c
*Log[f]))/Sqrt[(2*I)*f - c*Log[f]]])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*
I)*d + e^2/((2*I)*f + c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + x*((2*I)*f + c*Lo
g[f]))/Sqrt[(2*I)*f + c*Log[f]]])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
```

```
x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+cx^2}+\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int \exp\left(-2id-2iex+a\log(f)-x^2(2if-c\log(f))\right)dx\\
&\quad+\frac{1}{4}\int \exp\left(2id+2iex+a\log(f)+x^2(2if+c\log(f))\right)dx\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}\\
&\quad+\frac{1}{4}\left(e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\right)\int \exp\left(\frac{(-2ie+2x(-2if+c\log(f)))^2}{4(-2if+c\log(f))}\right)dx\\
&\quad+\frac{1}{4}\left(e^{2id+\frac{e^2}{2if+c\log(f)}}f^a\right)\int \exp\left(\frac{(2ie+2x(2if+c\log(f)))^2}{4(2if+c\log(f))}\right)dx\\
&= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{ie+x(2if-c\log(f))}{\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}\\
&\quad+\frac{e^{2id+\frac{e^2}{2if+c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie+x(2if+c\log(f))}{\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)dx=\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{\sqrt{c}\sqrt{\log(f)}}\right.$$
$$+\frac{\sqrt[4]{-1}\left(-e^{\frac{e^2}{-2if+c\log(f)}}\text{erfi}\left(\frac{(-1)^{3/4}(e+2fx+icx\log(f))}{\sqrt{2f+ic\log(f)}}\right)(2f-ic\log(f))\sqrt{2f+ic\log(f)}(\cos(2d)-i\sin(2d))+e\right.}{4f^2+c^2\log^2(}$$

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + ((
-1)^(1/4)*(-(E^(e^2/((-2*I)*f + c*Log[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + I*
c*x*Log[f]))/Sqrt[2*f + I*c*Log[f]]]*(2*f - I*c*Log[f])*Sqrt[2*f + I*c*Log[
f]]*(Cos[2*d] - I*Sin[2*d])) + E^(e^2/((2*I)*f + c*Log[f]))*Erfi[((-1)^(1/4
)*(e + 2*f*x - I*c*x*Log[f]))/Sqrt[2*f - I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]
]*(2*f + I*c*Log[f])*((-I)*Cos[2*d] + Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/
8
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{2 i d \ln(f) c+4 d f-e^{2}}{c \ln(f)-2 i f}} \operatorname{erf}\left(x \sqrt{2 i f-c \ln(f)}+\frac{i e}{\sqrt{2 i f-c \ln(f)}}\right)}{8 \sqrt{2 i f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{\frac{2 i d \ln(f) c-4 d f+e^{2}}{2 i f+c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-2 i f}\, x+\frac{i e}{\sqrt{-c \ln(f)-2}}\right.}{8 \sqrt{-c \ln(f)-2 i f}}$

```
[In] int(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-2*I*f))/(2*I*f-c*l
n(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(1/2)+I*e/(2*I*f-c*ln(f))^(1/2))-1/8*Pi^(
1/2)*f^a*exp((2*I*d*ln(f)*c-4*d*f+e^2)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(1
/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+I*e/(-c*ln(f)-2*I*f)^(1/2))+1/4*f^a*Pi^(1
/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(155) = 310.

Time = 0.27 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.71

$$\int f^{a+cx^2} \cos^2\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{2\sqrt{\pi}\left(c^2\log(f)^2+4f^2\right)\sqrt{-c\log(f)}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)+\sqrt{\pi}\left(c^2\log(f)^2-2icf\log(f)\right)\sqrt{-c\log(}}{}$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log
(f))*x) + sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*
erf((c^2*x*log(f)^2 + 4*f^2*x + I*c*e*log(f) + 2*e*f)*sqrt(-c*log(f) - 2*I*
f)/(c^2*log(f)^2 + 4*f^2))*e^((a*c^2*log(f)^3 + 2*I*c^2*d*log(f)^2 - 2*I*e^
2*f + 8*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) + sqrt(
```

```
pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf((c^2*x*log(
f)^2 + 4*f^2*x - I*c*e*log(f) + 2*e*f)*sqrt(-c*log(f) + 2*I*f)/(c^2*log(f)^
2 + 4*f^2))*e^((a*c^2*log(f)^3 - 2*I*c^2*d*log(f)^2 + 2*I*e^2*f - 8*I*d*f^2
 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)))/(c^3*log(f)^3 + 4*c*f
^2*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x + f*x**2)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 863, normalized size of antiderivative = 4.09

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^(c*e^2/(c^2*log(f)^2 + 4*
f^2))*f^a*cos(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2))
+ f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*sin(2*(c^2*d*log(f)^2 - e^2*f + 4*d*
f^2)/(c^2*log(f)^2 + 4*f^2)))*erf(((c*log(f) - 2*I*f)*x - I*e)/sqrt(-c*log(
f) + 2*I*f)) + (-I*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*cos(2*(c^2*d*log(f)
^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)) + f^(c*e^2/(c^2*log(f)^2 + 4*
f^2))*f^a*sin(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)))
*erf(((c*log(f) + 2*I*f)*x + I*e)/sqrt(-c*log(f) - 2*I*f)))*sqrt(c*log(f) +
 sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2
 + 8*f^2)*((f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*cos(2*(c^2*d*log(f)^2 - e^
2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)) - I*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))
*f^a*sin(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)))*erf(
((c*log(f) - 2*I*f)*x - I*e)/sqrt(-c*log(f) + 2*I*f)) + (f^(c*e^2/(c^2*log(
f)^2 + 4*f^2))*f^a*cos(2*(c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 +
 4*f^2)) + I*f^(c*e^2/(c^2*log(f)^2 + 4*f^2))*f^a*sin(2*(c^2*d*log(f)^2 - e
^2*f + 4*d*f^2)/(c^2*log(f)^2 + 4*f^2)))*erf(((c*log(f) + 2*I*f)*x + I*e)/s
qrt(-c*log(f) - 2*I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(
-c*log(f)) + 2*sqrt(pi)*((c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(x*conjugate(s
qrt(-c*log(f)))) + (c^2*f^a*log(f)^2 + 4*f^(a + 2))*erf(sqrt(-c*log(f))*x))
)/((c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f)))
```

Giac [F]

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+a}\cos\left(fx^2+ex+d\right)^2\,dx$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\cos\left(f\,x^2+e\,x+d\right)^2\,dx$$

```
[In] int(f^(a + c*x^2)*cos(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + c*x^2)*cos(d + e*x + f*x^2)^2, x)
```

3.124 $\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 369

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx = \frac{3e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} + \frac{e^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{3ie+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}} + \frac{3e^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} + \frac{e^{3id+\frac{9e^2}{4(3if+c\log(f))}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{3ie+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

```
[Out] 3/16*exp(-I*d-e^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e+2*x*(I*f-c*ln(f)))/(I
*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/16*exp(-3*I*d-9/4*e^2/(3*
I*f-c*ln(f)))*f^a*erf(1/2*(3*I*e+2*x*(3*I*f-c*ln(f)))/(3*I*f-c*ln(f))^(1/2)
)*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)+3/16*exp(I*d+e^2/(4*I*f+4*c*ln(f)))*f^a*er
fi(1/2*(I*e+2*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^
(1/2)+1/16*exp(3*I*d+9/4*e^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(3*I*e+2*x*(3*I*
f+c*ln(f)))/(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)dx=\frac{\sqrt{\pi}f^a\exp\left(-\frac{9e^2}{4(-c\log(f)+3if)}-3id\right)\text{erf}\left(\frac{2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3\sqrt{\pi}f^ae^{-\frac{e^2}{-4c\log(f)+4if}-id}\text{erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{3\sqrt{\pi}f^ae^{\frac{e^2}{4c\log(f)+4if}+id}\text{erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}+\frac{\sqrt{\pi}f^ae^{\frac{9e^2}{4(c\log(f)+3if)}+3id}\text{erfi}\left(\frac{2x(c\log(f)+3if)+3ie}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}$$

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*(I*f
 - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt[I*f - c*Log[f]]) + (E^((-
3*I)*d - (9*e^2)/(4*((3*I)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e + 2*x*
((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/(16*Sqrt[(3*I)*f - c*L
og[f]]) + (3*E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e +
2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]])
+ (E^((3*I)*d + (9*e^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[((3*I)*
e + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])])/(16*Sqrt[(3*I)
*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{8}e^{-3i(d+ex+fx^2)}f^{a+cx^2} + \frac{3}{8}\exp\left(2id+2iex+2ifx^2-3i(d+ex+fx^2)\right)f^{a+cx^2} + \frac{3}{8}\exp\left(4id+4iex+4ifx^2-3i(d+ex+fx^2)\right)f^{a+cx^2} + \frac{1}{8}\exp\left(6id+6iex+6ifx^2-3i(d+ex+fx^2)\right)f^{a+cx^2}\right)dx$$

$$= \frac{1}{8}\int e^{-3i(d+ex+fx^2)}f^{a+cx^2}\,dx + \frac{1}{8}\int \exp\left(6id+6iex+6ifx^2-3i(d+ex+fx^2)\right)f^{a+cx^2}\,dx + \frac{3}{8}\int \exp\left(2id+2iex+2ifx^2-3i(d+ex+fx^2)\right)f^{a+cx^2}\,dx + \frac{3}{8}\int \exp\left(4id+4iex+4ifx^2-3i(d+ex+fx^2)\right)f^{a+cx^2}\,dx$$

$$= \frac{1}{8}\int \exp\left(-3id-3iex+a\log(f)-x^2(3if-c\log(f))\right)dx + \frac{1}{8}\int \exp\left(3id+3iex+a\log(f)+x^2(3if+c\log(f))\right)dx + \frac{3}{8}\int \exp\left(-id-iex+a\log(f)-x^2(if-c\log(f))\right)dx + \frac{3}{8}\int \exp\left(id+iex+a\log(f)+x^2(if+c\log(f))\right)dx$$

$$= \frac{1}{8}\left(3e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(-ie+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx + \frac{1}{8}\left(e^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a\right)\int \exp\left(\frac{(-3ie+2x(-3if+c\log(f)))^2}{4(-3if+c\log(f))}\right)dx + \frac{1}{8}\left(e^{3id+\frac{9e^2}{4(3if+c\log(f))}}f^a\right)\int \exp\left(\frac{(3ie+2x(3if+c\log(f)))^2}{4(3if+c\log(f))}\right)dx + \frac{1}{8}\left(3e^{id+\frac{e^2}{4if+4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+2x(if+c\log(f)))^2}{4(if+c\log(f))}\right)dx$$

$$= \frac{3e^{-id-\frac{e^2}{4if-4c\log(f)}} f^a \sqrt{\pi}\operatorname{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} + \frac{e^{-3id-\frac{9e^2}{4(3if-c\log(f))}} f^a \sqrt{\pi}\operatorname{erf}\left(\frac{3ie+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}}$$
$$+ \frac{3e^{id+\frac{e^2}{4if+4c\log(f)}} f^a \sqrt{\pi}\operatorname{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} + \frac{e^{3id+\frac{9e^2}{4(3if+c\log(f))}} f^a \sqrt{\pi}\operatorname{erfi}\left(\frac{3ie+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

Mathematica [A] (verified)

Time = 4.96 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.33

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\sqrt[4]{-1}f^a\sqrt{\pi}\left(3e^{\frac{e^2}{4if+4c\log(f)}}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(e+2fx-2icx\log(f))}{2\sqrt{f-ic\log(f)}}\right)\sqrt{f-ic\log(f)}\left(9f^3+9icf^2\log(f)+c^2f\log^2(f)+\right.\right.}{}$$

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^a*Sqrt[Pi]*(3*E^(e^2/((4*I)*f + 4*c*Log[f]))*Erfi[((-1)^(1/4)
*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log
[f]]*(9*f^3 + (9*I)*c*f^2*Log[f] + c^2*f*Log[f]^2 + I*c^3*Log[f]^3)*((-I)*C
os[d] + Sin[d]) + (f - I*c*Log[f])*(-((3*f - I*c*Log[f])*(3*E^(e^2/((-4*I)*
f + 4*c*Log[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f
 + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*(3*f + I*c*Log[f])*(Cos[d] - I*Sin[d]
) + E^((9*e^2)/(4*((-3*I)*f + c*Log[f])))*Erfi[((-1)^(3/4)*(3*e + 6*f*x + (
2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*(f + I*c*Log[f])*Sqrt[3*f + I
*c*Log[f]]*(Cos[3*d] - I*Sin[3*d]))) + E^((9*e^2)/(4*((3*I)*f + c*Log[f])))
*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f
]])]*Sqrt[3*f - I*c*Log[f]]*(3*f^2 + (4*I)*c*f*Log[f] - c^2*Log[f]^2)*((-I)
*Cos[3*d] + Sin[3*d]))))/(16*(9*f^4 + 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 i d \ln(f) c+12 d f-3 e^{2}\right)}{4(c\ln(f)-3 i f)}} \operatorname{erf}\left(x\sqrt{3 i f-c\ln(f)}+\frac{3 i e}{2\sqrt{3 i f-c\ln(f)}}\right)}{16\sqrt{3 i f-c\ln(f)}}+\frac{3\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 i d \ln(f) c+4 d f-e^{2}}{4(c\ln(f)-i f)}} \operatorname{erf}\left(x\sqrt{i f-c\ln(f)}+\frac{i e}{2\sqrt{i f-c}}\right.}{16\sqrt{i f-c\ln(f)}}$

```
[In] int(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3/4*(4*I*d*ln(f)*c+12*d*f-3*e^2)/(c*ln(f)-3*I*f))/(3
*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f-c*ln(f))^(1/2)+3/2*I*e/(3*I*f-c*ln(f))^(1/
```

```
2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-I*f))/(I*
f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(f))^(1/2))-3/1
6*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I
*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f)^(1/2))-1/16*Pi
^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c-12*d*f+3*e^2)/(3*I*f+c*ln(f)))/(-c*ln(f)-
3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+3/2*I*e/(-c*ln(f)-3*I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(269) = 538.

Time = 0.31 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.92

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\left(c^3\log(f)^3-3i\,c^2f\log(f)^2+cf^2\log(f)-3i\,f^3\right)\sqrt{-c\log(f)-3i\,f}\,\mathrm{erf}\left(\frac{\left(2\,c^2x\log(f)^2+18\,f^2x+3i\,ce\log(f)\right.}{2\left(c^2\log(f)^2+9\right.}\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 18*f^2*x + 3*I*c*e*lo
g(f) + 9*e*f)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c
^2*log(f)^3 + 12*I*c^2*d*log(f)^2 - 27*I*e^2*f + 108*I*d*f^2 + 9*(c*e^2 + 4
*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(c^3*log(f)^3 + 3*I*c^2*
f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(2*c^2
*x*log(f)^2 + 18*f^2*x - 3*I*c*e*log(f) + 9*e*f)*sqrt(-c*log(f) + 3*I*f)/(c
^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c^2*log(f)^3 - 12*I*c^2*d*log(f)^2 + 27*I
*e^2*f - 108*I*d*f^2 + 9*(c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^2))
+ 3*sqrt(pi)*(c^3*log(f)^3 - I*c^2*f*log(f)^2 + 9*c*f^2*log(f) - 9*I*f^3)*s
qrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x + I*c*e*log(f) + e
*f)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*log(f)^3 +
4*I*c^2*d*log(f)^2 - I*e^2*f + 4*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*l
og(f)^2 + f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 + I*c^2*f*log(f)^2 + 9*c*f^2*log
(f) + 9*I*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x -
I*c*e*log(f) + e*f)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a
*c^2*log(f)^3 - 4*I*c^2*d*log(f)^2 + I*e^2*f - 4*I*d*f^2 + (c*e^2 + 4*a*f^2
)*log(f))/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^
4)
```

Sympy [F]

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+e*x+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x + f*x**2)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2180 vs. 2(269) = 538.

Time = 0.30 (sec) , antiderivative size = 2180, normalized size of antiderivative = 5.91

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^(9/4*c*e^2/(c^2*log
(f)^2 + 9*f^2))*f^a*log(f)^2 + I*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^(a
+ 2))*cos(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2
)) + (c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^a*log(f)^2 + f^(9/4*c*e^2/
(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*sin(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36
*d*f^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*f)*x - 3*I*e)/s
qrt(-c*log(f) + 3*I*f)) + ((-I*c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^a
*log(f)^2 - I*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*cos(3/4*(4*c^
2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^(9/4*c*
e^2/(c^2*log(f)^2 + 9*f^2))*f^a*log(f)^2 + f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f
^2))*f^(a + 2))*sin(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)
^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + 3*I*e)/sqrt(-c*log(f) - 3*I
*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*l
og(f)^2 + 2*f^2)*(((-I*c^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*log(f)^2
- 9*I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^(a + 2))*cos(1/4*(4*c^2*d*log(f)
^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) - (c^2*f^(1/4*c*e^2/(c^2*log(f)
^2 + f^2))*f^a*log(f)^2 + 9*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^(a + 2))*s
in(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*
(2*(c*log(f) - I*f)*x - I*e)/sqrt(-c*log(f) + I*f)) + ((I*c^2*f^(1/4*c*e^2/
(c^2*log(f)^2 + f^2))*f^a*log(f)^2 + 9*I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))
*f^(a + 2))*cos(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^
2)) - (c^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*log(f)^2 + 9*f^(1/4*c*e^2
/(c^2*log(f)^2 + f^2))*f^(a + 2))*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f
^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log
```

```
(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^
2*log(f)^2 + 18*f^2)*(((c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^a*log(f)
^2 + f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*cos(3/4*(4*c^2*d*log(f
)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)) - (I*c^2*f^(9/4*c*e^2/(c^
2*log(f)^2 + 9*f^2))*f^a*log(f)^2 + I*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*
f^(a + 2))*sin(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 +
9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*f)*x - 3*I*e)/sqrt(-c*log(f) + 3*I*f))
+ ((c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^a*log(f)^2 + f^(9/4*c*e^2/(c
^2*log(f)^2 + 9*f^2))*f^(a + 2))*cos(3/4*(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d
*f^2)/(c^2*log(f)^2 + 9*f^2)) - (-I*c^2*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2)
)*f^a*log(f)^2 - I*f^(9/4*c*e^2/(c^2*log(f)^2 + 9*f^2))*f^(a + 2))*sin(3/4*
(4*c^2*d*log(f)^2 - 9*e^2*f + 36*d*f^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2
*(c*log(f) + 3*I*f)*x + 3*I*e)/sqrt(-c*log(f) - 3*I*f)))*sqrt(-c*log(f) + s
qrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((c^2
*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*log(f)^2 + 9*f^(1/4*c*e^2/(c^2*log(
f)^2 + f^2))*f^(a + 2))*cos(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*l
og(f)^2 + f^2)) + (-I*c^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*log(f)^2 -
 9*I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^(a + 2))*sin(1/4*(4*c^2*d*log(f)^
2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x -
 I*e)/sqrt(-c*log(f) + I*f)) + ((c^2*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a
*log(f)^2 + 9*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^(a + 2))*cos(1/4*(4*c^2*
d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2)) + (I*c^2*f^(1/4*c*e^2/(
c^2*log(f)^2 + f^2))*f^a*log(f)^2 + 9*I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*
f^(a + 2))*sin(1/4*(4*c^2*d*log(f)^2 - e^2*f + 4*d*f^2)/(c^2*log(f)^2 + f^2
)))*erf(1/2*(2*(c*log(f) + I*f)*x + I*e)/sqrt(-c*log(f) - I*f)))*sqrt(-c*lo
g(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f
^4)
```

Giac [F]

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+a}\cos\left(fx^2+ex+d\right)^3\,dx$$

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\cos\left(f\,x^2+e\,x+d\right)^3\,dx$$

```
[In] int(f^(a + c*x^2)*cos(d + e*x + f*x^2)^3,x)
[Out] int(f^(a + c*x^2)*cos(d + e*x + f*x^2)^3, x)
```

3.125 $\int f^{a+bx+cx^2} \cos(d+ex) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 172

$$\int f^{a+bx+cx^2} \cos(d+ex) \, dx = -\frac{e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}} f^a \sqrt{\pi}\operatorname{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}} f^a \sqrt{\pi}\operatorname{erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 1/4*exp(-I*d+1/4*(e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-I*e+b*ln(f)+2*c*x*
ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(I*d+1/4*(e
-I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)
^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {4561, 2325, 2266, 2235}

$$\int f^{a+bx+cx^2} \cos(d+ex) \, dx = \frac{\sqrt{\pi} f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi} f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x],x]
```

```
[Out] -1/4*(E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e -
 b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]])
 + (E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Lo
g[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-id-iex}f^{a+bx+cx^2}+\frac{1}{2}e^{id+iex}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{2}\int e^{-id-iex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{id+iex}f^{a+bx+cx^2}\,dx\\
&= \frac{1}{2}\int \exp\left(-id+a\log(f)+cx^2\log(f)-x(ie-b\log(f))\right)dx\\
&\quad+\frac{1}{2}\int \exp\left(id+a\log(f)+cx^2\log(f)+x(ie+b\log(f))\right)dx\\
&= \frac{1}{2}\left(e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\\
&\quad+\frac{1}{2}\left(e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\\
&= -\frac{e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\cos(d+ex)\,dx = \frac{e^{\frac{e(e-2ib\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(e^{\frac{ibe}{c}}\operatorname{erfi}\left(\frac{-ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)-i\sin(d))+\operatorname{erfi}\left(\frac{ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(d)+i\sin(d)\right.}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x],x]
```

```
[Out] (E^((e*(e - (2*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*(E^((
I*b*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos
[d] - I*Sin[d]) + Erfi[(I*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]
*(Cos[d] + I*Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{2i\ln(f)be-4id\ln(f)c+e^{2}}{4\ln(f)c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{b\ln(f)-ie}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{-\frac{2i\ln(f)be-4id\ln(f)c-e^{2}}{4\ln(f)c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\right.}{4\sqrt{-c\ln(f)}}$

```
[In] int(f^(c*x^2+b*x+a)*cos(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c+e^2)/
ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*e)/(-c*ln(
f))^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*(2*I*ln(f)*b*e-4*I*d*ln
(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(I*e+b*ln(
f))/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02

$$\int f^{a+bx+cx^2}\cos(d+ex)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2cx+b)\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{\left(b^2-4ac\right)\log(f)^2-e^2+2(2icd-ibe)\log(f)}{4c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}}{4c\log(f)}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - I*e)*sqrt(-c*l
og(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 + 2*(2*I*c*d - I*b
*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log
(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^
2 + 2*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2} \cos(d+ex)\,dx = \int f^{a+bx+cx^2} \cos{(d+ex)}\,dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*cos(e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + e*x), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.06

$$\int f^{a+bx+cx^2} \cos(d+ex)\,dx =$$

$$-\frac{\sqrt{\pi}\left(f^a\left(\cos\left(-\frac{2\,cd-be}{2\,c}\right)-i\,\sin\left(-\frac{2\,cd-be}{2\,c}\right)\right)\operatorname{erf}\left(x\overline{\sqrt{-c\log\left(f\right)}}-\frac{1}{2}\left(b\log\left(f\right)+i\,e\right)\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^2}{4\,c\log(f)}\right)}+\cdots\right.}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(f^a*(cos(-1/2*(2*c*d - b*e)/c) - I*sin(-1/2*(2*c*d - b*e)/c)
)*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + I*e)*conjugate(1/sqrt(
-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(cos(-1/2*(2*c*d - b*e)/c) + I*si
n(-1/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) -
 I*e)*conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + f^a*(cos(-1/2*
(2*c*d - b*e)/c) - I*sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*l
og(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))) + f^a*(cos(
-1/2*(2*c*d - b*e)/c) + I*sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f)
+ b*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))))*sqrt(
-c*log(f))/(c*f^(1/4*b^2/c)*log(f))
```

Giac [F]

$$\int f^{a+bx+cx^2}\cos(d+ex)\,dx=\int f^{cx^2+bx+a}\cos\left(ex+d\right)\,dx$$

[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d),x, algorithm="giac")

[Out] integrate(f^(c*x^2 + b*x + a)*cos(e*x + d), x)

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\,\cos\left(d+e\,x\right)\,dx$$

[In] int(f^(a + b*x + c*x^2)*cos(d + e*x),x)

[Out] int(f^(a + b*x + c*x^2)*cos(d + e*x), x)

3.126 $\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 231

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{2ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2id-\frac{(2ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{2ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 1/8*exp(-2*I*d+1/4*(2*e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-2*I*e+b*ln(f)+
2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*I*
d-1/4*(2*I*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(2*I*e+b*ln(f)+2*c*x*ln(f))/c
^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^(a-1/4*b^2/c)*erfi(1
/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

$= \{4561, 2266, 2235, 2325\}$

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx = \frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi}f^a e^{\frac{(2e+ib\log(f))^2}{4c\log(f)}-2id}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi}f^a e^{2id-\frac{(b\log(f)+2ie)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d + (2*e + I*b*Log[f])^2/(4*c*Log[f]))
*f^a*Sqrt[Pi]*Erfi[((2*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[
f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^((2*I)*d - ((2*I)*e + b*Log[f])^2/(4*c
*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2id-2iex}f^{a+bx+cx^2}+\frac{1}{4}e^{2id+2iex}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2id-2iex}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2id+2iex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\
&= \frac{1}{4}\int \exp\left(-2id+a\log(f)+cx^2\log(f)-x(2ie-b\log(f))\right)dx\\
&\quad+\frac{1}{4}\int \exp\left(2id+a\log(f)+cx^2\log(f)+x(2ie+b\log(f))\right)dx\\
&\quad+\frac{1}{2}f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}\,dx\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(\exp\left(-2id\right.\right.\\
&\qquad\left.\left.+\frac{(2e+ib\log(f))^2}{4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(-2ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\\
&\quad+\frac{1}{4}\left(e^{2id-\frac{(2ie+b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(2ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\\
&\quad-\frac{\exp\left(-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}\right)f^a\sqrt{\pi}\,\text{erfi}\left(\frac{2ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}\\
&\quad+\frac{e^{2id-\frac{(2ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{2ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx$$

$$= \frac{e^{-\frac{ibe}{c}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(2e^{\frac{ibe}{c}}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+e^{\frac{e(e+2ib\log(f))}{c\log(f)}}\text{erfi}\left(\frac{-2ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cos(2d)-i\sin(2d))+e^{\frac{e^2}{c\log(}}\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(2*E^((I*b*e)/c)*Erfi[((b + 2*c*x)*Sqrt[Log[f]]
)/(2*Sqrt[c])] + E^((e*(e + (2*I)*b*Log[f]))/(c*Log[f]))*Erfi[((-2*I)*e + (
```

b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + E^(e
^2/(c*Log[f]))*Erfi[((2*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])
]*(Cos[2*d] + I*Sin[2*d])))/(8*Sqrt[c]*E^((I*b*e)/c)*Sqrt[Log[f]])

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{i \ln(f) b e-2 i d \ln(f) c+e^{2}}{\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{b\ln(f)-2ie}{2\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{-\frac{i \ln(f) b e-2 i d \ln(f) c-e^{2}}{\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x-\right.}{8\sqrt{-c\ln(f)}}$

[In] int(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -1/8*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp((I*ln(f)*b*e-2*I*d*ln(f)*c+e^2)/ln(f)/
c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*I*e)/(-c*ln(f))^
(1/2))-1/8*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-(I*ln(f)*b*e-2*I*d*ln(f)*c-e^2)
/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-c*
ln(f))^(1/2))-1/4*Pi^(1/2)*f^(-1/4*b^2/c)*f^a/(-c*ln(f))^(1/2)*erf(-(-c*ln(
f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.97

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx = -\frac{\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2\,cx+b)\log(f)-2i\,e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)e^{\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-4\,e^2+4\,(2i\,cd-i\,be)\log(f)}{4\,c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{er}}{8\,c\log(}$$

[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x, algorithm="fricas")

[Out] -1/8*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - 2*I*e)*sqrt(-c
*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*e^2 + 4*(2*I*c*d -
 I*b*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)
*log(f) + 2*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^
2 - 4*e^2 + 4*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))) + 2*sqrt(pi)*sqrt(-c*l
og(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c*l
og(f))

Sympy [F]

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx = \int f^{a+bx+cx^2}\cos^2(d+ex)\,dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*cos(e*x+d)**2,x)
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + e*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.73

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx$$
$$= \frac{\sqrt{\pi}\left(f^a\left(\cos\left(-\frac{2\,cd-be}{c}\right)-i\,\sin\left(-\frac{2\,cd-be}{c}\right)\right)\operatorname{erf}\left(x\overline{\sqrt{-c\log(f)}}-\frac{1}{2}\left(b\log(f)+2i\,e\right)\overline{\frac{1}{\sqrt{-c\log(f)}}}\right)e^{\left(\frac{e^2}{c\log(f)}\right)}+f\right.}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x, algorithm="maxima")
[Out] 1/16*sqrt(pi)*(f^a*(cos(-(2*c*d - b*e)/c) - I*sin(-(2*c*d - b*e)/c))*erf(x*
conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + 2*I*e)*conjugate(1/sqrt(-c*log
(f))))*e^(e^2/(c*log(f))) + f^a*(cos(-(2*c*d - b*e)/c) + I*sin(-(2*c*d - b*
e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) - 2*I*e)*conjugate(
1/sqrt(-c*log(f))))*e^(e^2/(c*log(f))) + f^a*(cos(-(2*c*d - b*e)/c) - I*sin
(-(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) + 2*I*e)*sqrt(-c*log(f
))/(c*log(f)))*e^(e^2/(c*log(f))) + f^a*(cos(-(2*c*d - b*e)/c) + I*sin(-(2*
c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f) - 2*I*e)*sqrt(-c*log(f))/(c
*log(f)))*e^(e^2/(c*log(f))) + 2*f^a*erf(-1/2*b*conjugate(1/sqrt(-c*log(f))
)*log(f) + x*conjugate(sqrt(-c*log(f)))) - 2*f^a*erf(1/2*(2*c*x*log(f) + b*
log(f))/sqrt(-c*log(f))))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Giac [F]

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx = \int f^{cx^2+bx+a}\cos(ex+d)^2\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*cos(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\cos\left(d+e\,x\right)^2\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*cos(d + e*x)^2,x)
[Out] int(f^(a + b*x + c*x^2)*cos(d + e*x)^2, x)
```

3.127 $\int f^{a+bx+cx^2} \cos^3(d+ex)\, dx$

Optimal result

Integrand size = 21, antiderivative size = 346

$$\int f^{a+bx+cx^2} \cos^3(d+ex)\, dx = -\frac{3e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}} f^a \sqrt{\pi}\operatorname{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-3id+\frac{(3e+ib\log(f))^2}{4c\log(f)}} f^a \sqrt{\pi}\operatorname{erfi}\left(\frac{3ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{3e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}} f^a \sqrt{\pi}\operatorname{erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{3id-\frac{(3ie+b\log(f))^2}{4c\log(f)}} f^a \sqrt{\pi}\operatorname{erfi}\left(\frac{3ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 3/16*exp(-I*d+1/4*(e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-I*e+b*ln(f)+2*c*x
*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*I*d+1
/4*(3*e+I*b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-3*I*e+b*ln(f)+2*c*x*ln(f))/c^(
1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+3/16*exp(I*d+1/4*(e-I*b*ln(f
))^2/c/ln(f))*f^a*erfi(1/2*(I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*P
i^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*I*d-1/4*(3*I*e+b*ln(f))^2/c/ln(f))*f
^a*erfi(1/2*(3*I*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4561, 2325, 2266, 2235}

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx = -\frac{3\sqrt{\pi}f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi}f^a e^{\frac{(3e+ib\log(f))^2}{4c\log(f)}-3id}\text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{3\sqrt{\pi}f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\text{erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi}f^a e^{3id-\frac{(b\log(f)+3ie)^2}{4c\log(f)}}\text{erfi}\left(\frac{b\log(f)+2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x]^3,x]
```

```
[Out] (-3*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - b
*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]
) - (E^((-3*I)*d + (3*e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3
*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqr
t[Log[f]]) + (3*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi
[(I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqr
t[Log[f]]) + (E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi
]*Erfi[((3*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*S
qrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{8}e^{-id-iex}f^{a+bx+cx^2}+\frac{3}{8}e^{id+iex}f^{a+bx+cx^2}+\frac{1}{8}e^{-3id-3iex}f^{a+bx+cx^2} \right. \\
&\qquad \left. +\frac{1}{8}e^{3id+3iex}f^{a+bx+cx^2}\right)dx \\
&= \frac{1}{8}\int e^{-3id-3iex}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int e^{3id+3iex}f^{a+bx+cx^2}\,dx \\
&\quad +\frac{3}{8}\int e^{-id-iex}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{id+iex}f^{a+bx+cx^2}\,dx \\
&= \frac{1}{8}\int \exp\left(-3id+a\log(f)+cx^2\log(f)-x(3ie-b\log(f))\right)dx \\
&\quad +\frac{1}{8}\int \exp\left(3id+a\log(f)+cx^2\log(f)+x(3ie+b\log(f))\right)dx \\
&\quad +\frac{3}{8}\int \exp\left(-id+a\log(f)+cx^2\log(f)-x(ie-b\log(f))\right)dx \\
&\quad +\frac{3}{8}\int \exp\left(id+a\log(f)+cx^2\log(f)+x(ie+b\log(f))\right)dx \\
&= \frac{1}{8}\left(3e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx \\
&\quad +\frac{1}{8}\left(3e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx \\
&\quad +\frac{1}{8}\left(\exp\left(-3id \right.\right. \\
&\qquad \left.\left. +\frac{(3e+ib\log(f))^2}{4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(-3ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx \\
&\quad +\frac{1}{8}\left(e^{3id-\frac{(3ie+b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(3ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx
\end{aligned}$$

$$= -\frac{3e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{\exp\left(-3id+\frac{(3e+ib\log(f))^2}{4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{3e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{3id-\frac{(3ie+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.12

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx = \frac{e^{\frac{e(e-6ib\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(e^{\frac{e(2e+3ib\log(f))}{c\log(f)}}\cos(3d)\operatorname{erfi}\left(\frac{-3ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+e^{\frac{2e^2}{c\log(f)}}\cos(3d)\operatorname{erfi}\left(\frac{3ie+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3\right.}{}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x]^3,x]
```

```
[Out] (E^((e*(e - (6*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*(E^((
e*(2*e + (3*I)*b*Log[f]))/(c*Log[f]))*Cos[3*d]*Erfi[((-3*I)*e + (b + 2*c*x)
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + E^((2*e^2)/(c*Log[f]))*Cos[3*d]*Erfi[(
(3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^(((2*I)*b*e)/
c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[d] - I
*Sin[d]) + 3*E^((I*b*e)/c)*Erfi[(I*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[
Log[f]])]*(Cos[d] + I*Sin[d]) - I*E^((e*(2*e + (3*I)*b*Log[f]))/(c*Log[f]))
*Erfi[((-3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*Sin[3*d] +
I*E^((2*e^2)/(c*Log[f]))*Erfi[((3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqr
t[Log[f]])]*Sin[3*d]))/(16*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{\frac{3i\ln(f) b e}{2}-3 i d \ln(f) c+\frac{9e^{2}}{4}}{c\ln(f)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{b\ln(f)-3ie}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}-\frac{3\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{2i\ln(f) b e-4 i d\ln(f) c+e^{2}}{4\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c\ln\right.}{16\sqrt{-c\ln(f)}}$

[In] int(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x,method=_RETURNVERBOSE)

[Out] -1/16*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(3/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*I*e)/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*e)/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-3/4*(2*I*ln(f)*b*e-4*I*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-c*ln(f))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.99

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx =$$

$$-\frac{3\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{((2cx+b)\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^2-4ac)\log(f)^2-e^2+2(2icd-ibe)\log(f)}{4c\log(f)}\right)}+3\sqrt{\pi}\sqrt{-c\log(f)}}{}$$

[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x, algorithm="fricas")

[Out] -1/16*(3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 + 2*(2*I*c*d - I*b*e)*log(f))/(c*log(f))) + 3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 + 2*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 9*e^2 + 6*(2*I*c*d - I*b*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 9*e^2 + 6*(-2*I*c*d + I*b*e)*log(f))/(c*log(f))))/(c*log(f))

Sympy [F]

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx = \int f^{a+bx+cx^2}\cos^3(d+ex)\,dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*cos(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + e*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.97

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/32*sqrt(pi)*(f^a*(cos(-3/2*(2*c*d - b*e)/c) - I*sin(-3/2*(2*c*d - b*e)/c
))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + 3*I*e)*conjugate(1/sq
rt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(cos(-3/2*(2*c*d - b*e)/c) + I
*sin(-3/2*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f
) - 3*I*e)*conjugate(1/sqrt(-c*log(f))))*e^(9/4*e^2/(c*log(f))) + f^a*(cos(
-3/2*(2*c*d - b*e)/c) - I*sin(-3/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f)
+ b*log(f) + 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(9/4*e^2/(c*log(f))) + f^
a*(cos(-3/2*(2*c*d - b*e)/c) + I*sin(-3/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*
log(f) + b*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(9/4*e^2/(c*log(f)
)) + 3*f^a*(cos(-1/2*(2*c*d - b*e)/c) - I*sin(-1/2*(2*c*d - b*e)/c))*erf(x*
conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) + I*e)*conjugate(1/sqrt(-c*log(f
))))*e^(1/4*e^2/(c*log(f))) + 3*f^a*(cos(-1/2*(2*c*d - b*e)/c) + I*sin(-1/2
*(2*c*d - b*e)/c))*erf(x*conjugate(sqrt(-c*log(f))) - 1/2*(b*log(f) - I*e)*
conjugate(1/sqrt(-c*log(f))))*e^(1/4*e^2/(c*log(f))) + 3*f^a*(cos(-1/2*(2*c
*d - b*e)/c) - I*sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) + b*log(f
) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))) + 3*f^a*(cos(-1
/2*(2*c*d - b*e)/c) + I*sin(-1/2*(2*c*d - b*e)/c))*erf(1/2*(2*c*x*log(f) +
b*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(1/4*e^2/(c*log(f))))*sqrt(-c
*log(f))/(c*f^(1/4*b^2/c)*log(f))
```

Giac [F]

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx=\int f^{cx^2+bx+a}\cos{(ex+d)}^3\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*cos(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\cos{(d+e\,x)}^3\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*cos(d + e*x)^3,x)
[Out] int(f^(a + b*x + c*x^2)*cos(d + e*x)^3, x)
```

3.128 $\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 189

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx=-\frac{e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] -1/4*exp(-I*d+b^2*ln(f)^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(I*f-
c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/4*exp(I*d-b^2
*ln(f)^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(I*f+c*ln(f)))/(I*f+c
*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx=\frac{\sqrt{\pi}f^ae^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\mathrm{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}-\frac{\sqrt{\pi}f^ae^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\mathrm{erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + f*x^2],x]
```

```
[Out] -1/4*(E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(
b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*Lo
g[f]] + (E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[
(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(4*Sqrt[I*f +
c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-id-ifx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{id+ifx^2}f^{a+bx+cx^2}\right)dx \\
&= \frac{1}{2}\int e^{-id-ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{id+ifx^2}f^{a+bx+cx^2}\,dx \\
&= \frac{1}{2}\int \exp\left(-id+a\log(f)+bx\log(f)-x^2(if-c\log(f))\right)dx \\
&\quad +\frac{1}{2}\int \exp\left(id+a\log(f)+bx\log(f)+x^2(if+c\log(f))\right)dx
\end{aligned}$$

$$=\frac{1}{2}\left(e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx+\frac{1}{2}\left(e^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(if+c\log(f)))^2}{4(if+c\log(f))}\right)dx$$

$$=-\frac{e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\text{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.22

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)dx=-\frac{(-1)^{3/4}e^{\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\left(\text{erfi}\left(\frac{(-1)^{3/4}(2fx+i(b+2cx)\log(f))}{2\sqrt{f+ic\log(f)}}\right)(f-ic\log(f))\sqrt{f+ic\log(f)}(-i\cos(d)-\sin(d$$

$$4\left(f^2+c^2\log\right.$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + f*x^2],x]
```

```
[Out] -1/4*((-1)^(3/4)*E^((b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*(Er
fi[((-1)^(3/4)*(2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*(f
 - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*((-I)*Cos[d] - Sin[d]) + E^(((I/2)*b^2*
f*Log[f]^2)/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(1/4)*(2*f*x - I*(b + 2*c*x)*L
og[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*(Co
s[d] + I*Sin[d])))/(f^2 + c^2*Log[f]^2)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2+4id\ln(f)c+4df}{4(c\ln(f)-if)}}\text{erf}\left(-x\sqrt{if-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{if-c\ln(f)}}\right)}{4\sqrt{if-c\ln(f)}}-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2-4id\ln(f)c+4df}{4(if+c\ln(f))}}\text{erf}\left(-\sqrt{-c\ln(f)-if}\,x+\right.}{4\sqrt{-c\ln(f)-if}}$

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*d*ln(f)*c+4*d*f)/(c*ln(f)-I*f))
/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(I*f-c*ln(f))^(
1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*d*ln(f)*c+4*d*f)/(I*f+c*ln
(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f
)-I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(145) = 290.

Time = 0.26 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.65

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}(c\log(f)-i\,f)\sqrt{-c\log(f)-i\,f}\,\mathrm{erf}\left(\frac{\left(2\,f^2x-i\,bf\log(f)+\left(2\,c^2x+bc\right)\log(f)^2\right)\sqrt{-c\log(f)-i\,f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(\frac{4\,af^2\log(f)-\left(b^2c-\cdots\right.}{\cdots}\right.}}{\cdots}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*
b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2
+ f^2))*e^(1/4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (
4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c*log(f) +
 I*f)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*log(f) + (2*c^2*x + b*
c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^2*lo
g(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I*c^2*d - I*b^2*f)*log(
f)^2)/(c^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + f*x**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(145) = 290.

Time = 0.25 (sec) , antiderivative size = 648, normalized size of antiderivative = 3.43

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx$$

$$=\frac{\sqrt{\pi}\sqrt{2\,c^2\log(f)^2+2\,f^2}\left(\left(i\,f^a\cos\left(\frac{4\,df^2+\left(4\,c^2d+b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2+f^2\right)}\right)+f^a\sin\left(\frac{4\,df^2+\left(4\,c^2d+b^2f\right)\log(f)^2}{4\left(c^2\log(f)^2+f^2\right)}\right)\right)\mathrm{erf}\left(\frac{2\,(c\log(\cdots}{2\,\sqrt{\cdots}}\right.\cdots}{\cdots}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*f^a*cos(1/4*(4*d*f^2 + (4*c^
2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^a*sin(1/4*(4*d*f^2 + (4*c^
2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x
 + b*log(f))/sqrt(-c*log(f) + I*f)) + (-I*f^a*cos(1/4*(4*d*f^2 + (4*c^2*d +
 b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + f^a*sin(1/4*(4*d*f^2 + (4*c^2*d +
 b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*
log(f))/sqrt(-c*log(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) -
 sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((f^a*cos(1/4*(4*d*f^2 + (4*c^2*d +
b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - I*f^a*sin(1/4*(4*d*f^2 + (4*c^2*d
+ b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b
*log(f))/sqrt(-c*log(f) + I*f)) + (f^a*cos(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)
*log(f)^2)/(c^2*log(f)^2 + f^2)) + I*f^a*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*
f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f
))/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^2
*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^2*e^(1/4*b^2*c*lo
g(f)^3/(c^2*log(f)^2 + f^2)))
```

Giac [F]

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx=\int f^{cx^2+bx+a}\cos\left(fx^2+d\right)\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cos\left(f\,x^2+d\right)\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*cos(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cos(d + f*x^2), x)
```

3.129 $\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 245

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log(f)-2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id-\frac{b^2\log^2(f)}{8if+4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{b\log(f)+2x(2if+c\log(f))}{2\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}$$

```
[Out] 1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)-1/8*exp(-2*I*d+b^2*ln(f)^2/(8*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*
ln(f)-2*x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f-c*ln(f))^
(1/2)+1/8*exp(2*I*d-b^2*ln(f)^2/(8*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*
x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{4561, 2266, 2235, 2325, 2236\}$

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+8if}-2id}\operatorname{erf}\left(\frac{b\log(f)-2x(-c\log(f)+2if)}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}} + \frac{\sqrt{\pi}f^a e^{2id-\frac{b^2\log^2(f)}{4c\log(f)+8if}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}} + \frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d + (b^2*Log[f]^2)/((8*I)*f - 4*c*Log[
f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((2*I)*f - c*Log[f]))/(2*Sqrt[(2*I)*f
 - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*I)*d - (b^2*Log[f]^2)
/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((2*I)*f + c*Log
[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
```

```
x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2id+2ifx^2}f^{a+bx+cx^2}\right)\,dx\\
&= \frac{1}{4}\int e^{-2id-2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2id+2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\
&= \frac{1}{4}\int \exp\left(-2id+a\log(f)+bx\log(f)-x^2(2if-c\log(f))\right)\,dx\\
&\quad+\frac{1}{4}\int \exp\left(2id+a\log(f)+bx\log(f)+x^2(2if+c\log(f))\right)\,dx\\
&\quad+\frac{1}{2}f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}\,dx\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\\
&\quad+\frac{1}{4}\left(e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-2if+c\log(f)))^2}{4(-2if+c\log(f))}\right)\,dx\\
&\quad+\frac{1}{4}\left(e^{2id-\frac{b^2\log^2(f)}{8if+4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(2if+c\log(f)))^2}{4(2if+c\log(f))}\right)\,dx\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log(f)-2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}\\
&\quad+\frac{e^{2id-\frac{b^2\log^2(f)}{8if+4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{b\log(f)+2x(2if+c\log(f))}{2\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 2.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.23

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx=\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2f^{-\frac{b^2}{4c}}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}\right.$$
$$+\frac{\sqrt[4]{-1}e^{\frac{b^2\log^2(f)}{8if-4c\log(f)}}\left(-\text{erfi}\left(\frac{(-1)^{3/4}(4fx+i(b+2cx)\log(f))}{2\sqrt{2f+ic\log(f)}}\right)(2f-ic\log(f))\sqrt{2f+ic\log(f)}(\cos(2d)-i\sin(2d))\right.}{4f^2+c^2}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f^
(b^2/(4*c))*Sqrt[Log[f]]) + ((-1)^(1/4)*E^((b^2*Log[f]^2)/((8*I)*f - 4*c*Lo
g[f]))*(-(Erfi[((-1)^(3/4)*(4*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f + I*
c*Log[f]])]*(2*f - I*c*Log[f])*Sqrt[2*f + I*c*Log[f]]*(Cos[2*d] - I*Sin[2*d
])) + E^((I*b^2*f*Log[f]^2)/(4*f^2 + c^2*Log[f]^2))*Erfi[((-1)^(1/4)*(4*f*x
 - I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Sqrt[2*f - I*c*Log[f]
]*(2*f + I*c*Log[f])*((-I)*Cos[2*d] + Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/
8
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+8 i d \ln(f) c+16 d f}{4(c\ln(f)-2 i f)}} \operatorname{erf}\left(-x\sqrt{2 i f-c\ln(f)}+\frac{\ln(f) b}{2\sqrt{2 i f-c\ln(f)}}\right)}{8\sqrt{2 i f-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-8 i d \ln(f) c+16 d f}{4(2 i f+c\ln(f))}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-2\right.}{8\sqrt{-c\ln(f)-2 i f}}$

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+8*I*d*ln(f)*c+16*d*f)/(c*ln(f)-2*I*
f))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*I*f-c
*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-8*I*d*ln(f)*c+16*d*f)
/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2*
ln(f)*b/(-c*ln(f)-2*I*f)^(1/2))-1/4*Pi^(1/2)*f^(-1/4*b^2/c)*f^a/(-c*ln(f))^
(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(185) = 370.

Time = 0.27 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.64

$$\int f^{a+bx+cx^2} \cos^2\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\left(c^2\log\left(f\right)^2-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-2i\,f}\,\operatorname{erf}\left(\frac{\left(8\,f^2x-2i\,bf\log(f)+\left(2\,c^2x+bc\right)\log(f)^2\right)\sqrt{-c\log(f)-2i\,f}}{2\left(c^2\log(f)^2+4\,f^2\right)}\right)e^{\left(\right.}}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(
1/2*(8*f^2*x - 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) -
2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c^2)*
```

```
log(f)^3 + 32*I*d*f^2 - 2*(-4*I*c^2*d - I*b^2*f)*log(f)^2)/(c^2*log(f)^2 +
4*f^2)) + sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*
erf(1/2*(8*f^2*x + 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f
) + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c
^2)*log(f)^3 - 32*I*d*f^2 - 2*(4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2
 + 4*f^2)) + 2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c
*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2
*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+d)**2,x)
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + f*x**2)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 997, normalized size of antiderivative = 4.07

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x, algorithm="maxima")
[Out] 1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*f^(1/4*b^2/c)*cos(1/2*(
16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)) + f^a*f^(1/4
*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f
^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f))/sqrt(-c*log(f) + 2*I*f))
+ (-I*f^a*f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^
2*log(f)^2 + 4*f^2)) + f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2
*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) + 2*I*f)*x + b*
log(f))/sqrt(-c*log(f) - 2*I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2
))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((f^a*f^(1/4*b^2
/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))
 - I*f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2
*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f))/sqrt(-c*lo
g(f) + 2*I*f)) + (f^a*f^(1/4*b^2/c)*cos(1/2*(16*d*f^2 + (4*c^2*d + b^2*f)*l
og(f)^2)/(c^2*log(f)^2 + 4*f^2)) + I*f^a*f^(1/4*b^2/c)*sin(1/2*(16*d*f^2 +
(4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) +
```

```
 2*I*f)*x + b*log(f))/sqrt(-c*log(f) - 2*I*f)))*sqrt(-c*log(f) + sqrt(c^2*l
og(f)^2 + 4*f^2))*sqrt(-c*log(f)) + 2*sqrt(pi)*((c^2*f^a*e^(1/4*b^2*c*log(f
)^3/(c^2*log(f)^2 + 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c
^2*log(f)^2 + 4*f^2)))*erf(-1/2*b*conjugate(1/sqrt(-c*log(f)))*log(f) + x*c
onjugate(sqrt(-c*log(f)))) - (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 +
 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2
)))*erf(1/2*(2*c*x*log(f) + b*log(f))/sqrt(-c*log(f)))))/((c^2*e^(1/4*b^2*c
*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*log(f)^2 + 4*f^2*e^(1/
4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c))*sqrt(-c*log(f)
))
```

Giac [F]

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx=\int f^{cx^2+bx+a}\cos\left(fx^2+d\right)^2\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cos\left(f\,x^2+d\right)^2\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*cos(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cos(d + f*x^2)^2, x)
```

3.130 $\int f^{a+bx+cx^2} \cos^3\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 378

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx = -\frac{3e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} - \frac{e^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}} + \frac{3e^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} + \frac{e^{3id-\frac{b^2\log^2(f)}{4(3if+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

```
[Out] -3/16*exp(-I*d+b^2*ln(f)^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(I*f
-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)-1/16*exp(-3*I*
d+b^2*ln(f)^2/(12*I*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(3*I*f-c*ln(f)))
/(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)+3/16*exp(I*d-b^2*ln(
f)^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(I*f+c*ln(f)))/(I*f+c*ln(
f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)+1/16*exp(3*I*d-1/4*b^2*ln(f)^2/(3*I
*f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln(f))^(1/
2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx=-\frac{3\sqrt{\pi}f^ae^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\operatorname{erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{\sqrt{\pi}f^ae^{\frac{b^2\log^2(f)}{-4c\log(f)+12if}-3id}\operatorname{erf}\left(\frac{b\log(f)-2x(-c\log(f)+3if)}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{\sqrt{\pi}f^a\exp\left(3id-\frac{b^2\log^2(f)}{4(c\log(f)+3if)}\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3if)}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}+\frac{3\sqrt{\pi}f^ae^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^3,x]
```

```
[Out] (-3*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*
Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt[I*f - c*
Log[f]]) - (E^((-3*I)*d + (b^2*Log[f]^2)/((12*I)*f - 4*c*Log[f]))*f^a*Sqrt[
Pi]*Erf[(b*Log[f] - 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])]
)/(16*Sqrt[(3*I)*f - c*Log[f]]) + (3*E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c
*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f +
 c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]]) + (E^((3*I)*d - (b^2*Log[f]^2)/(4*(
(3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((3*I)*f + c*Log[f]
))/(2*Sqrt[(3*I)*f + c*Log[f]])])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{8}e^{-id-ifx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{id+ifx^2}f^{a+bx+cx^2}+\frac{1}{8}e^{-3id-3ifx^2}f^{a+bx+cx^2}\right.\\
&\qquad\left.+\frac{1}{8}e^{3id+3ifx^2}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{8}\int e^{-3id-3ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int e^{3id+3ifx^2}f^{a+bx+cx^2}\,dx\\
&\quad+\frac{3}{8}\int e^{-id-ifx^2}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{id+ifx^2}f^{a+bx+cx^2}\,dx\\
&= \frac{1}{8}\int \exp\left(-3id+a\log(f)+bx\log(f)-x^2(3if-c\log(f))\right)dx\\
&\quad+\frac{1}{8}\int \exp\left(3id+a\log(f)+bx\log(f)+x^2(3if+c\log(f))\right)dx\\
&\quad+\frac{3}{8}\int \exp\left(-id+a\log(f)+bx\log(f)-x^2(if-c\log(f))\right)dx\\
&\quad+\frac{3}{8}\int \exp\left(id+a\log(f)+bx\log(f)+x^2(if+c\log(f))\right)dx\\
&= \frac{1}{8}\left(3e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx\\
&\quad+\frac{1}{8}\left(e^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-3if+c\log(f)))^2}{4(-3if+c\log(f))}\right)dx\\
&\quad+\frac{1}{8}\left(\exp\left(3id-\frac{b^2\log^2(f)}{4(3if+c\log(f))}\right)f^a\right)\int \exp\left(\frac{(b\log(f)+2x(3if+c\log(f)))^2}{4(3if+c\log(f))}\right)dx\\
&\quad+\frac{1}{8}\left(3e^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(if+c\log(f)))^2}{4(if+c\log(f))}\right)dx
\end{aligned}$$

$$= -\frac{3e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} - \frac{e^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}} + \frac{3e^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} + \frac{\exp\left(3id-\frac{b^2\log^2(f)}{4(3if+c\log(f))}\right)f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3285 vs. 2(378) = 756.

Time = 7.58 (sec) , antiderivative size = 3285, normalized size of antiderivative = 8.69

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx = \text{Result too large to show}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^3
*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f
 - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*(-1)^(1/4)*c*E^(((I/4)*b^2*Log[f
]^2)/(f - I*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2
*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*
(-1)^(3/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f*Cos[d]*Erfi[((-1
)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*
Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(f
 - I*c*Log[f]))*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log
[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4
)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[((-1)^(1/4)
*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Sqrt[
3*f - I*c*Log[f]] + (-1)^(1/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f])
)*f^2*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2
*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]] - 3*(-1)^(3/4)*c^2*
E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(1/4)*(6*
f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^2*
Sqrt[3*f - I*c*Log[f]] + (-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*
Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))
/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]] - (27*(-1)^(1/
4)*f^3*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*
```

```
Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I
*c*Log[f])) + (27*(-1)^(3/4)*c*f^2*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log
[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[
f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (3*(-1)^(1/4)*c^2*f*Cos[d]
*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*
Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*L
og[f])) + (3*(-1)^(3/4)*c^3*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (
2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]])/
E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (3*(-1)^(1/4)*f^3*Cos[3*d]*Erfi
[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log
[f]])]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f]))
 + ((-1)^(3/4)*c*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c
*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]])/E^((
(I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) - (3*(-1)^(1/4)*c^2*f*Cos[3*d]*Erfi
[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log
[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*
Log[f])) + ((-1)^(3/4)*c^3*Cos[3*d]*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] +
(2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[
f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + 27*(-1)^(1/4)*E^(((I/4)*
b^2*Log[f]^2)/(f - I*c*Log[f]))*f^3*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] -
(2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*Sin[d] +
27*(-1)^(3/4)*c*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^2*Erfi[((-1)^(1
/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[
f]*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(
f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f])
)/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(
3/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(2*f*x
- I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f
 - I*c*Log[f]]*Sin[d] + (27*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(2*f*x + I*b*Lo
g[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*Si
n[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) + (27*(-1)^(1/4)*c*f^2*Erfi
[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f
]])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*L
og[f])) + (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*
c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d]
)/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) + (3*(-1)^(1/4)*c^3*Erfi[((-1)^
(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Lo
g[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f]
)) + 3*(-1)^(1/4)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Erfi[((-1
)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])
]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (-1)^(3/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3
*f - I*c*Log[f]))*f^2*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + 3
*(-1)^(1/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Erfi[((-1)^(1
/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Lo
```

g[f]^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (-1)^(3/4)*c^3*E^(((I/4)*b^2*Log[f
]^2)/(3*f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*L
og[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]]*Sin[3*d
] + (3*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)
*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4)*(6*
f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sq
rt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f]))
+ (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/
E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c^3*Erfi[((-1)^(3
/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Lo
g[f]^3*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*
Log[f]))))/(16*(f - I*c*Log[f])*(3*f - I*c*Log[f])*(f + I*c*Log[f])*(3*f +
I*c*Log[f]))

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{\ln\left(f\right)^{2} b^{2}+12 i d \ln\left(f\right) c+36 d f}{4 \left(c \ln\left(f\right)-3 i f\right)}} \operatorname{erf}\left(-x \sqrt{3 i f-c \ln\left(f\right)}+\frac{\ln\left(f\right) b}{2 \sqrt{3 i f-c \ln\left(f\right)}}\right)}{16 \sqrt{3 i f-c \ln\left(f\right)}}-\frac{3 \sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{\ln\left(f\right)^{2} b^{2}+4 i d \ln\left(f\right) c+4 d f}{4 \left(c \ln\left(f\right)-i f\right)}} \operatorname{erf}\left(-x \sqrt{i f-c \ln(\right.}{16 \sqrt{i f-c \ln\left(f\right)}}$

[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x,method=_RETURNVERBOSE)

[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+12*I*d*ln(f)*c+36*d*f)/(c*ln(f)-3*
I*f))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*I*f
-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*d*ln(f)*c+4*d*
f)/(c*ln(f)-I*f))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*ln(f)*
b/(I*f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*d*ln(f)*
c+4*d*f)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/
2*ln(f)*b/(-c*ln(f)-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-12*
I*d*ln(f)*c+36*d*f)/(3*I*f+c*ln(f)))/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-
3*I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-3*I*f)^(1/2))

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 725 vs. 2(289) = 578.

Time = 0.29 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.92

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\left(c^3\log\left(f\right)^3-3i\,c^2f\log\left(f\right)^2+cf^2\log\left(f\right)-3i\,f^3\right)\sqrt{-c\log\left(f\right)-3i\,f}\,\mathrm{erf}\left(\frac{\left(18\,f^2x-3i\,bf\log(f)+\left(2\,c^2x+bc\right.\right.}{2\left(c^2\log(f)^2\right.}\right.}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x - 3*I*b*f*log(f) + (2*c^2*x + b
*c)*log(f)^2)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*
f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 108*I*d*f^2 - 3*(-4*I*c^2*d - I*b
^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(c^3*log(f)^3 + 3*I*c^2*
f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(18*f^
2*x + 3*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + 3*I*f)/(c
^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3
- 108*I*d*f^2 - 3*(4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) +
 3*sqrt(pi)*(c^3*log(f)^3 - I*c^2*f*log(f)^2 + 9*c*f^2*log(f) - 9*I*f^3)*sq
rt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*b*f*log(f) + (2*c^2*x + b*c)*log(f
)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^2*log(f) - (
b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^
2*log(f)^2 + f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 + I*c^2*f*log(f)^2 + 9*c*f^2*
log(f) + 9*I*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*log(f) + (
2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4
*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I*c^2*d - I
*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^
2 + 9*f^4)
```

Sympy [F]

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx = \int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + f*x**2)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2456 vs. 2(289) = 578.

Time = 0.28 (sec) , antiderivative size = 2456, normalized size of antiderivative = 6.50

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^a*e^(1/4*b^2*c*log(
f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^
2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*lo
g(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log
(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/4*(36*
d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*
log(f) - 3*I*f)*x + b*log(f))/sqrt(-c*log(f) + 3*I*f)) + ((-I*c^2*f^a*e^(1/
4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 - I*f^(a + 2)*e^(1/4*b^2*c*
log(f)^3/(c^2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f
)^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2
 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*
sin(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*er
f(1/2*(2*(c*log(f) + 3*I*f)*x + b*log(f))/sqrt(-c*log(f) - 3*I*f)))*sqrt(c*
log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f
^2)*(((-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 -
9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1/4*(4*d*f
^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - (c^2*f^a*e^(1/4*b^
2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*lo
g(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)
^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f))/sqrt(-
c*log(f) + I*f)) + ((I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)
)*log(f)^2 + 9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*c
os(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - (c^2*
f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a + 2)*e^
(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d +
b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*l
og(f))/sqrt(-c*log(f) - I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) -
sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^
2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2
 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9
*f^2)) - (I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 +
I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/4*(36*d*f^2
+ (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f)
 - 3*I*f)*x + b*log(f))/sqrt(-c*log(f) + 3*I*f)) + ((c^2*f^a*e^(1/4*b^2*c*l
```

```
og(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c
^2*log(f)^2 + f^2)))*cos(3/4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*l
og(f)^2 + 9*f^2)) - (-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))
*log(f)^2 - I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2)))*sin(3/
4*(36*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*
(2*(c*log(f) + 3*I*f)*x + b*log(f))/sqrt(-c*log(f) - 3*I*f)))*sqrt(-c*log(f
) + sqrt(c^2*log(f)^2 + 9*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(
((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a +
 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1/4*(4*d*f^2 + (4*c^
2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (-I*c^2*f^a*e^(1/4*b^2*c*log
(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - 9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^
3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(
c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f))/sqrt(-c*log
(f) + I*f)) + ((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f
)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*cos(1/4*(4
*d*f^2 + (4*c^2*d + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (I*c^2*f^a*e^(
1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*I*f^(a + 2)*e^(1/4*
b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2)))*sin(1/4*(4*d*f^2 + (4*c^2*d + b^2*f
)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f)
)/sqrt(-c*log(f) - I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*
e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(
f)^2 + f^2))*log(f)^4 + 10*c^2*f^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*
f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^4*e^(1/4*b^2
*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2
)))
```

Giac [F]

$$\int f^{a+bx+cx^2} \cos^3\left(d+fx^2\right)\,dx = \int f^{cx^2+bx+a} \cos\left(fx^2+d\right)^3\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \cos^3\left(d+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a} \cos\left(f\,x^2+d\right)^3\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*cos(d + f*x^2)^3,x)
[Out] int(f^(a + b*x + c*x^2)*cos(d + f*x^2)^3, x)
```

3.131 $\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 208

$$\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)\,dx=\frac{e^{-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

```
[Out] 1/4*exp(-I*d-(e+I*b*ln(f))^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e-b*ln(f)+2*
x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/4*exp(
I*d+(e-I*b*ln(f))^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*e+b*ln(f)+2*x*(I*f+c
*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)\,dx=\frac{\sqrt{\pi}f^a\exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\mathrm{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}f^a\exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\mathrm{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2],x]
```

```
[Out] (E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I
*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(4*Sqrt[I*
f - c*Log[f]]) + (E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(I*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]
])])/(4*Sqrt[I*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-id-iex-ifx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{id+iex+ifx^2}f^{a+bx+cx^2}\right)\,dx\\
&= \frac{1}{2}\int e^{-id-iex-ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{id+iex+ifx^2}f^{a+bx+cx^2}\,dx\\
&= \frac{1}{2}\int \exp\left(-id+a\log(f)-x(ie-b\log(f))-x^2(if-c\log(f))\right)\,dx\\
&\quad+\frac{1}{2}\int \exp\left(id+a\log(f)+x(ie+b\log(f))+x^2(if+c\log(f))\right)\,dx
\end{aligned}$$

$$= \frac{1}{2}\left(\exp\left(-id - \frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(-ie+b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx$$
$$+\frac{1}{2}\left(\exp\left(id + \frac{(e-ib\log(f))^2}{4if+4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2x(if+c\log(f)))^2}{4(if+c\log(f))}\right)dx$$

$$= \frac{\exp\left(-id - \frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}$$
$$+\frac{\exp\left(id + \frac{(e-ib\log(f))^2}{4if+4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}$$

Mathematica [A] (warning: unable to verify)

Time = 1.75 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.67

$$\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\sqrt[4]{-1}e^{-\frac{1}{4}i\left(\frac{e^2}{f-ic\log(f)}+\frac{b^2\log^2(f)}{f+ic\log(f)}\right)}f^{\frac{f(-be+af)+ac^2\log^2(f)}{f^2+c^2\log^2(f)}}\sqrt{\pi}\left(-e^{\frac{ie^2f}{2\left(f^2+c^2\log^2(f)\right)}}f^{\frac{be}{2f-2ic\log(f)}}\operatorname{erfi}\left(\frac{(-1)^{3/4}(e+2fx+i(b+2cx)\log(f}{2\sqrt{f+ic\log(f)}}\right.\right.}{}$$

[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2],x]

[Out] ((-1)^(1/4)*f^((f*(-(b*e) + a*f) + a*c^2*Log[f]^2)/(f^2 + c^2*Log[f]^2))*Sqrt[Pi]*(-(E^(((I/2)*e^2*f)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f - (2*I)*c*Log[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(Cos[d] - I*Sin[d])) + E^(((I/2)*b^2*f*Log[f]^2)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f + (2*I)*c*Log[f]))*Erfi[((-1)^(1/4)*(e + 2*f*x - I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*((-I)*Cos[d] + Sin[d])))/(4*E^((I/4)*(e^2/(f - I*c*Log[f]) + (b^2*Log[f]^2)/(f + I*c*Log[f])))*(f^2 + c^2*Log[f]^2))

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2-2i\ln(f)be+4id\ln(f)c+4df-e^2}{4(c\ln(f)-if)}}\operatorname{erf}\left(-x\sqrt{if-c\ln(f)}+\frac{b\ln(f)-ie}{2\sqrt{if-c\ln(f)}}\right)}{4\sqrt{if-c\ln(f)}} - \frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2+2i\ln(f)be-4id\ln(f)c+4df-}{4(if+c\ln(f))}}}{4\sqrt{-c}}$

[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x,method=_RETURNVERBOSE)

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*ln(f)*c+4*d*f-e
^2)/(c*ln(f)-I*f))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*(b*ln
(f)-I*e)/(I*f-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln
(f)*b*e-4*I*d*ln(f)*c+4*d*f-e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(
-c*ln(f)-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f)-I*f)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(155) = 310$.

Time = 0.26 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.81

$$\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)\,dx = -\frac{\sqrt{\pi}(c\log(f)-i\,f)\sqrt{-c\log(f)-i\,f}\,\text{erf}\left(\frac{\left(2\,f^2x+\left(2\,c^2x+bc\right)\log(f)^2+ef+\left(i\,ce-i\,bf\right)\log(f)\right)\sqrt{-c\log(f)-i\,f}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(-\frac{\left(b^2c-\right.}{}\right.}}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x + (2
*c^2*x + b*c)*log(f)^2 + e*f + (I*c*e - I*b*f)*log(f))*sqrt(-c*log(f) - I*f
)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*e^2*f - 4*I
*d*f^2 - (4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f + 4*
a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c*log(f) + I*f)*sqrt(-c*lo
g(f) + I*f)*erf(1/2*(2*f^2*x + (2*c^2*x + b*c)*log(f)^2 + e*f + (-I*c*e + I
*b*f)*log(f))*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c -
 4*a*c^2)*log(f)^3 - I*e^2*f + 4*I*d*f^2 - (-4*I*c^2*d + 2*I*b*c*e - I*b^2*
f)*log(f)^2 - (c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)))/(c
^2*log(f)^2 + f^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)\,dx = \int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)\,dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + e*x + f*x**2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(155) = 310$.

Time = 0.25 (sec) , antiderivative size = 1008, normalized size of antiderivative = 4.85

$$\int f^{a+bx+cx^2} \cos\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*((I*f^(1/4*c*e^2/(c^2*log(f)^2
+ f^2))*f^a*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^
2)/(c^2*log(f)^2 + f^2)) + f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(-1/4*
(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^
2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I*f)/
(c*log(f) - I*f)) + (-I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(-1/4*(e^
2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2))
 + f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*sin(-1/4*(e^2*f - 4*d*f^2 - (4*c^
2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f
) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f) - I*f)/(c*log(f) + I*f)))*sqrt(
c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2
)*((f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c
^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - I*f^(1/4*c*e^2/(c
^2*log(f)^2 + f^2))*f^a*sin(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^
2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) - I*f)*x + b*log
(f) - I*e)*sqrt(-c*log(f) + I*f)/(c*log(f) - I*f)) + (f^(1/4*c*e^2/(c^2*log
(f)^2 + f^2))*f^a*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*l
og(f)^2)/(c^2*log(f)^2 + f^2)) + I*f^(1/4*c*e^2/(c^2*log(f)^2 + f^2))*f^a*s
in(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f
)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f)
 - I*f)/(c*log(f) + I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^2
*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^
2 + f^2))*log(f)^2 + f^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 1/2*b
*e*f*log(f)/(c^2*log(f)^2 + f^2)))
```

Giac [F]

$$\int f^{a+bx+cx^2} \cos\left(d+ex+fx^2\right)\,dx = \int f^{cx^2+bx+a} \cos\left(fx^2+ex+d\right)\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \cos\left(d+ex+fx^2\right) dx = \int f^{c\,x^2+b\,x+a} \cos\left(f\,x^2+e\,x+d\right) dx$$

```
[In] int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2),x)
[Out] int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2), x)
```

3.132 $\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 268

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{2ie-b\log(f)+2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id+\frac{(2e-ib\log(f))^2}{8if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2ie+b\log(f)+2x(2if+c\log(f))}{2\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}$$

```
[Out] 1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)+1/8*exp(-2*I*d-(2*e+I*b*ln(f))^2/(8*I*f-4*c*ln(f)))*f^a*erf(1
/2*(2*I*e-b*ln(f)+2*x*(2*I*f-c*ln(f)))/(2*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*I
*f-c*ln(f))^(1/2)+1/8*exp(2*I*d+(2*e-I*b*ln(f))^2/(8*I*f+4*c*ln(f)))*f^a*er
fi(1/2*(2*I*e+b*ln(f)+2*x*(2*I*f+c*ln(f)))/(2*I*f+c*ln(f))^(1/2))*Pi^(1/2)/
(2*I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{4561, 2266, 2235, 2325, 2236\}$

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi}f^a\exp\left(-\frac{(2e+ib\log(f))^2}{-4c\log(f)+8if}-2id\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+2if)+2ie}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}} + \frac{\sqrt{\pi}f^a\exp\left(\frac{(2e-ib\log(f))^2}{4c\log(f)+8if}+2id\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)+2ie}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d - (2*e + I*b*Log[f])^2/((8*I)*f - 4*
c*Log[f]))*f^a*Sqrt[Pi]*Erf[((2*I)*e - b*Log[f] + 2*x*((2*I)*f - c*Log[f]))
/(2*Sqrt[(2*I)*f - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*I)*d
+ (2*e - I*b*Log[f])^2/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e +
 b*Log[f] + 2*x*((2*I)*f + c*Log[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqr
t[(2*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\
&= \frac{1}{4}\int \exp\left(-2id+a\log(f)-x(2ie-b\log(f))-x^2(2if-c\log(f))\right)dx\\
&\quad+\frac{1}{4}\int \exp\left(2id+a\log(f)+x(2ie+b\log(f))+x^2(2if+c\log(f))\right)dx\\
&\quad+\frac{1}{2}f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}\,dx\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(\exp\left(-2id\right.\right.\\
&\quad\left.\left.-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(-2ie+b\log(f)+2x(-2if+c\log(f)))^2}{4(-2if+c\log(f))}\right)dx\\
&\quad+\frac{1}{4}\left(\exp\left(2id\right.\right.\\
&\quad\quad\left.\left.+\frac{(2e-ib\log(f))^2}{8if+4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(2ie+b\log(f)+2x(2if+c\log(f)))^2}{4(2if+c\log(f))}\right)dx\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\\
&\quad+\frac{\exp\left(-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}\right)f^a\sqrt{\pi}\,\text{erf}\left(\frac{2ie-b\log(f)+2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}\\
&\quad+\frac{\exp\left(2id+\frac{(2e-ib\log(f))^2}{8if+4c\log(f)}\right)f^a\sqrt{\pi}\,\text{erfi}\left(\frac{2ie+b\log(f)+2x(2if+c\log(f))}{2\sqrt{2if+c\log(f)}}\right)}{8\sqrt{2if+c\log(f)}}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1118 vs. 2(268) = 536.

Time = 6.58 (sec) , antiderivative size = 1118, normalized size of antiderivative = 4.17

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx$$

$$= \frac{f^a\sqrt{\pi}\left(8\sqrt{c}f^{2-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}+2c^{5/2}f^{-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\log^{\frac{5}{2}}(f)-2(-1)^{3/4}ce^{\frac{i(-4e^2+}{}}\right.}{}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(8*Sqrt[c]*f^(2 - b^2/(4*c))*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/
(2*Sqrt[c])]*Sqrt[Log[f]] + (2*c^(5/2)*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*S
qrt[c])]*Log[f]^(5/2))/f^(b^2/(4*c)) - 2*(-1)^(3/4)*c*E^(((I/4)*(-4*e^2 + (
4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Cos[2*d]*Erfi[((-1)^
(1/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[
f]])]*Log[f]*Sqrt[2*f - I*c*Log[f]] + (-1)^(1/4)*c^2*E^(((I/4)*(-4*e^2 + (4
*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Cos[2*d]*Erfi[((-1)^(1/
4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]]
)]*Log[f]^2*Sqrt[2*f - I*c*Log[f]] - (2*(-1)^(1/4)*c*f*Cos[2*d]*Erfi[((-1)^
(3/4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[
f]])]*Log[f]*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] +
 b^2*Log[f]^2))/(2*f + I*c*Log[f])) + ((-1)^(3/4)*c^2*Cos[2*d]*Erfi[((-1)^(
3/4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f
]])]*Log[f]^2*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f]
+ b^2*Log[f]^2))/(2*f + I*c*Log[f])) + 2*(-1)^(1/4)*c*E^(((I/4)*(-4*e^2 + (
4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(2*
e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Log
[f]*Sqrt[2*f - I*c*Log[f]]*Sin[2*d] + (-1)^(3/4)*c^2*E^(((I/4)*(-4*e^2 + (4
*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(2*e +
 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Log[f]
^2*Sqrt[2*f - I*c*Log[f]]*Sin[2*d] + (2*(-1)^(3/4)*c*f*Erfi[((-1)^(3/4)*(2*
e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]*Log
[f]*Sqrt[2*f + I*c*Log[f]]*Sin[2*d])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] +
 b^2*Log[f]^2))/(2*f + I*c*Log[f])) + ((-1)^(1/4)*c^2*Erfi[((-1)^(3/4)*(2*e
 + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]*Log[
f]^2*Sqrt[2*f + I*c*Log[f]]*Sin[2*d])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f]
+ b^2*Log[f]^2))/(2*f + I*c*Log[f]))))/(8*c*Log[f]*(2*f - I*c*Log[f])*(2*f
+ I*c*Log[f]))
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-4 i \ln(f) b e+8 i d \ln(f) c+16 d f-4 e^{2}}{4(c \ln(f)-2 i f)}} \operatorname{erf}\left(-x \sqrt{2 i f-c \ln(f)}+\frac{b \ln(f)-2 i e}{2 \sqrt{2 i f-c \ln(f)}}\right)}{8 \sqrt{2 i f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i \ln(f) b e-8 i d \ln(f) c+}{4(2 i f+c \ln(f))}}}{8}$

[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*b*e+8*I*d*ln(f)*c+16*d*f-4*e^2)/(c*ln(f)-2*I*f))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-2*I*e)/(2*I*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*ln(f)*b*e-8*I*d*ln(f)*c+16*d*f-4*e^2)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-c*ln(f)-2*I*f)^(1/2))-1/4*Pi^(1/2)*f^(-1/4*b^2/c)*f^a/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(199) = 398.

Time = 0.26 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.75

$$\int f^{a+bx+cx^2} \cos^2\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\left(c^{2} \log(f)^{2}-2 i c f \log(f)\right) \sqrt{-c \log(f)-2 i f} \operatorname{erf}\left(\frac{\left(8 f^{2} x+\left(2 c^{2} x+b c\right) \log(f)^{2}+4 e f-2(-i c e+i b f) \log(f)\right) \sqrt{-c \log(}}{2\left(c^{2} \log(f)^{2}+4 f^{2}\right)}\right.}{}$$

[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="fricas")

[Out] -1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(1/2*(8*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*e*f - 2*(-I*c*e + I*b*f)*log(f))*sqrt(-c*log(f) - 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 8*I*e^2*f - 32*I*d*f^2 + 2*(-4*I*c^2*d + 2*I*b*c*e - I*b^2*f)*log(f)^2 - 4*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) + sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(1/2*(8*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*e*f - 2*(I*c*e - I*b*f)*log(f))*sqrt(-c*log(f) + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 - 8*I*e^2*f + 32*I*d*f^2 + 2*(4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - 4*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) + 2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2*log(f))

Sympy [F]

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + e*x + f*x**2)**2, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 1487, normalized size of antiderivative = 5.55

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^2)*((I*f^a*cos(-1/2*(4*e^2*f - 16
*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))*e^(c
*e^2*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c) + f^a*e^(c*e^2*log(f
)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*sin(-1/2*(4*e^2*f - 16*d*f^2 -
 (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*
(c*log(f) - 2*I*f)*x + b*log(f) - 2*I*e)*sqrt(-c*log(f) + 2*I*f)/(c*log(f)
- 2*I*f)) + (-I*f^a*cos(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^2
*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2
) + 1/4*b^2*log(f)/c) + f^a*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^
2*log(f)/c)*sin(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(
f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) + 2*I*f)*x + b*log(f) +
 2*I*e)*sqrt(-c*log(f) - 2*I*f)/(c*log(f) + 2*I*f)))*sqrt(c*log(f) + sqrt(c
^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - sqrt(pi)*sqrt(2*c^2*log(f)^2 + 8*f^
2)*((f^a*cos(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^
2)/(c^2*log(f)^2 + 4*f^2))*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2
*log(f)/c) - I*f^a*e^(c*e^2*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/
c)*sin(-1/2*(4*e^2*f - 16*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^
2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(c*log(f) - 2*I*f)*x + b*log(f) - 2*I*e)*s
qrt(-c*log(f) + 2*I*f)/(c*log(f) - 2*I*f)) + (f^a*cos(-1/2*(4*e^2*f - 16*d*
f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2))*e^(c*e^
2*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c) + I*f^a*e^(c*e^2*log(f)
/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*sin(-1/2*(4*e^2*f - 16*d*f^2 -
(4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*(
c*log(f) + 2*I*f)*x + b*log(f) + 2*I*e)*sqrt(-c*log(f) - 2*I*f)/(c*log(f) +
 2*I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 4*f^2))*sqrt(-c*log(f)) - 2*
```

```
sqrt(pi)*((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*b*e*f*l
og(f)/(c^2*log(f)^2 + 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/
(c^2*log(f)^2 + 4*f^2) + 2*b*e*f*log(f)/(c^2*log(f)^2 + 4*f^2)))*erf(-1/2*b
*conjugate(1/sqrt(-c*log(f)))*log(f) + x*conjugate(sqrt(-c*log(f)))) - (c^2
*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*b*e*f*log(f)/(c^2*log
(f)^2 + 4*f^2))*log(f)^2 + 4*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2
+ 4*f^2) + 2*b*e*f*log(f)/(c^2*log(f)^2 + 4*f^2)))*erf(1/2*(2*c*x*log(f) +
b*log(f))/sqrt(-c*log(f)))))/((c^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*
f^2) + 2*b*e*f*log(f)/(c^2*log(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c)*log(f)^2 +
 4*f^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 4*f^2) + 2*b*e*f*log(f)/(c^2*l
og(f)^2 + 4*f^2) + 1/4*b^2*log(f)/c))*sqrt(-c*log(f)))
```

Giac [F]

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{cx^2+bx+a}\cos\left(fx^2+ex+d\right)^2\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cos\left(f\,x^2+e\,x+d\right)^2\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2)^2, x)
```

3.133 $\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 422

$$\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\frac{3e^{-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}+\frac{e^{-3id-\frac{(3e+ib\log(f))^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{3ie-b\log(f)+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}}+\frac{3e^{id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}}+\frac{e^{3id-\frac{(3ie+b\log(f))^2}{4(3if+c\log(f))}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{3ie+b\log(f)+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}$$

```
[Out] 3/16*exp(-I*d-(e+I*b*ln(f))^2/(4*I*f-4*c*ln(f)))*f^a*erf(1/2*(I*e-b*ln(f)+2
*x*(I*f-c*ln(f)))/(I*f-c*ln(f))^(1/2))*Pi^(1/2)/(I*f-c*ln(f))^(1/2)+1/16*ex
p(-3*I*d-1/4*(3*e+I*b*ln(f))^2/(3*I*f-c*ln(f)))*f^a*erf(1/2*(3*I*e-b*ln(f)+
2*x*(3*I*f-c*ln(f)))/(3*I*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f-c*ln(f))^(1/2)+
3/16*exp(I*d+(e-I*b*ln(f))^2/(4*I*f+4*c*ln(f)))*f^a*erfi(1/2*(I*e+b*ln(f)+2
*x*(I*f+c*ln(f)))/(I*f+c*ln(f))^(1/2))*Pi^(1/2)/(I*f+c*ln(f))^(1/2)+1/16*ex
p(3*I*d-1/4*(3*I*e+b*ln(f))^2/(3*I*f+c*ln(f)))*f^a*erfi(1/2*(3*I*e+b*ln(f)+
2*x*(3*I*f+c*ln(f)))/(3*I*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*I*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx = \frac{3\sqrt{\pi}f^a\exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}} + \frac{\sqrt{\pi}f^a\exp\left(-\frac{(3e+ib\log(f))^2}{4(-c\log(f)+3if)}-3id\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}} + \frac{3\sqrt{\pi}f^a\exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{16\sqrt{c\log(f)+if}} + \frac{\sqrt{\pi}f^a\exp\left(3id-\frac{(b\log(f)+3ie)^2}{4(c\log(f)+3if)}\right)\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3if)+3ie}{2\sqrt{c\log(f)+3if}}\right)}{16\sqrt{c\log(f)+3if}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[
(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt
[I*f - c*Log[f]]) + (E^((-3*I)*d - (3*e + I*b*Log[f])^2/(4*((3*I)*f - c*Log
[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e - b*Log[f] + 2*x*((3*I)*f - c*Log[f]))/(2*
Sqrt[(3*I)*f - c*Log[f]])])/(16*Sqrt[(3*I)*f - c*Log[f]]) + (3*E^(I*d + (e
- I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] +
 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]])
 + (E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[
Pi]*Erfi[((3*I)*e + b*Log[f] + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f +
c*Log[f]])])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{8}e^{-3i(d+ex+fx^2)}f^{a+bx+cx^2} + \frac{3}{8}\exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx+cx^2} + \frac{3}{8}\exp\left(4id+4iex+4ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx+cx^2} + \frac{1}{8}\exp\left(6id+6iex+6ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx+cx^2}\right)dx \\
&= \frac{1}{8}\int e^{-3i(d+ex+fx^2)}f^{a+bx+cx^2}\,dx + \frac{1}{8}\int \exp\left(6id+6iex+6ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx+cx^2}\,dx + \frac{3}{8}\int \exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx+cx^2}\,dx + \frac{3}{8}\int \exp\left(4id+4iex+4ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx+cx^2}\,dx \\
&= \frac{1}{8}\int \exp\left(-3id+a\log(f)-x(3ie-b\log(f))-x^2(3if-c\log(f))\right)dx + \frac{1}{8}\int \exp\left(3id+a\log(f)+x(3ie+b\log(f))+x^2(3if+c\log(f))\right)dx + \frac{3}{8}\int \exp\left(-id+a\log(f)-x(ie-b\log(f))-x^2(if-c\log(f))\right)dx + \frac{3}{8}\int \exp\left(id+a\log(f)+x(ie+b\log(f))+x^2(if+c\log(f))\right)dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{8}\Bigg(3\exp\Bigg(-id \\
&\quad -\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\Bigg)f^a\Bigg)\int\exp\left(\frac{(-ie+b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx \\
&\quad +\frac{1}{8}\Bigg(\exp\Bigg(-3id \\
&\quad -\frac{(3e+ib\log(f))^2}{4(3if-c\log(f))}\Bigg)f^a\Bigg)\int\exp\left(\frac{(-3ie+b\log(f)+2x(-3if+c\log(f)))^2}{4(-3if+c\log(f))}\right)dx \\
&\quad +\frac{1}{8}\Bigg(\exp\Bigg(3id \\
&\quad -\frac{(3ie+b\log(f))^2}{4(3if+c\log(f))}\Bigg)f^a\Bigg)\int\exp\left(\frac{(3ie+b\log(f)+2x(3if+c\log(f)))^2}{4(3if+c\log(f))}\right)dx \\
&\quad +\frac{1}{8}\Bigg(3\exp\Bigg(id \\
&\quad +\frac{(e-ib\log(f))^2}{4if+4c\log(f)}\Bigg)f^a\Bigg)\int\exp\left(\frac{(ie+b\log(f)+2x(if+c\log(f)))^2}{4(if+c\log(f))}\right)dx \\
&= \frac{3\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}} \\
&\quad +\frac{\exp\left(-3id-\frac{(3e+ib\log(f))^2}{4(3if-c\log(f))}\right)f^a\sqrt{\pi}\operatorname{erf}\left(\frac{3ie-b\log(f)+2x(3if-c\log(f))}{2\sqrt{3if-c\log(f)}}\right)}{16\sqrt{3if-c\log(f)}} \\
&\quad +\frac{3\exp\left(id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+b\log(f)+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{16\sqrt{if+c\log(f)}} \\
&\quad +\frac{\exp\left(3id-\frac{(3ie+b\log(f))^2}{4(3if+c\log(f))}\right)f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie+b\log(f)+2x(3if+c\log(f))}{2\sqrt{3if+c\log(f)}}\right)}{16\sqrt{3if+c\log(f)}}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3829 vs. 2(422) = 844.

Time = 6.98 (sec) , antiderivative size = 3829, normalized size of antiderivative = 9.07

$$
\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)dx = \text{Result too large to show}
$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[
f]^2))/(f - I*c*Log[f]))*f^3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f
] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*
```

```
(-1)^(1/4)*c*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*L
og[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log
[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4)*
c^2*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f
*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((
I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*Cos[d]*Erf
i[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*
Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4)*E^(((I/4)*(-9*e^2 +
(6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[((-
1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*L
og[f]])]*Sqrt[3*f - I*c*Log[f]] + (-1)^(1/4)*c*E^(((I/4)*(-9*e^2 + (6*I)*b*
e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^2*Cos[3*d]*Erfi[((-1)^(1/4)
*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]
*Log[f]*Sqrt[3*f - I*c*Log[f]] - 3*(-1)^(3/4)*c^2*E^(((I/4)*(-9*e^2 + (6*I)
*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(1/4
)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])
]*Log[f]^2*Sqrt[3*f - I*c*Log[f]] + (-1)^(1/4)*c^3*E^(((I/4)*(-9*e^2 + (6*I
)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)
*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]
*Log[f]^3*Sqrt[3*f - I*c*Log[f]] - (27*(-1)^(1/4)*f^3*Cos[d]*Erfi[((-1)^(3/
4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*S
qrt[f + I*c*Log[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f
 + I*c*Log[f])) + (27*(-1)^(3/4)*c*f^2*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x +
 I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f +
I*c*Log[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*L
og[f])) - (3*(-1)^(1/4)*c^2*f*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[
f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log
[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f]))
 + (3*(-1)^(3/4)*c^3*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I
)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]])/E^(
((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - (3*(-1
)^(1/4)*f^3*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x
*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*(-9
*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(3/4)*
c*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f
]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*(-
9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) - (3*(-1)^(1/
4)*c^2*f*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Lo
g[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]])/E^(((I/
4)*(-9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^
(3/4)*c^3*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*L
og[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[f]])/E^(((I
/4)*(-9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + 27*(-
1)^(1/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f
```

```
]))*f^3*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*Sin[d] + 27*(-1)^(3/4)*c*E^(((I/4
)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f^2*Erfi[((-1
)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]
])]*Log[f]*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*(-e^2 +
 (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(e
+ 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^
2*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(3/4)*c^3*E^(((I/4)*(-e^2 + (2*I)*b*
e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(e + 2*f*x - I
*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f -
I*c*Log[f]]*Sin[d] + (27*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*L
og[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*S
in[d])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])
) + (27*(-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c
*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d])/E
^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) + (3*(
-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]
))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4
)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) + (3*(-1)^(1/
4)*c^3*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqr
t[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*(-e^2 -
 (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) + 3*(-1)^(1/4)*E^(((I/
4)*(-9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^3*Erfi
[((-1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I
*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (-1)^(3/4)*c*E^(((I/4)*(-9*e
^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^2*Erfi[((-1)^(
1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f
]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*(-
9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f*Erfi[((-1)^
(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[
f]])]*Log[f]^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (-1)^(3/4)*c^3*E^(((I/4)*(
-9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*Erfi[((-1)^(
1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f
]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (3*(-1)^(3/4)*f^3*Erfi[((-1
)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Lo
g[f]])]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b*e*Log[
f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4
)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])
]*Log[f]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b*e*Log
[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(
3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f
]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b*
e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c^3*Erfi[((-1)^
(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[
f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b
```

```
*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f]))))/(16*(f - I*c*Log[f])*(3*f
- I*c*Log[f])*(f + I*c*Log[f])*(3*f + I*c*Log[f]))
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-6 i \ln(f) b e+12 i d \ln(f) c+36 d f-9 e^{2}}{4(c \ln(f)-3 i f)}} \operatorname{erf}\left(-x \sqrt{3 i f-c \ln(f)}+\frac{b \ln(f)-3 i e}{2 \sqrt{3 i f-c \ln(f)}}\right)}{16 \sqrt{3 i f-c \ln(f)}}-\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-2 i \ln(f) b e+4 i d \ln(}{4(c \ln(f)-i f)}}}{1}$

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*I*ln(f)*b*e+12*I*d*ln(f)*c+36*d*
f-9*e^2)/(c*ln(f)-3*I*f))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(1/2
)+1/2*(b*ln(f)-3*I*e)/(3*I*f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln
(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*ln(f)*c+4*d*f-e^2)/(c*ln(f)-I*f))/(I*f-c*ln(f
))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-I*e)/(I*f-c*ln(f))^(1/2))-
3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c+4*d*f-e
^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*(I*
e+b*ln(f))/(-c*ln(f)-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*
I*ln(f)*b*e-12*I*d*ln(f)*c+36*d*f-9*e^2)/(3*I*f+c*ln(f)))/(-c*ln(f)-3*I*f)^
(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-c*ln(f)-3*I*f)^(1
/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 859 vs. 2(312) = 624.

Time = 0.30 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.04

$$\int f^{a+bx+cx^2} \cos^3\left(d+ex+fx^2\right)\, dx = \text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*
e*f - 3*(-I*c*e + I*b*f)*log(f))*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*
f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 27*I*e^2*f - 108*I*d*f^2 + 3*(-
4*I*c^2*d + 2*I*b*c*e - I*b^2*f)*log(f)^2 - 9*(c*e^2 - 2*b*e*f + 4*a*f^2)*l
og(f))/(c^2*log(f)^2 + 9*f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 - I*c^2*f*log(f)^
2 + 9*c*f^2*log(f) - 9*I*f^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x + (2*c
^2*x + b*c)*log(f)^2 + e*f + (I*c*e - I*b*f)*log(f))*sqrt(-c*log(f) - I*f)/
```

```
(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*e^2*f - 4*I*d
*f^2 - (4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f + 4*a*
f^2)*log(f))/(c^2*log(f)^2 + f^2)) + 3*sqrt(pi)*(c^3*log(f)^3 + I*c^2*f*log
(f)^2 + 9*c*f^2*log(f) + 9*I*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x +
(2*c^2*x + b*c)*log(f)^2 + e*f + (-I*c*e + I*b*f)*log(f))*sqrt(-c*log(f) +
I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 - I*e^2*f +
4*I*d*f^2 - (-4*I*c^2*d + 2*I*b*c*e - I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f
+ 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c^3*log(f)^3 + 3*I*c^2
*f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(18*f
^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*e*f - 3*(I*c*e - I*b*f)*log(f))*sqrt(-c
*log(f) + 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^
3 - 27*I*e^2*f + 108*I*d*f^2 + 3*(4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2
 - 9*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^2)))/(c^4*log(
f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\text{Timed out}$$

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4348 vs. $2(312) = 624$.

Time = 0.31 (sec) , antiderivative size = 4348, normalized size of antiderivative = 10.30

$$\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(sqrt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((I*c^2*f^a*e^(1/4*b^2*c*log
(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*
b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(
f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b
*e*f*log(f)/(c^2*log(f)^2 + f^2)))*cos(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^2*d
- 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*
c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) +
 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*l
og(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/
```

2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*sin(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^2
*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(
f) - 3*I*f)*x + b*log(f) - 3*I*e)*sqrt(-c*log(f) + 3*I*f)/(c*log(f) - 3*I*f
)) + ((-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*lo
g(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)
^2 - I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log
(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*cos(-3
/4*(9*e^2*f - 36*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^
2 + 9*f^2)) + (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e
^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*l
og(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*
log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*sin
(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(
f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + b*log(f) + 3*I*e)*sqrt(-c
*log(f) - 3*I*f)/(c*log(f) + 3*I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + 9
*f^2)) - 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((-I*c^2*f^a*e^(1/4*b^2*c
*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2) +
9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - 9*I*f^(a + 2)*e^(1/4*b^
2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2)
 + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*cos(-1/4*(e^2*f - 4*d*f^2 - (4
*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) - (c^2*f^a*e^(1/4
*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f
^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a + 2)*e^(1/
4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 +
f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*sin(-1/4*(e^2*f - 4*d*f^2
- (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(
c*log(f) - I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I*f)/(c*log(f) - I*f))
 + ((I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log
(f)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^
2 + 9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*
log(f)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*cos
(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^
2 + f^2)) - (c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e
^2*log(f)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*l
og(f)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*
e^2*log(f)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))
sin(-1/4(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log
(f)^2 + f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(
f) - I*f)/(c*log(f) + I*f)))*sqrt(c*log(f) + sqrt(c^2*log(f)^2 + f^2)) - sq
rt(pi)*sqrt(2*c^2*log(f)^2 + 18*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*
log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f
)/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log
(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(
c^2*log(f)^2 + f^2)))*cos(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^2*d - 2*b*c*e + b
^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) - (I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/

```
(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*
log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(
c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*l
og(f)/(c^2*log(f)^2 + f^2)))*sin(-3/4*(9*e^2*f - 36*d*f^2 - (4*c^2*d - 2*b*
c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)))*erf(1/2*(2*(c*log(f) - 3*I*
f)*x + b*log(f) - 3*I*e)*sqrt(-c*log(f) + 3*I*f)/(c*log(f) - 3*I*f)) + ((c^
2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*lo
g(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + f^(a +
2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c^2*log(f
)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*cos(-3/4*(9*e^2*f -
36*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) -
(-I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(
c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 - I
*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/4*c*e^2*log(f)/(c
^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2)))*sin(-3/4*(9*
e^2*f - 36*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*
f^2)))*erf(1/2*(2*(c*log(f) + 3*I*f)*x + b*log(f) + 3*I*e)*sqrt(-c*log(f) -
 3*I*f)/(c*log(f) + 3*I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + 9*f^2)) -
 3*sqrt(pi)*sqrt(2*c^2*log(f)^2 + 2*f^2)*(((c^2*f^a*e^(1/4*b^2*c*log(f)^3/(
c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*l
og(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/
(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*
log(f)/(c^2*log(f)^2 + 9*f^2)))*cos(-1/4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*
c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + (-I*c^2*f^a*e^(1/4*b^2*c*log
(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2) + 9/2*
b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 - 9*I*f^(a + 2)*e^(1/4*b^2*c*
log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*log(f)^2 + f^2) + 9
/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*sin(-1/4*(e^2*f - 4*d*f^2 - (4*c^2
*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))*erf(1/2*(2*(c*log(f)
 - I*f)*x + b*log(f) - I*e)*sqrt(-c*log(f) + I*f)/(c*log(f) - I*f)) + ((c^2
*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*l
og(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 + 9*f^(a
 + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/(c^2*
log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*cos(-1/4*(e^2*f
 - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) +
(I*c^2*f^a*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(f)/
(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2))*log(f)^2 +
9*I*f^(a + 2)*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*c*e^2*log(
f)/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2)))*sin(-1/
4*(e^2*f - 4*d*f^2 - (4*c^2*d - 2*b*c*e + b^2*f)*log(f)^2)/(c^2*log(f)^2 +
f^2)))*erf(1/2*(2*(c*log(f) + I*f)*x + b*log(f) + I*e)*sqrt(-c*log(f) - I*f
)/(c*log(f) + I*f)))*sqrt(-c*log(f) + sqrt(c^2*log(f)^2 + f^2)))/(c^4*e^(1/
4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f)^2
+ f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*lo
g(f)^2 + f^2))*log(f)^4 + 10*c^2*f^2*e^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 +
```

```
9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*lo
g(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^2*log(f)^2 + f^2))*log(f)^2 + 9*f^4*e
^(1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + 9*f^2) + 1/4*b^2*c*log(f)^3/(c^2*log(f
)^2 + f^2) + 9/2*b*e*f*log(f)/(c^2*log(f)^2 + 9*f^2) + 1/2*b*e*f*log(f)/(c^
2*log(f)^2 + f^2)))
```

Giac [F]

$$\int f^{a+bx+cx^2} \cos^3\left(d+ex+fx^2\right)\,dx = \int f^{cx^2+bx+a} \cos\left(fx^2+ex+d\right)^3\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \cos^3\left(d+ex+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a} \cos\left(f\,x^2+e\,x+d\right)^3\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2)^3,x)
[Out] int(f^(a + b*x + c*x^2)*cos(d + e*x + f*x^2)^3, x)
```

3.134 $\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 209

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx = \frac{e^{-\left((i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)}\sqrt{\pi}\operatorname{erf}\left(\frac{b(i-\log(f))+2x(ie-c\log(f))}{2\sqrt{ie-c\log(f)}}\right)}{4\sqrt{ie-c\log(f)}} + \frac{e^{(i+\log(f))\left(a-\frac{b^2(i+\log(f))}{4ie+4c\log(f)}\right)}\sqrt{\pi}\operatorname{erfi}\left(\frac{b(i+\log(f))+2x(ie+c\log(f))}{2\sqrt{ie+c\log(f)}}\right)}{4\sqrt{ie+c\log(f)}}$$

```
[Out] -1/4*erf(1/2*(-b*(I-ln(f))-2*x*(I*e-c*ln(f)))/(I*e-c*ln(f))^(1/2))*Pi^(1/2)
/exp((I-ln(f))*(a-b^2*(I-ln(f))/(4*I*e-4*c*ln(f))))/(I*e-c*ln(f))^(1/2)+1/4
*exp((I+ln(f))*(a-b^2*(I+ln(f))/(4*I*e+4*c*ln(f))))*erfi(1/2*(b*(I+ln(f))+2
*x*(I*e+c*ln(f)))/(I*e+c*ln(f))^(1/2))*Pi^(1/2)/(I*e+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4561, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx = \frac{\sqrt{\pi}\exp\left(-\left((-\log(f)+i)\left(a-\frac{b^2(-\log(f)+i)}{-4c\log(f)+4ie}\right)\right)\right)\operatorname{erf}\left(\frac{b(-\log(f)+i)+2x(-c\log(f)+ie)}{2\sqrt{-c\log(f)+ie}}\right)}{4\sqrt{-c\log(f)+ie}} + \frac{\sqrt{\pi}\exp\left((\log(f)+i)\left(a-\frac{b^2(\log(f)+i)}{4c\log(f)+4ie}\right)\right)\operatorname{erfi}\left(\frac{b(\log(f)+i)+2x(c\log(f)+ie)}{2\sqrt{c\log(f)+ie}}\right)}{4\sqrt{c\log(f)+ie}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Cos[a + b*x + e*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erf[(b*(I - Log[f]) + 2*x*(I*e - c*Log[f]))/(2*Sqrt[I*e - c*Log[f
]])])/(4*E^((I - Log[f])*(a - (b^2*(I - Log[f]))/((4*I)*e - 4*c*Log[f])))*S
qrt[I*e - c*Log[f]]) + (E^((I + Log[f])*(a - (b^2*(I + Log[f]))/((4*I)*e +
4*c*Log[f])))*Sqrt[Pi]*Erfi[(b*(I + Log[f]) + 2*x*(I*e + c*Log[f]))/(2*Sqrt
[I*e + c*Log[f]])])/(4*Sqrt[I*e + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4561

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}e^{-ia-ibx-iex^2}f^{a+bx+cx^2}+\frac{1}{2}e^{ia+ibx+iex^2}f^{a+bx+cx^2}\right)\,dx\\
&= \frac{1}{2}\int e^{-ia-ibx-iex^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{ia+ibx+iex^2}f^{a+bx+cx^2}\,dx\\
&= \frac{1}{2}\int \exp\left(-a(i-\log(f))-bx(i-\log(f))-x^2(ie-c\log(f))\right)\,dx\\
&\quad+\frac{1}{2}\int \exp\left(a(i+\log(f))+bx(i+\log(f))+x^2(ie+c\log(f))\right)\,dx
\end{aligned}$$

$$= \frac{1}{2}\exp\left(-\left((i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)\right)\int\exp\left(\frac{(-b(i-\log(f))+2x(-ie+c\log(f)))^2}{4(-ie+c\log(f))}\right)$$
$$+\frac{1}{2}\exp\left((i+\log(f))\left(a-\frac{b^2(i+\log(f))}{4ie+4c\log(f)}\right)\right)\int\exp\left(\frac{(b(i+\log(f))+2x(ie+c\log(f)))^2}{4(ie+c\log(f))}\right)dx$$
$$=\frac{\exp\left(-\left((i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)\right)\sqrt{\pi}\operatorname{erf}\left(\frac{b(i-\log(f))+2x(ie-c\log(f))}{2\sqrt{ie-c\log(f)}}\right)}{4\sqrt{ie-c\log(f)}}$$
$$+\frac{\exp\left((i+\log(f))\left(a-\frac{b^2(i+\log(f))}{4ie+4c\log(f)}\right)\right)\sqrt{\pi}\operatorname{erfi}\left(\frac{b(i+\log(f))+2x(ie+c\log(f))}{2\sqrt{ie+c\log(f)}}\right)}{4\sqrt{ie+c\log(f)}}$$

Mathematica [A] (warning: unable to verify)

Time = 1.64 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.56

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx =$$
$$-\frac{ie^{-\frac{b^2c\log^3(f)}{2\left(e^2+c^2\log^2(f)\right)}}f^{a-\frac{b^2}{2(e-ic\log(f))}}\sqrt{\pi}\left(-e^{\frac{1}{4}b^2\left(\frac{1}{-ie+c\log(f)}+\frac{\log^2(f)}{ie+c\log(f)}\right)}f^{\frac{ib^2c\log(f)}{e^2+c^2\log^2(f)}}\operatorname{erfi}\left(\frac{-i(b+2ex)+(b+2cx)\log(f)}{2\sqrt{-ie+c\log(f)}}\right)(e-\right.}{}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[a + b*x + e*x^2],x]
```

```
[Out] ((-1/4*I)*f^(a - b^2/(2*(e - I*c*Log[f])))*Sqrt[Pi]*(-(E^((b^2*(((-I)*e + c
*Log[f])^(-1) + Log[f]^2/(I*e + c*Log[f])))/4)*f^((I*b^2*c*Log[f])/(e^2 + c
^2*Log[f]^2))*Erfi[((-I)*(b + 2*e*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[(-I)*e +
 c*Log[f]])]*(e - I*c*Log[f])*Sqrt[(-I)*e + c*Log[f]]*(Cos[a] - I*Sin[a]))
+ E^((b^2*(Log[f]^2/((-I)*e + c*Log[f]) + (I*e + c*Log[f])^(-1)))/4)*Erfi[(
I*(b + 2*e*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[I*e + c*Log[f]])]*(e + I*c*Log[
f])*Sqrt[I*e + c*Log[f]]*(Cos[a] + I*Sin[a])))/(E^((b^2*c*Log[f]^3)/(2*(e^2
 + c^2*Log[f]^2)))*(e^2 + c^2*Log[f]^2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 i \ln(f) a c-2 i \ln(f) b^{2}+4 a e-b^{2}}{4(c \ln(f)-i e)}} \operatorname{erf}\left(-\sqrt{i e-c \ln(f)}\, x+\frac{b \ln(f)-i b}{2 \sqrt{i e-c \ln(f)}}\right)}{4 \sqrt{i e-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{-\ln(f)^{2} b^{2}+4 i \ln(f) a c-2 i \ln(f) b^{2}-4}{4 i e+4 c \ln(f)}}}{4 \sqrt{-}}$

[In] int(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x,method=_RETURNVERBOSE)

[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*ln(f)*a*c-2*I*ln(f)*b^2+4*a*e-b^2)/(c*ln(f)-I*e))/(I*e-c*ln(f))^(1/2)*erf(-(I*e-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*b)/(I*e-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*(-ln(f)^2*b^2+4*I*ln(f)*a*c-2*I*ln(f)*b^2-4*a*e+b^2)/(I*e+c*ln(f)))/(-c*ln(f)-I*e)^(1/2)*erf(-(-c*ln(f)-I*e)^(1/2)*x+1/2*(b*ln(f)+I*b)/(-c*ln(f)-I*e)^(1/2))

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(153) = 306.

Time = 0.27 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.82

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}(c\log(f)-i\,e)\sqrt{-c\log(f)-i\,e}\,\operatorname{erf}\left(\frac{\left(2\,e^2x+\left(2\,c^2x+bc\right)\log(f)^2+be+\left(i\,bc-i\,be\right)\log(f)\right)\sqrt{-c\log(f)-i\,e}}{2\left(c^2\log(f)^2+e^2\right)}\right)e^{\left(-\frac{\left(b^2c-4\right.}{}\right.}}{}$$

[In] integrate(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x, algorithm="fricas")

[Out] -1/4*(sqrt(pi)*(c*log(f) - I*e)*sqrt(-c*log(f) - I*e)*erf(1/2*(2*e^2*x + (2*c^2*x + b*c)*log(f)^2 + b*e + (I*b*c - I*b*e)*log(f))*sqrt(-c*log(f) - I*e)/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*b^2*e - 4*I*a*e^2 - (-2*I*b^2*c + 4*I*a*c^2 + I*b^2*e)*log(f)^2 - (b^2*c - 2*b^2*e + 4*a*e^2)*log(f))/(c^2*log(f)^2 + e^2)) + sqrt(pi)*(c*log(f) + I*e)*sqrt(-c*log(f) + I*e)*erf(1/2*(2*e^2*x + (2*c^2*x + b*c)*log(f)^2 + b*e + (-I*b*c + I*b*e)*log(f))*sqrt(-c*log(f) + I*e)/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 - I*b^2*e + 4*I*a*e^2 - (2*I*b^2*c - 4*I*a*c^2 - I*b^2*e)*log(f)^2 - (b^2*c - 2*b^2*e + 4*a*e^2)*log(f))/(c^2*log(f)^2 + e^2)))/(c^2*log(f)^2 + e^2)

Sympy [F]

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx=\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*cos(e*x**2+b*x+a),x)
[Out] Integral(f**(a + b*x + c*x**2)*cos(a + b*x + e*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 1058, normalized size of antiderivative = 5.06

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x, algorithm="maxima")
[Out] 1/8*sqrt(pi)*((f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(cos(1/2*arctan2(e, -
c*log(f))) + I*sin(1/2*arctan2(e, -c*log(f))))*cos(-1/4*(b^2*e - 4*a*e^2 +
(2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)) - f^(1/4*b^2*c/
(c^2*log(f)^2 + e^2))*f^a*(-I*cos(1/2*arctan2(e, -c*log(f))) + sin(1/2*arct
an2(e, -c*log(f))))*sin(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)
*log(f)^2)/(c^2*log(f)^2 + e^2)))*erf(x*conjugate(sqrt(-c*log(f) + I*e)) -
1/2*(b*log(f) + I*b)*conjugate(1/sqrt(-c*log(f) + I*e))) + (f^(1/4*b^2*c/(c
^2*log(f)^2 + e^2))*f^a*(cos(1/2*arctan2(e, -c*log(f))) - I*sin(1/2*arctan2
(e, -c*log(f))))*cos(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)*lo
g(f)^2)/(c^2*log(f)^2 + e^2)) - f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(I*c
os(1/2*arctan2(e, -c*log(f))) + sin(1/2*arctan2(e, -c*log(f))))*sin(-1/4*(b
^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)
))*erf(x*conjugate(sqrt(-c*log(f) - I*e)) - 1/2*(b*log(f) - I*b)*conjugate(
1/sqrt(-c*log(f) - I*e))) + (f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(cos(1/
2*arctan2(e, -c*log(f))) - I*sin(1/2*arctan2(e, -c*log(f))))*cos(-1/4*(b^2*
e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)) -
 f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(I*cos(1/2*arctan2(e, -c*log(f))) +
 sin(1/2*arctan2(e, -c*log(f))))*sin(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a
*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 + e^2)))*erf(1/2*(2*(c*log(f) - I*e)*
x + b*log(f) - I*b)*sqrt(-c*log(f) + I*e)/(c*log(f) - I*e)) + (f^(1/4*b^2*c
/(c^2*log(f)^2 + e^2))*f^a*(cos(1/2*arctan2(e, -c*log(f))) + I*sin(1/2*arct
an2(e, -c*log(f))))*cos(-1/4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)
*log(f)^2)/(c^2*log(f)^2 + e^2)) - f^(1/4*b^2*c/(c^2*log(f)^2 + e^2))*f^a*(
-I*cos(1/2*arctan2(e, -c*log(f))) + sin(1/2*arctan2(e, -c*log(f))))*sin(-1/
4*(b^2*e - 4*a*e^2 + (2*b^2*c - 4*a*c^2 - b^2*e)*log(f)^2)/(c^2*log(f)^2 +
```

```
e^2)))*erf(1/2*(2*(c*log(f) + I*e)*x + b*log(f) + I*b)*sqrt(-c*log(f) - I*e
)/(c*log(f) + I*e)))*e^(-1/4*b^2*c*log(f)^3/(c^2*log(f)^2 + e^2) - 1/2*b^2*
e*log(f)/(c^2*log(f)^2 + e^2))/(c^2*log(f)^2 + e^2)^(1/4)
```

Giac [F]

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx=\int f^{cx^2+bx+a}\cos\left(ex^2+bx+a\right)\,dx$$

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x, algorithm="giac")
[Out] integrate(f^(c*x^2 + b*x + a)*cos(e*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cos\left(e\,x^2+b\,x+a\right)\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*cos(a + b*x + e*x^2),x)
[Out] int(f^(a + b*x + c*x^2)*cos(a + b*x + e*x^2), x)
```

3.135 $\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 245

$$\begin{aligned}\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx &= \frac{f^2F^{ac+bcx}}{bc\log(F)} - \frac{2ef^2F^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)} \\ &+ \frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)} \\ &+ \frac{2bcf^2F^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)} \\ &- \frac{2ef^2F^{ac+bcx}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)} \\ &+ \frac{bcf^2F^{ac+bcx}\log(F)\sin^2(d+ex)}{4e^2+b^2c^2\log^2(F)}\end{aligned}$$

```
[Out] f^2*F^(b*c*x+a*c)/b/c/ln(F)-2*e*f^2*F^(b*c*x+a*c)*cos(e*x+d)/(e^2+b^2*c^2*l
n(F)^2)+2*e^2*f^2*F^(b*c*x+a*c)/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+2*b*c*f^2
*F^(b*c*x+a*c)*ln(F)*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)-2*e*f^2*F^(b*c*x+a*c)
*cos(e*x+d)*sin(e*x+d)/(4*e^2+b^2*c^2*ln(F)^2)+b*c*f^2*F^(b*c*x+a*c)*ln(F)*
sin(e*x+d)^2/(4*e^2+b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6873, 12, 6874, 2225, 4517, 4519}

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx=\frac{bcf^2\log(F)\sin^2(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2bcf^2\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{2ef^2\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{2ef^2\sin(d+ex)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}+\frac{f^2F^{ac+bcx}}{bc\log(F)}$$

```
[In] Int[F^(c*(a + b*x))*(f + f*Sin[d + e*x])^2,x]
```

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) - (2*e*f^2*F^(a*c + b*c*x)*Cos[d + e*x])
/(e^2 + b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[F]*(4*e^2
+ b^2*c^2*Log[F]^2)) + (2*b*c*f^2*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x])/(e^2
 + b^2*c^2*Log[F]^2) - (2*e*f^2*F^(a*c + b*c*x)*Cos[d + e*x]*Sin[d + e*x])/
(4*e^2 + b^2*c^2*Log[F]^2) + (b*c*f^2*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x]^2
)/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4519

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d +
 e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int f^2 F^{ac+bcx}(1+\sin(d+ex))^2\,dx \\
&= f^2 \int F^{ac+bcx}(1+\sin(d+ex))^2\,dx \\
&= f^2 \int \left(F^{ac+bcx}+2F^{ac+bcx}\sin(d+ex)+F^{ac+bcx}\sin^2(d+ex)\right)\,dx \\
&= f^2 \int F^{ac+bcx}\,dx + f^2 \int F^{ac+bcx}\sin^2(d+ex)\,dx + \left(2f^2\right)\int F^{ac+bcx}\sin(d+ex)\,dx \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)} - \frac{2ef^2F^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)} + \frac{2bcf^2F^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)} \\
&\quad - \frac{2ef^2F^{ac+bcx}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)} \\
&\quad + \frac{bcf^2F^{ac+bcx}\log(F)\sin^2(d+ex)}{4e^2+b^2c^2\log^2(F)} + \frac{\left(2e^2f^2\right)\int F^{ac+bcx}\,dx}{4e^2+b^2c^2\log^2(F)} \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)} - \frac{2ef^2F^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)} \\
&\quad + \frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)} + \frac{2bcf^2F^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)} \\
&\quad - \frac{2ef^2F^{ac+bcx}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)} + \frac{bcf^2F^{ac+bcx}\log(F)\sin^2(d+ex)}{4e^2+b^2c^2\log^2(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 7.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx$$

$$= \frac{f^2F^{c(a+bx)}(1+\sin(d+ex))^2\left(\frac{3}{bc\log(F)}-\frac{4e\cos(d+ex)}{e^2+b^2c^2\log^2(F)}-\frac{bc\cos(2(d+ex))\log(F)}{4e^2+b^2c^2\log^2(F)}+\frac{4bc\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}-\frac{2e\sin(2(d+e}{4e^2+b^2c^2\log^2}\right.}{2\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^4}$$

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Sin[d + e*x])^2,x]
```

```
[Out] (f^2*F^(c*(a + b*x))*(1 + Sin[d + e*x])^2*(3/(b*c*Log[F]) - (4*e*Cos[d + e*
x])/(e^2 + b^2*c^2*Log[F]^2) - (b*c*Cos[2*(d + e*x)]*Log[F])/(4*e^2 + b^2*c
^2*Log[F]^2) + (4*b*c*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) - (2*e*
Sin[2*(d + e*x)])/(4*e^2 + b^2*c^2*Log[F]^2)))/(2*(Cos[(d + e*x)/2] + Sin[(
d + e*x)/2])^4)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.79

method	result
risch	$\frac{3f^2F^{c(xb+a)}}{2bc\ln(F)}-\frac{2F^{c(xb+a)}ef^2\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{2F^{c(xb+a)}\ln(F)bcf^2\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}-\frac{\ln(F)cbf^2F^{c(xb+a)}\cos(2ex+2d)}{2\left(4e^2+b^2c^2\ln(F)^2\right)}-\frac{ef^2}{}$
parallelrisch	$-\frac{2F^{c(xb+a)}f^2\left(\frac{b^2c^2\ln(F)^2\left(e^2+b^2c^2\ln(F)^2\right)\cos(2ex+2d)}{4}+\frac{cbe\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\sin(2ex+2d)}{2}+\left(4e^2+b^2c^2\ln(F)^2\right)\left(-\sin(e\right.\right.}{bc\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\left(4e^2+b^2c^2\ln(F)^2\right)}$
default	$\frac{F^{ac}f^2\left(\frac{4F^{bcx}}{bc\ln(F)}+\frac{-\frac{8e\,e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4e^2+b^2c^2\ln(F)^2}+\frac{8e\,e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{4e^2+b^2c^2\ln(F)^2}-\frac{2\left(b^2c^2\ln(F)^2+2e^2\right)e^{bcx\ln(F)}}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{4\left(b^2c^2\ln(F)^2-2e^2\right)e^{bcx}}{bc\ln(F)\left(4e^2+b^2\right.}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}\right.}{}$
parts	$\frac{f^2F^{c(xb+a)}}{bc\ln(F)}+\frac{-\frac{4ef^2e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4e^2+b^2c^2\ln(F)^2}+\frac{4ef^2e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{4e^2+b^2c^2\ln(F)^2}+\frac{2e^2f^2e^{c(xb+a)\ln(F)}}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{2e^2f^2e^{c(xb+a)}}{bc\ln(F)\left(4e\right.}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}$
norman	$\frac{\frac{\left(b^4c^4\ln(F)^4-2\ln(F)^3b^3c^3e+7b^2c^2e^2\ln(F)^2-8e^3bc\ln(F)+6e^4\right)f^2e^{c(xb+a)\ln(F)}}{bc\ln(F)\left(b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2+4e^4\right)}+\frac{f^2\left(b^4c^4\ln(F)^4+2\ln(F)^3b^3c^3e+7b^2c^2e^2\ln(F)^2+8e^3\right.}{\left(b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F\right.}}{}$

```
[In] int(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2/b/c/ln(F)*f^2*F^(c*(b*x+a))-2*F^(c*(b*x+a))*e*f^2/(e^2+b^2*c^2*ln(F)^2)
*cos(e*x+d)+2*F^(c*(b*x+a))*ln(F)*b*c*f^2/(e^2+b^2*c^2*ln(F)^2)*sin(e*x+d)-
```

```
1/2/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*c*b*f^2*F^(c*(b*x+a))*cos(2*e*x+2*d)-e*f^
2*F^(c*(b*x+a))/(4*e^2+b^2*c^2*ln(F)^2)*sin(2*e*x+2*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.04

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx =$$
$$-\frac{\left(2\,b^3c^3ef^2\cos\left(ex+d\right)\log\left(F\right)^3+8\,bce^3f^2\cos\left(ex+d\right)\log\left(F\right)-6\,e^4f^2+\left(b^4c^4f^2\cos\left(ex+d\right)^2-2\,b^4c^4f\right.\right.}{}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -(2*b^3*c^3*e*f^2*cos(e*x + d)*log(F)^3 + 8*b*c*e^3*f^2*cos(e*x + d)*log(F)
 - 6*e^4*f^2 + (b^4*c^4*f^2*cos(e*x + d)^2 - 2*b^4*c^4*f^2)*log(F)^4 + (b^2
*c^2*e^2*f^2*cos(e*x + d)^2 - 8*b^2*c^2*e^2*f^2)*log(F)^2 - 2*(b^4*c^4*f^2*
log(F)^4 - b^3*c^3*e*f^2*cos(e*x + d)*log(F)^3 + 4*b^2*c^2*e^2*f^2*log(F)^2
 - b*c*e^3*f^2*cos(e*x + d)*log(F))*sin(e*x + d))*F^(b*c*x + a*c)/(b^5*c^5*
log(F)^5 + 5*b^3*c^3*e^2*log(F)^3 + 4*b*c*e^4*log(F))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.75 (sec) , antiderivative size = 2465, normalized size of antiderivative = 10.06

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx = \text{Too large to display}$$

```
[In] integrate(F**(c*(b*x+a))*(f+f*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((x*(f*sin(d) + f)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)),
 (f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x - f**2*sin(d
 + e*x)*cos(d + e*x)/(2*e) - 2*f**2*cos(d + e*x)/e, Eq(F, 1)), (F**(a*c)*(f
**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x - f**2*sin(d +
e*x)*cos(d + e*x)/(2*e) - 2*f**2*cos(d + e*x)/e), Eq(b, 0)), (f**2*x*sin(d
+ e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x - f**2*sin(d + e*x)*cos(d +
 e*x)/(2*e) - 2*f**2*cos(d + e*x)/e, Eq(c, 0)), (-F**(a*c + b*c*x)*f**2*x*s
in(I*b*c*x*log(F) - d) + I*F**(a*c + b*c*x)*f**2*x*cos(I*b*c*x*log(F) - d)
+ F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) - d)**2/(3*b*c*log(F)) + 2*I*F**
(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) - d)*cos(I*b*c*x*log(F) - d)/(3*b*c*l
og(F)) + F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) - d)/(b*c*log(F)) + 2*F**
(a*c + b*c*x)*f**2*cos(I*b*c*x*log(F) - d)**2/(3*b*c*log(F)) - 2*I*F**(a*c
```

```
+ b*c*x)*f**2*cos(I*b*c*x*log(F) - d)/(b*c*log(F)) + F**(a*c + b*c*x)*f**2/
(b*c*log(F)), Eq(e, -I*b*c*log(F))), (F**(a*c + b*c*x)*f**2*x*sin(I*b*c*x*l
og(F)/2 - d)**2/4 - I*F**(a*c + b*c*x)*f**2*x*sin(I*b*c*x*log(F)/2 - d)*cos
(I*b*c*x*log(F)/2 - d)/2 - F**(a*c + b*c*x)*f**2*x*cos(I*b*c*x*log(F)/2 - d
)**2/4 + 3*I*F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F)/2 - d)*cos(I*b*c*x*lo
g(F)/2 - d)/(2*b*c*log(F)) - 8*F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F)/2 -
 d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2*cos(I*b*c*x*log(F)/2 - d)**2/(b*
c*log(F)) + 4*I*F**(a*c + b*c*x)*f**2*cos(I*b*c*x*log(F)/2 - d)/(3*b*c*log(
F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, -I*b*c*log(F)/2)), (F**(a*c
 + b*c*x)*f**2*x*sin(I*b*c*x*log(F)/2 + d)**2/4 - I*F**(a*c + b*c*x)*f**2*x
*sin(I*b*c*x*log(F)/2 + d)*cos(I*b*c*x*log(F)/2 + d)/2 - F**(a*c + b*c*x)*f
**2*x*cos(I*b*c*x*log(F)/2 + d)**2/4 + F**(a*c + b*c*x)*f**2*sin(I*b*c*x*lo
g(F)/2 + d)**2/(b*c*log(F)) - I*F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F)/2
+ d)*cos(I*b*c*x*log(F)/2 + d)/(2*b*c*log(F)) + 8*F**(a*c + b*c*x)*f**2*sin
(I*b*c*x*log(F)/2 + d)/(3*b*c*log(F)) - 4*I*F**(a*c + b*c*x)*f**2*cos(I*b*c
*x*log(F)/2 + d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e,
 I*b*c*log(F)/2)), (F**(a*c + b*c*x)*f**2*x*sin(I*b*c*x*log(F) + d) - I*F**
(a*c + b*c*x)*f**2*x*cos(I*b*c*x*log(F) + d) + F**(a*c + b*c*x)*f**2*sin(I*
b*c*x*log(F) + d)**2/(3*b*c*log(F)) + 2*I*F**(a*c + b*c*x)*f**2*sin(I*b*c*x
*log(F) + d)*cos(I*b*c*x*log(F) + d)/(3*b*c*log(F)) - F**(a*c + b*c*x)*f**2
*sin(I*b*c*x*log(F) + d)/(b*c*log(F)) + 2*F**(a*c + b*c*x)*f**2*cos(I*b*c*x
*log(F) + d)**2/(3*b*c*log(F)) + 2*I*F**(a*c + b*c*x)*f**2*cos(I*b*c*x*log(
F) + d)/(b*c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, I*b*c*log(
F))), (F**(a*c + b*c*x)*b**4*c**4*f**2*log(F)**4*sin(d + e*x)**2/(b**5*c**5
*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c +
b*c*x)*b**4*c**4*f**2*log(F)**4*sin(d + e*x)/(b**5*c**5*log(F)**5 + 5*b**3*
c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + F**(a*c + b*c*x)*b**4*c**4*f**2*
log(F)**4/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*lo
g(F)) - 2*F**(a*c + b*c*x)*b**3*c**3*e*f**2*log(F)**3*sin(d + e*x)*cos(d +
e*x)/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F))
 - 2*F**(a*c + b*c*x)*b**3*c**3*e*f**2*log(F)**3*cos(d + e*x)/(b**5*c**5*lo
g(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 3*F**(a*c + b*c
*x)*b**2*c**2*e**2*f**2*log(F)**2*sin(d + e*x)**2/(b**5*c**5*log(F)**5 + 5*
b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 8*F**(a*c + b*c*x)*b**2*c**
2*e**2*f**2*log(F)**2*sin(d + e*x)/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*
log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c + b*c*x)*b**2*c**2*e**2*f**2*log
(F)**2*cos(d + e*x)**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 +
4*b*c*e**4*log(F)) + 5*F**(a*c + b*c*x)*b**2*c**2*e**2*f**2*log(F)**2/(b**5
*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 2*F**(a
*c + b*c*x)*b*c*e**3*f**2*log(F)*sin(d + e*x)*cos(d + e*x)/(b**5*c**5*log(F
)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) - 8*F**(a*c + b*c*x)
*b*c*e**3*f**2*log(F)*cos(d + e*x)/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*
log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c + b*c*x)*e**4*f**2*sin(d + e*x)*
*2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) +
 2*F**(a*c + b*c*x)*e**4*f**2*cos(d + e*x)**2/(b**5*c**5*log(F)**5 + 5*b**3
```

```
*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 4*F**(a*c + b*c*x)*e**4*f**2/(b
**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)), True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(245) = 490$.

Time = 0.26 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.37

$$\int F^{c(a+bx)}(f + f\sin(d+ex))^2\,dx =$$

$$-\frac{\left(\left(F^{ac}b^2c^2\cos\left(2\,d\right)\log\left(F\right)^2 + 2\,F^{ac}bce\log\left(F\right)\sin\left(2\,d\right)\right)F^{bcx}\cos\left(2\,ex\right) + \left(F^{ac}b^2c^2\cos\left(2\,d\right)\log\left(F\right)^2 - 2\,F\right.}{}$$

$$-\frac{\left(\left(F^{ac}bc\log\left(F\right)\sin\left(d\right) + F^{ac}e\cos\left(d\right)\right)F^{bcx}\cos\left(ex+2\,d\right) - \left(F^{ac}bc\log\left(F\right)\sin\left(d\right) - F^{ac}e\cos\left(d\right)\right)F^{bcx}\cos\left(\right.}{b^2c^2\cos\left(d\right)^2\log\left(F\right)^2 + b^2c^2}$$

$$+\frac{F^{bcx+ac}f^2}{bc\log\left(F\right)}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))
*F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c*
e*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2*s
in(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a*c)*
b^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(
2*e*x + 4*d) - 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c^2*log
(F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*F^(b*c*
x))*f^2/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 + 4*(b*c
*cos(2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2) - ((F^(a*c)*b*c*log(F)*sin
(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2*d) - (F^(a*c)*b*c*log(F)*sin(
d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) - (F^(a*c)*b*c*cos(d)*log(F) - F^
(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b*c*cos(d)*log(F) + F^(
a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))*f^2/(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2
*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2) + F^(b*c*x + a*c)*f^2/(b*c*
log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 1738, normalized size of antiderivative = 7.09

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F)
- 1/2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*
sgn(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*f^2*sin(1/2*pi*b
*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)/
(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(
abs(F)) + a*c*log(abs(F))) - 1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*p
i*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)*log(abs(F))/(4*b^2*
c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi
*b*c - 4*e)*f^2*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F)
- 1/2*pi*a*c - 2*e*x - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c - 4*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*(2*b*c*f^2*cos(-
1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(ab
s(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(
F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F
) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(
b*c*x*log(abs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f^2*log(abs(F))*sin(1/2*pi*
b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*
b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) - (pi*b*c*sgn(F)
- pi*b*c + 2*e)*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn
(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 2*(2*b*c*f^2*log(a
bs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*
a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2)
 - (pi*b*c*sgn(F) - pi*b*c - 2*e)*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*
x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (p
i*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
I*(-I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) -
 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*lo
g(abs(F)) + 16*I*e) + I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/
2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(-4*I*pi*b*c*sgn(F) + 4
*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs
(F))) - (-I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sg
n(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(
abs(F)) + 2*I*e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I
*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-I*pi*b*c*sgn(F) + I*pi*b*c +
```

```
2*b*c*log(abs(F)) - 2*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - (I*f
^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*
pi*a*c - I*e*x - I*d)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) - 2*I
*e) + I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F
) + 1/2*I*pi*a*c + I*e*x + I*d)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(ab
s(F)) + 2*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(-I*f^2*e^(1/2*
I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - 2
*I*e*x - 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*
e) + I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F)
 + 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c
*log(abs(F)) + 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f^2*
e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*
a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*f^2*e^(-1/2*I
*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2
*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) +
a*c*log(abs(F))) + I*(I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2
*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(ab
s(F))) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(
b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 27.57 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx = -\frac{F^{a\,c+b\,c\,x}\,f^2\left(\frac{b^4\,c^4\,\ln(F)^4\,\cos(2\,d+2\,e\,x)}{2}-\frac{3\,b^4\,c^4\,\ln(F)^4}{2}-2\,b^4\,c^4\,\sin\left(d+e\,x\right)\,\ln\left(F\right)^4-6\,e^4-\frac{15\,b^2\,c^2\,e^2\,\ln(F)^2}{2}+b^3\,c\right.}{}$$

```
[In] int(F^(c*(a + b*x))*(f + f*sin(d + e*x))^2,x)
```

```
[Out] -(F^(a*c + b*c*x)*f^2*((b^4*c^4*log(F)^4*cos(2*d + 2*e*x))/2 - (3*b^4*c^4*l
og(F)^4)/2 - 2*b^4*c^4*sin(d + e*x)*log(F)^4 - 6*e^4 - (15*b^2*c^2*e^2*log(
F)^2)/2 + b^3*c^3*e*log(F)^3*sin(2*d + 2*e*x) - 8*b^2*c^2*e^2*sin(d + e*x)*
log(F)^2 + b*c*e^3*log(F)*sin(2*d + 2*e*x) + (b^2*c^2*e^2*log(F)^2*cos(2*d
+ 2*e*x))/2 + 2*b^3*c^3*e*cos(d + e*x)*log(F)^3 + 8*b*c*e^3*cos(d + e*x)*lo
g(F)))/(b*c*log(F)*(4*e^4 + b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2))
```

3.136 $\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 99

$$\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx=\frac{fF^{ac+bcx}}{bc\log(F)}-\frac{efF^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{bcfF^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

```
[Out] f*F^(b*c*x+a*c)/b/c/ln(F)-e*f*F^(b*c*x+a*c)*cos(e*x+d)/(e^2+b^2*c^2*ln(F)^2
)+b*c*f*F^(b*c*x+a*c)*ln(F)*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6873, 12, 6874, 2225, 4517}

$$\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx=\frac{bcf\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{ef\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

```
[In] Int[F^(c*(a + b*x))*(f + f*Sin[d + e*x]),x]
```

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) - (e*f*F^(a*c + b*c*x)*Cos[d + e*x])/(e^2
+ b^2*c^2*Log[F]^2) + (b*c*f*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x])/(e^2 + b^
2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int fF^{ac+bcx}(1+\sin(d+ex))\,dx \\
&= f\int F^{ac+bcx}(1+\sin(d+ex))\,dx \\
&= f\int \left(F^{ac+bcx}+F^{ac+bcx}\sin(d+ex)\right)\,dx \\
&= f\int F^{ac+bcx}\,dx+f\int F^{ac+bcx}\sin(d+ex)\,dx \\
&= \frac{fF^{ac+bcx}}{bc\log(F)}-\frac{efF^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{bcfF^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx = \frac{fF^{c(a+bx)}\left(e^2-bce\cos(d+ex)\log(F)+b^2c^2\log^2(F)+b^2c^2\log^2(F)\sin(d+ex)\right)}{bc\log(F)\left(e^2+b^2c^2\log^2(F)\right)}$$

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Sin[d + e*x]),x]
```

```
[Out] (f*F^(c*(a + b*x))*(e^2 - b*c*e*Cos[d + e*x]*Log[F] + b^2*c^2*Log[F]^2 + b^
2*c^2*Log[F]^2*Sin[d + e*x]))/(b*c*Log[F]*(e^2 + b^2*c^2*Log[F]^2))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

method	result
parallelrisch	$\frac{f\,F^{c(xb+a)}\left(\sin(ex+d)b^2c^2\ln(F)^2+b^2c^2\ln(F)^2-bc\ln(F)e\cos(ex+d)+e^2\right)}{\left(e^2+b^2c^2\ln(F)^2\right)bc\ln(F)}$
risch	$\frac{f\,F^{c(xb+a)}}{bc\ln(F)}+\frac{e\,F^{c(xb+a)}f\cos(ex+d)}{-e^2-b^2c^2\ln(F)^2}-\frac{\ln(F)cb\,F^{c(xb+a)}f\sin(ex+d)}{-e^2-b^2c^2\ln(F)^2}$
parts	$\frac{f\,F^{c(xb+a)}}{bc\ln(F)}+\frac{\frac{ef\,\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}-\frac{ef\,\mathrm{e}^{c(xb+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2fbc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$
norman	$\frac{\frac{f\left(b^2c^2\ln(F)^2-\ln(F)bce+e^2\right)\mathrm{e}^{c(xb+a)\ln(F)}}{\left(e^2+b^2c^2\ln(F)^2\right)bc\ln(F)}+\frac{f\left(b^2c^2\ln(F)^2+\ln(F)bce+e^2\right)\mathrm{e}^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{\left(e^2+b^2c^2\ln(F)^2\right)bc\ln(F)}+\frac{2fbc\ln(F)\mathrm{e}^{c(xb+a)\ln(F)}\tan}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$

```
[In] int(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

```
[Out] f*F^(c*(b*x+a))*(sin(e*x+d)*b^2*c^2*ln(F)^2+b^2*c^2*ln(F)^2-b*c*ln(F)*e*cos
(e*x+d)+e^2)/(e^2+b^2*c^2*ln(F)^2)/b/c/ln(F)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx = \frac{\left(b^2c^2f\log\left(F\right)^2\sin\left(ex+d\right)+b^2c^2f\log\left(F\right)^2-bcef\cos\left(ex+d\right)\log\left(F\right)+e^2f\right)F^{bcx+ac}}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] (b^2*c^2*f*log(F)^2*sin(e*x + d) + b^2*c^2*f*log(F)^2 - b*c*e*f*cos(e*x + d
)*log(F) + e^2*f)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + b*c*e^2*log(F))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 542, normalized size of antiderivative = 5.47

$$\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx = \begin{cases} x(f\sin\left(d\right)+f) \\ fx-\frac{f\cos\left(d+ex\right)}{e} \\ F^{ac}\left(fx-\frac{f\cos\left(d+ex\right)}{e}\right) \\ fx-\frac{f\cos\left(d+ex\right)}{e} \\ -\frac{F^{ac+bcx}fx\sin\left(ibcx\log\left(F\right)-d\right)}{2}+\frac{iF^{ac+bcx}fx\cos\left(ibcx\log\left(F\right)-d\right)}{2}+\frac{F^{ac+bcx}f\sin\left(ibcx\log\left(F\right)-d\right)}{2bc\log\left(F\right)}-\frac{iF^{ac+bcx}f\cos\left(ibcx\log\left(F\right)-d\right)}{bc\log\left(F\right)} \\ \frac{F^{ac+bcx}fx\sin\left(ibcx\log\left(F\right)+d\right)}{2}-\frac{iF^{ac+bcx}fx\cos\left(ibcx\log\left(F\right)+d\right)}{2}-\frac{F^{ac+bcx}f\sin\left(ibcx\log\left(F\right)+d\right)}{2bc\log\left(F\right)}+\frac{iF^{ac+bcx}f\cos\left(ibcx\log\left(F\right)+d\right)}{bc\log\left(F\right)}+ \\ \frac{F^{ac+bcx}b^2c^2f\log\left(F\right)^2\sin\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)}+\frac{F^{ac+bcx}b^2c^2f\log\left(F\right)^2}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)}-\frac{F^{ac+bcx}bcef\log\left(F\right)\cos\left(d+ex\right)}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)}+\frac{F^{ac+bcx}e^2f}{b^3c^3\log\left(F\right)^3+bce^2\log\left(F\right)} \end{cases}$$

```
[In] integrate(F**(c*(b*x+a))*(f+f*sin(e*x+d)),x)
```

```
[Out] Piecewise((x*(f*sin(d) + f), Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (f
*x - f*cos(d + e*x)/e, Eq(F, 1)), (F**(a*c)*(f*x - f*cos(d + e*x)/e), Eq(b,
 0)), (f*x - f*cos(d + e*x)/e, Eq(c, 0)), (-F**(a*c + b*c*x)*f*x*sin(I*b*c*
x*log(F) - d)/2 + I*F**(a*c + b*c*x)*f*x*cos(I*b*c*x*log(F) - d)/2 + F**(a*
c + b*c*x)*f*sin(I*b*c*x*log(F) - d)/(2*b*c*log(F)) - I*F**(a*c + b*c*x)*f*
cos(I*b*c*x*log(F) - d)/(b*c*log(F)) + F**(a*c + b*c*x)*f/(b*c*log(F)), Eq(
e, -I*b*c*log(F))), (F**(a*c + b*c*x)*f*x*sin(I*b*c*x*log(F) + d)/2 - I*F**
(a*c + b*c*x)*f*x*cos(I*b*c*x*log(F) + d)/2 - F**(a*c + b*c*x)*f*sin(I*b*c*
```

```
x*log(F) + d)/(2*b*c*log(F)) + I*F**(a*c + b*c*x)*f*cos(I*b*c*x*log(F) + d)
/(b*c*log(F)) + F**(a*c + b*c*x)*f/(b*c*log(F)), Eq(e, I*b*c*log(F))), (F**
(a*c + b*c*x)*b**2*c**2*f*log(F)**2*sin(d + e*x)/(b**3*c**3*log(F)**3 + b*c
*e**2*log(F)) + F**(a*c + b*c*x)*b**2*c**2*f*log(F)**2/(b**3*c**3*log(F)**3
 + b*c*e**2*log(F)) - F**(a*c + b*c*x)*b*c*e*f*log(F)*cos(d + e*x)/(b**3*c*
*3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c + b*c*x)*e**2*f/(b**3*c**3*log(F)
**3 + b*c*e**2*log(F)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(99) = 198$.

Time = 0.22 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.20

$$\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx = -\frac{((F^{ac}bc\log(F)\sin(d)+F^{ac}e\cos(d))F^{bcx}\cos(ex+2d)-(F^{ac}bc\log(F)\sin(d)-F^{ac}e\cos(d))F^{bcx}\cos}{2\left(b^2c^2\cos(d)^2\log(F)^2+b^2c\right.} + \frac{F^{bcx+ac}f}{bc\log(F)}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] -1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2*
d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) - (F
^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^
(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))*f/(b^2*c^2*
cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2)
+ F^(b*c*x + a*c)*f/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 923, normalized size of antiderivative = 9.32

$$\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/
2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2
) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - p
i*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*f*log(abs(F))*s
```

```
in(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*
x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) - (pi*b
*c*sgn(F) - pi*b*c + 2*e)*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi
*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(
F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - (2*b*c*f*l
og(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2
*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e
)^2) - (pi*b*c*sgn(F) - pi*b*c - 2*e)*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*
c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 +
(pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
- (-I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1
/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(
F)) + 4*I*e) - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*
c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4
*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - (I*f*e
^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a
*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I
*e) + I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F)
+ 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(
abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f*e^(1/2*I
*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*
pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) - I*f*e^(-1/2*I*pi*b*c*x*sgn(
F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F)
 + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 25.80 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx = \frac{F^{a\,c+b\,c\,x}\,f\,\left(e^2+b^2\,c^2\ln\left(F\right)^2+b^2\,c^2\,\sin\left(d+e\,x\right)\,\ln\left(F\right)^2-b\,c\,e\,\cos\left(d+e\,x\right)\,\ln\left(F\right)\right)}{b\,c\,\ln\left(F\right)\,\left(b^2\,c^2\ln\left(F\right)^2+e^2\right)}$$

```
[In] int(F^(c*(a + b*x))*(f + f*sin(d + e*x)),x)
```

```
[Out] (F^(a*c + b*c*x)*f*(e^2 + b^2*c^2*log(F)^2 + b^2*c^2*sin(d + e*x)*log(F)^2
- b*c*e*cos(d + e*x)*log(F)))/(b*c*log(F)*(e^2 + b^2*c^2*log(F)^2))
```

3.137 $\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},ie^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

```
[Out] -2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-I*b*c*ln(F)/e],[2-I*b*c*ln(
F)/e],I*exp(I*(e*x+d)))/f/(e-I*b*c*ln(F))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4541, 4535}

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},ie^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

```
[In] Int[F^(c*(a + b*x))/(f + f*Sin[d + e*x]),x]
```

```
[Out] (-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])
/e, 2 - (I*b*c*Log[F])/e, I*E^(I*(d + e*x))])/(f*(e - I*b*c*Log[F]))
```

Rule 4535

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/
(I*e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 +
n/2 - I*b*c*(Log[F]/(2*e)), (-E^(2*I*k*Pi))*E^(2*I*(d + e*x))], x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

Rule 4541

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4*
g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f
^2 - g^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int F^{c(a+bx)}\sec^2\left(\frac{d}{2}-\frac{\pi}{4}+\frac{ex}{2}\right)dx}{2f}$$

$$= -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},ie^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

Mathematica [A] (verified)

Time = 3.87 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.60

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx$$

$$= \frac{2F^{c(a+bx)}\left(-i\,\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{e},1-\frac{ibc\log(F)}{e},i\cos(d+ex)-\sin(d+ex)\right)-\frac{1}{\cos(d)+i(1+\sin(d}\right.}{ef}$$

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Sin[d + e*x]),x]
```

```
[Out] (2*F^(c*(a + b*x))*((-I)*Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b
*c*Log[F])/e, I*Cos[d + e*x] - Sin[d + e*x]] - (Cos[d] + I*(1 + Sin[d]))^(-
1) + Sin[(e*x)/2]/((Cos[d/2] + Sin[d/2])*(Cos[(d + e*x)/2] + Sin[(d + e*x)/
2]))))/(e*f)
```

Maple [F]

$$\int \frac{F^{c(xb+a)}}{f+f\sin(ex+d)}dx$$

```
[In] int(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x)
[Out] int(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x)
```

Fricas [F]

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\sin(ex+d)+f}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)/(f*sin(e*x + d) + f), x)
```

Sympy [F]

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = \frac{\int \frac{F^{ac+bcx}}{\sin(d+ex)+1}\,dx}{f}$$

```
[In] integrate(F**(c*(b*x+a))/(f+f*sin(e*x+d)),x)
[Out] Integral(F**(a*c + b*c*x)/(sin(d + e*x) + 1), x)/f
```

Maxima [F]

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\sin(ex+d)+f}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x, algorithm="maxima")
[Out] 2*(6*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3 + 4*F^(
a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(e*x + d)^2 + 2*(F^(a*c)*b^3*c^3*log(F)^3
 + 4*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(e*x + d)^2 + (5*F^(a*c)*b^2*c^2*
e*log(F)^2 - 4*F^(a*c)*e^3)*F^(b*c*x)*cos(e*x + d) + (F^(a*c)*b^3*c^3*log(F
)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(e*x + d) - (6*F^(b*c*x)*F^(a
*c)*b*c*e^2*log(F) + (F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*e^3)*F^(b*c*x)
*cos(e*x + d) + (F^(a*c)*b^3*c^3*log(F)^3 + 4*F^(a*c)*b*c*e^2*log(F))*F^(b*
c*x)*sin(e*x + d))*cos(2*e*x + 2*d) - 2*((F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^
```

```
(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*cos(2*e*x + 2*d)^2
 + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*
c)*b*c*e^5*log(F))*f*cos(e*x + d)^2 + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(
a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*cos(e*x + d)*sin(2*
e*x + 2*d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 +
 4*F^(a*c)*b*c*e^5*log(F))*f*sin(2*e*x + 2*d)^2 + 4*(F^(a*c)*b^5*c^5*e*log(
F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*sin(e*x
 + d)^2 + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 +
4*F^(a*c)*b*c*e^5*log(F))*f*sin(e*x + d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 5*
F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f - 2*(2*(F^(a*c)*
b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log
(F))*f*sin(e*x + d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*l
og(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f)*cos(2*e*x + 2*d))*integrate((3*F^(b*
c*x)*b*c*e*cos(3*e*x + 3*d)*log(F) - 9*F^(b*c*x)*b*c*e*cos(e*x + d)*log(F)
- 9*F^(b*c*x)*b*c*e*log(F)*sin(2*e*x + 2*d) - 3*(b^2*c^2*log(F)^2 - 2*e^2)*
F^(b*c*x)*cos(2*e*x + 2*d) - (b^2*c^2*log(F)^2 - 2*e^2)*F^(b*c*x)*sin(3*e*x
 + 3*d) + 3*(b^2*c^2*log(F)^2 - 2*e^2)*F^(b*c*x)*sin(e*x + d) + (b^2*c^2*lo
g(F)^2 - 2*e^2)*F^(b*c*x))/((b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*
e^4)*f*cos(3*e*x + 3*d)^2 + 9*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 +
4*e^4)*f*cos(2*e*x + 2*d)^2 + 9*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2
+ 4*e^4)*f*cos(e*x + d)^2 + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*
e^4)*f*sin(3*e*x + 3*d)^2 + 18*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 +
 4*e^4)*f*cos(e*x + d)*sin(2*e*x + 2*d) + 9*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e
^2*log(F)^2 + 4*e^4)*f*sin(2*e*x + 2*d)^2 + 9*(b^4*c^4*log(F)^4 + 5*b^2*c^2
*e^2*log(F)^2 + 4*e^4)*f*sin(e*x + d)^2 + 6*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e
^2*log(F)^2 + 4*e^4)*f*sin(e*x + d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log
(F)^2 + 4*e^4)*f - 6*((b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f
*cos(e*x + d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(2
*e*x + 2*d))*cos(3*e*x + 3*d) - 6*(3*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(
F)^2 + 4*e^4)*f*sin(e*x + d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 +
 4*e^4)*f)*cos(2*e*x + 2*d) + 2*(3*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)
^2 + 4*e^4)*f*cos(2*e*x + 2*d) - 3*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)
^2 + 4*e^4)*f*sin(e*x + d) - (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4
*e^4)*f)*sin(3*e*x + 3*d)), x) + ((F^(a*c)*b^3*c^3*log(F)^3 + 4*F^(a*c)*b*c
*e^2*log(F))*F^(b*c*x)*cos(e*x + d) - (F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*
c)*e^3)*F^(b*c*x)*sin(e*x + d) + 2*(F^(a*c)*b^2*c^2*e*log(F)^2 - 2*F^(a*c)*
e^3)*F^(b*c*x))*sin(2*e*x + 2*d))/((b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)
^2 + 4*e^4)*f*cos(2*e*x + 2*d)^2 + 4*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(
F)^2 + 4*e^4)*f*cos(e*x + d)^2 + 4*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)
^2 + 4*e^4)*f*cos(e*x + d)*sin(2*e*x + 2*d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2
*e^2*log(F)^2 + 4*e^4)*f*sin(2*e*x + 2*d)^2 + 4*(b^4*c^4*log(F)^4 + 5*b^2*c
^2*e^2*log(F)^2 + 4*e^4)*f*sin(e*x + d)^2 + 4*(b^4*c^4*log(F)^4 + 5*b^2*c^2
*e^2*log(F)^2 + 4*e^4)*f*sin(e*x + d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*l
og(F)^2 + 4*e^4)*f - 2*(2*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^
4)*f*sin(e*x + d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f)*
```

```
cos(2*e*x + 2*d))
```

Giac [F]

$$\int \frac{F^{c(a+bx)}}{f + f\sin(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\sin(ex+d)+f}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)/(f*sin(e*x + d) + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{f + f\sin(d+ex)}\,dx = \int \frac{F^{c\,(a+b\,x)}}{f+f\,\sin(d+e\,x)}\,dx$$

```
[In] int(F^(c*(a + b*x))/(f + f*sin(d + e*x)),x)
[Out] int(F^(c*(a + b*x))/(f + f*sin(d + e*x)), x)
```

3.138 $\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 184

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = -\frac{F^{c(a+bx)}\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)}{6ef^2}-\frac{bcF^{c(a+bx)}\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\log(F)}{6e^2f^2}-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},ie^{i(d+ex)}\right)(e+ibc\log(F))}{3e^2f^2}$$

```
[Out] -1/6*F^(c*(b*x+a))*cot(1/2*d+1/4*Pi+1/2*e*x)*csc(1/2*d+1/4*Pi+1/2*e*x)^2/e/
f^2-1/6*b*c*F^(c*(b*x+a))*csc(1/2*d+1/4*Pi+1/2*e*x)^2*ln(F)/e^2/f^2-2/3*exp
(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-I*b*c*ln(F)/e],[2-I*b*c*ln(F)/e],
I*exp(I*(e*x+d)))*(e+I*b*c*ln(F))/e^2/f^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4541, 4533, 4535}

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = -\frac{2e^{i(d+ex)}F^{c(a+bx)}(e+ibc\log(F))\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},ie^{i(d+ex)}\right)}{3e^2f^2}-\frac{bc\log(F)\csc^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)F^{c(a+bx)}}{6e^2f^2}-\frac{\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\csc^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)F^{c(a+bx)}}{6ef^2}$$

```
[In] Int[F^(c*(a + b*x))/(f + f*Sin[d + e*x])^2,x]
```

```
[Out] -1/6*(F^(c*(a + b*x))*Cot[d/2 + Pi/4 + (e*x)/2]*Csc[d/2 + Pi/4 + (e*x)/2]^2
)/(e*f^2) - (b*c*F^(c*(a + b*x))*Csc[d/2 + Pi/4 + (e*x)/2]^2*Log[F])/(6*e^2
*f^2) - (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*
Log[F])/e, 2 - (I*b*c*Log[F])/e, I*E^(I*(d + e*x))]*(e + I*b*c*Log[F]))/(3*
e^2*f^2)
```

Rule 4533

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4535

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/
(I*e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 +
n/2 - I*b*c*(Log[F]/(2*e)), (-E^(2*I*k*Pi))*E^(2*I*(d + e*x))], x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

Rule 4541

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 - f*(Pi/(4*
g)) + e*(x/2)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f
^2 - g^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int F^{c(a+bx)}\sec^4\left(\frac{d}{2}-\frac{\pi}{4}+\frac{ex}{2}\right)\,dx}{4f^2} \\
&= -\frac{F^{c(a+bx)}\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)}{6ef^2}-\frac{bcF^{c(a+bx)}\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\log(F)}{6e^2f^2} \\
&\quad+\frac{\left(1+\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\sec^2\left(\frac{d}{2}-\frac{\pi}{4}+\frac{ex}{2}\right)\,dx}{6f^2} \\
&= -\frac{F^{c(a+bx)}\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)}{6ef^2}-\frac{bcF^{c(a+bx)}\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\log(F)}{6e^2f^2} \\
&\quad-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},ie^{i(d+ex)}\right)(e+ibc\log(F))}{3e^2f^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.30

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx$$
$$= \frac{F^{c(a+bx)}\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(2e^2\sin\left(\frac{1}{2}(d+ex)\right)-e(e+bc\log(F))\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\right.\right.}{}$$

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Sin[d + e*x])^2,x]
```

```
[Out] (F^(c*(a + b*x))*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*(2*e^2*Sin[(d + e*x)
/2] - e*(e + b*c*Log[F])*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) + 2*(e^2 + b
^2*c^2*Log[F]^2)*Sin[(d + e*x)/2]*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^2 -
 (1 - I)*(1 - (1 - I)*Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*
Log[F])/e, I*Cos[d + e*x] - Sin[d + e*x]])*(e^2 + b^2*c^2*Log[F]^2)*(Cos[(d
 + e*x)/2] + Sin[(d + e*x)/2])^3))/(3*e^3*f^2*(1 + Sin[d + e*x])^2)
```

Maple [F]

$$\int \frac{F^{c(xb+a)}}{(f+f\sin(ex+d))^2}dx$$

```
[In] int(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x)
```

Fricas [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\sin(ex+d)+f)^2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] integral(-F^(b*c*x + a*c)/(f^2*cos(e*x + d)^2 - 2*f^2*sin(e*x + d) - 2*f^2)
, x)
```

Sympy [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \frac{\int \frac{F^{ac+bcx}}{\sin^2(d+ex)+2\sin(d+ex)+1}\,dx}{f^2}$$

```
[In] integrate(F**(c*(b*x+a))/(f+f*sin(e*x+d))**2,x)
```

```
[Out] Integral(F**(a*c + b*c*x)/(sin(d + e*x)**2 + 2*sin(d + e*x) + 1), x)/f**2
```

Maxima [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\sin(ex+d)+f)^2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 4*(6*(F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a
*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 + 80*(F^(a*c)*b^3*c^3*e^2*
log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d)^2 + 6*(F^(a*c)
*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*l
og(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16
*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(e*x + d)^2 - 20*(F^(a*c)*b^4*c^4*e*l
og(F)^4 - 26*F^(a*c)*b^2*c^2*e^3*log(F)^2)*F^(b*c*x)*cos(e*x + d) - 140*(F^
(a*c)*b^3*c^3*e^2*log(F)^3 - 8*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(e*x +
d) - 40*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)
 - ((F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*
c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 4*(F^(a*c)*b^4*c^4*e*log(F)
^4 + 10*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x)*cos(e*x +
d) - 2*(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 144*
F^(a*c)*e^5)*F^(b*c*x)*sin(2*e*x + 2*d) - 20*(F^(a*c)*b^3*c^3*e^2*log(F)^3
+ 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(e*x + d) + 40*(F^(a*c)*b^3*c^3*e
^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(4*e*x + 4*d) - 4*(2*
(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 144*F^(a*c)
*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) + 20*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16*F^
(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d) + (F^(a*c)*b^5*c^5*log(F)^5 +
25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin
(2*e*x + 2*d) + 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 10*F^(a*c)*b^2*c^2*e^3*log(
F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x)*sin(e*x + d) - 4*(F^(a*c)*b^4*c^4*e*log(F)
^4 - 35*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 24*F^(a*c)*e^5)*F^(b*c*x))*cos(3*e*x
 + 3*d) + (8*(4*F^(a*c)*b^4*c^4*e*log(F)^4 + 55*F^(a*c)*b^2*c^2*e^3*log(F)^
2 - 144*F^(a*c)*e^5)*F^(b*c*x)*cos(e*x + d) - 4*(F^(a*c)*b^5*c^5*log(F)^5 +
 55*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 624*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*si
```

n(e*x + d) - (F^(a*c)*b^5*c^5*log(F)^5 - 215*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 1344*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(2*e*x + 2*d) - 4*((F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(4*e*x + 4*d)^2 + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(3*e*x + 3*d)^2 + 36*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(2*e*x + 2*d)^2 + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(e*x + d)^2 + (F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(4*e*x + 4*d)^2 + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(3*e*x + 3*d)^2 + 48*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(e*x + d)*sin(2*e*x + 2*d) + 36*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(2*e*x + 2*d)^2 + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(e*x + d)^2 + 8*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(e*x + d) + (F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2 - 2*(6*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(2*e*x + 2*d) + 4*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(3*e*x + 3*d) - 4*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(e*x + d) - (F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2)*cos(4*e*x + 4*d) - 16*(2*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(e*x + d) + 3*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(2*e*x + 2*d))*cos(3*e*x + 3*d) - 12*(4*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(e*x + d) + (F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2)*cos(2*e*x + 2*d) + 4*(2*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(3*e*x + 3*d) - 2*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log

(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2
)*f^2*cos(e*x + d) - 3*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3
*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(
F)^2)*f^2*sin(2*e*x + 2*d))*sin(4*e*x + 4*d) + 8*(6*(F^(a*c)*b^8*c^8*e*log(
F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 +
576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(2*e*x + 2*d) - 4*(F^(a*c)*b^8*c^
8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*lo
g(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(e*x + d) - (F^(a*c)*b^8*
c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*
log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2)*sin(3*e*x + 3*d))*integra
te(-((b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*cos(5*e*x + 5*d) + 15
*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*cos(4*e*x + 4*d) - 10*(b^3*c^3*lo
g(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*cos(3*e*x + 3*d) - 30*(3*b^2*c^2*e*lo
g(F)^2 - 8*e^3)*F^(b*c*x)*cos(2*e*x + 2*d) + 5*(b^3*c^3*log(F)^3 - 26*b*c*e
^2*log(F))*F^(b*c*x)*cos(e*x + d) + 3*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c
*x)*sin(5*e*x + 5*d) - 5*(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*s
in(4*e*x + 4*d) - 30*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*sin(3*e*x + 3
d) + 10(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)
+ 15*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*sin(e*x + d) + 3*(3*b^2*c^2*e
*log(F)^2 - 8*e^3)*F^(b*c*x))/((b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4
+ 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(5*e*x + 5*d)^2 + 25*(b^6*c^6*
log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^
2*cos(4*e*x + 4*d)^2 + 100*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 24
4*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(3*e*x + 3*d)^2 + 100*(b^6*c^6*log
(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*c
os(2*e*x + 2*d)^2 + 25*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^
2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d)^2 + (b^6*c^6*log(F)^6 + 29*b
^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(5*e*x + 5
d)^2 + 25(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*lo
g(F)^2 + 576*e^6)*f^2*sin(4*e*x + 4*d)^2 + 100*(b^6*c^6*log(F)^6 + 29*b^4*c
^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d)^
2 + 100*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F
)^2 + 576*e^6)*f^2*cos(e*x + d)*sin(2*e*x + 2*d) + 100*(b^6*c^6*log(F)^6 +
29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*x
+ 2*d)^2 + 25*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^
4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d)^2 + 10*(b^6*c^6*log(F)^6 + 29*b^4*c^
4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d) + (b^
6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e
^6)*f^2 - 10*(2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e
^4*log(F)^2 + 576*e^6)*f^2*cos(3*e*x + 3*d) - (b^6*c^6*log(F)^6 + 29*b^4*c^
4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d) + (b^
6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e
^6)*f^2*sin(4*e*x + 4*d) - 2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 +
244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*x + 2*d))*cos(5*e*x + 5*d)
- 10*(10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(

```
F)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d) + 10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^
2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d) - 5*(
b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576
*e^6)*f^2*sin(e*x + d) - (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*
b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2)*cos(4*e*x + 4*d) - 100*((b^6*c^6*log(F
)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos
(e*x + d) + 2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4
*log(F)^2 + 576*e^6)*f^2*sin(2*e*x + 2*d))*cos(3*e*x + 3*d) - 20*(5*(b^6*c^
6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*
f^2*sin(e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^
2*e^4*log(F)^2 + 576*e^6)*f^2)*cos(2*e*x + 2*d) + 2*(5*(b^6*c^6*log(F)^6 +
29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(4*e*x
 + 4*d) - 10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*
log(F)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d) - 10*(b^6*c^6*log(F)^6 + 29*b^4*c^
4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d) +
 5*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 +
 576*e^6)*f^2*sin(e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 +
244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2)*sin(5*e*x + 5*d) + 50*(2*(b^6*c^6*
log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^
2*cos(3*e*x + 3*d) - (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*
c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d) - 2*(b^6*c^6*log(F)^6 + 29*b^4
*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*x + 2*d
))*sin(4*e*x + 4*d) + 20*(10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 +
244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d) - 5*(b^6*c^6*log(F
)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin
(e*x + d) - (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*l
og(F)^2 + 576*e^6)*f^2)*sin(3*e*x + 3*d)), x) - 4*((F^(a*c)*b^7*c^7*e^2*log
(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3
+ 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(4*e*x + 4*d)^2 + 16*(F^(a*c)*b^7*c^7*
e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*lo
g(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(3*e*x + 3*d)^2 + 36*(F^(a*c)*b
^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3
*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(2*e*x + 2*d)^2 + 16*(F^
(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*
b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(e*x + d)^2 + (F^
(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*
b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(4*e*x + 4*d)^2 +
 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F
^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(3*e*x + 3
*d)^2 + 48*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5
+ 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(e*
x + d)*sin(2*e*x + 2*d) + 36*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5
*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*
log(F))*f^2*sin(2*e*x + 2*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a
*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b
```

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*c*e^8*log(F))*f^2*sin(e*x + d)^2 + 8*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^
(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)
*b*c*e^8*log(F))*f^2*sin(e*x + d) + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a
*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b
*c*e^8*log(F))*f^2 - 2*(6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^
5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log
(F))*f^2*cos(2*e*x + 2*d) + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^
5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8
*log(F))*f^2*sin(3*e*x + 3*d) - 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c
)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c
*e^8*log(F))*f^2*sin(e*x + d) - (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*
b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e
^8*log(F))*f^2)*cos(4*e*x + 4*d) - 16*(2*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29
*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a
*c)*b*c*e^8*log(F))*f^2*cos(e*x + d) + 3*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29
*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a
*c)*b*c*e^8*log(F))*f^2*sin(2*e*x + 2*d))*cos(3*e*x + 3*d) - 12*(4*(F^(a*c)
*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c
^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(e*x + d) + (F^(a*c)*b
^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3
*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2)*cos(2*e*x + 2*d) + 4*(2*(F
^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)
*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(3*e*x + 3*d) -
2*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(
a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(e*x + d) -
3*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(
a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(2*e*x + 2*d
))*sin(4*e*x + 4*d) + 8*(6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c
^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*lo
g(F))*f^2*cos(2*e*x + 2*d) - 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b
^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^
8*log(F))*f^2*sin(e*x + d) - (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5
*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*
log(F))*f^2)*sin(3*e*x + 3*d))*integrate((3*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*
F^(b*c*x)*cos(5*e*x + 5*d) - 5*(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*
c*x)*cos(4*e*x + 4*d) - 30*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*cos(3*e
*x + 3*d) + 10*(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x +
 2*d) + 15*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*cos(e*x + d) - (b^3*c^3
*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*sin(5*e*x + 5*d) - 15*(3*b^2*c^2*e
*log(F)^2 - 8*e^3)*F^(b*c*x)*sin(4*e*x + 4*d) + 10*(b^3*c^3*log(F)^3 - 26*b
*c*e^2*log(F))*F^(b*c*x)*sin(3*e*x + 3*d) + 30*(3*b^2*c^2*e*log(F)^2 - 8*e^
3)*F^(b*c*x)*sin(2*e*x + 2*d) - 5*(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^
(b*c*x)*sin(e*x + d) - (b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x))/((
b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576
*e^6)*f^2*cos(5*e*x + 5*d)^2 + 25*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)
```

```
^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(4*e*x + 4*d)^2 + 100*(b^6*
c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6
)*f^2*cos(3*e*x + 3*d)^2 + 100*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4
+ 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d)^2 + 25*(b^6*c^6*
log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^
2*cos(e*x + d)^2 + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^
2*e^4*log(F)^2 + 576*e^6)*f^2*sin(5*e*x + 5*d)^2 + 25*(b^6*c^6*log(F)^6 + 2
9*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(4*e*x
+ 4*d)^2 + 100*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^
4*log(F)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d)^2 + 100*(b^6*c^6*log(F)^6 + 29*b
^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d)*
sin(2*e*x + 2*d) + 100*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^
2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*x + 2*d)^2 + 25*(b^6*c^6*log(F)^6
 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*
x + d)^2 + 10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4
*log(F)^2 + 576*e^6)*f^2*sin(e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*
log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2 - 10*(2*(b^6*c^6*log(F)^
6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(3
*e*x + 3*d) - (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4
*log(F)^2 + 576*e^6)*f^2*cos(e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*
log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(4*e*x + 4*d) - 2*(b^
6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e
^6)*f^2*sin(2*e*x + 2*d))*cos(5*e*x + 5*d) - 10*(10*(b^6*c^6*log(F)^6 + 29*
b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(2*e*x +
2*d) + 10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log
(F)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d) - 5*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^
2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d) - (b^6*c^
6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*
f^2)*cos(4*e*x + 4*d) - 100*((b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 +
244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d) + 2*(b^6*c^6*log(F)^6
+ 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e
*x + 2*d))*cos(3*e*x + 3*d) - 20*(5*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(
F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d) + (b^6*c^6*log(
F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2)*c
os(2*e*x + 2*d) + 2*(5*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^
2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(4*e*x + 4*d) - 10*(b^6*c^6*log(F)^6 +
 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(2*e*
x + 2*d) - 10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4
*log(F)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d) + 5*(b^6*c^6*log(F)^6 + 29*b^4*c^
4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d) + (b^
6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e
^6)*f^2)*sin(5*e*x + 5*d) + 50*(2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)
^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(3*e*x + 3*d) - (b^6*c^6*lo
g(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*
cos(e*x + d) - 2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*
```

e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*x + 2*d))*sin(4*e*x + 4*d) + 20*(10*(b^
6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e
^6)*f^2*cos(2*e*x + 2*d) - 5*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 +
244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d) - (b^6*c^6*log(F)^6 +
29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2)*sin(3*e*
x + 3*d)), x) - (2*(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log
(F)^2 + 144*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) + 20*(F^(a*c)*b^3*c^3*e
^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d) + (F^(a*c)*
b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*lo
g(F))*F^(b*c*x)*sin(2*e*x + 2*d) + 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 10*F^(a*
c)*b^2*c^2*e^3*log(F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x)*sin(e*x + d) - 4*(F^(a*
c)*b^4*c^4*e*log(F)^4 - 35*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 24*F^(a*c)*e^5)*F
^(b*c*x))*sin(4*e*x + 4*d) + 4*((F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*
c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) +
4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 10*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 96*F^(a*
c)*e^5)*F^(b*c*x)*cos(e*x + d) - 2*(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)
*b^2*c^2*e^3*log(F)^2 + 144*F^(a*c)*e^5)*F^(b*c*x)*sin(2*e*x + 2*d) - 20*(F
^(a*c)*b^3*c^3*e^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(e*x
+ d) + 40*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*
x))*sin(3*e*x + 3*d) + 2*(2*(F^(a*c)*b^5*c^5*log(F)^5 + 55*F^(a*c)*b^3*c^3*
e^2*log(F)^3 + 624*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d) + 4*(4*F^
(a*c)*b^4*c^4*e*log(F)^4 + 55*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 144*F^(a*c)*e^
5)*F^(b*c*x)*sin(e*x + d) - (11*F^(a*c)*b^4*c^4*e*log(F)^4 - 445*F^(a*c)*b^
2*c^2*e^3*log(F)^2 + 144*F^(a*c)*e^5)*F^(b*c*x))*sin(2*e*x + 2*d))/((b^6*c^
6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*
f^2*cos(4*e*x + 4*d)^2 + 16*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 2
44*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(3*e*x + 3*d)^2 + 36*(b^6*c^6*log
(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*c
os(2*e*x + 2*d)^2 + 16*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^
2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d)^2 + (b^6*c^6*log(F)^6 + 29*b
^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(4*e*x + 4
d)^2 + 16(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*lo
g(F)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d)^2 + 48*(b^6*c^6*log(F)^6 + 29*b^4*c^
4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d)*sin(2
*e*x + 2*d) + 36*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*
e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*x + 2*d)^2 + 16*(b^6*c^6*log(F)^6 + 29*
b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d)
^2 + 8*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)
^2 + 576*e^6)*f^2*sin(e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^
4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2 - 2*(6*(b^6*c^6*log(F)^6 + 29*b
^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(2*e*x + 2
d) + 4(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F
)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d) - 4*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*
log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d) - (b^6*c^6*
log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^

```
2)*cos(4*e*x + 4*d) - 16*(2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 2
44*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d) + 3*(b^6*c^6*log(F)^6 +
 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*
x + 2*d))*cos(3*e*x + 3*d) - 12*(4*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F
)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d) + (b^6*c^6*log(F
)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2)*co
s(2*e*x + 2*d) + 4*(2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2
*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(3*e*x + 3*d) - 2*(b^6*c^6*log(F)^6 + 2
9*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x +
d) - 3*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)
^2 + 576*e^6)*f^2*sin(2*e*x + 2*d))*sin(4*e*x + 4*d) + 8*(6*(b^6*c^6*log(F)
^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(
2*e*x + 2*d) - 4*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*
e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d) - (b^6*c^6*log(F)^6 + 29*b^4*c^4*e
^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2)*sin(3*e*x + 3*d))
```

Giac [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\sin(ex+d)+f)^2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)/(f*sin(e*x + d) + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \int \frac{F^{c\,(a+b\,x)}}{(f+f\,\sin(d+e\,x))^2}\,dx$$

```
[In] int(F^(c*(a + b*x))/(f + f*sin(d + e*x))^2,x)
[Out] int(F^(c*(a + b*x))/(f + f*sin(d + e*x))^2, x)
```

3.139 $\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 245

$$\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx=\frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2bcf^2F^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}+\frac{bcf^2F^{ac+bcx}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)}+\frac{2ef^2F^{ac+bcx}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{2ef^2F^{ac+bcx}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}$$

```
[Out] f^2*F^(b*c*x+a*c)/b/c/ln(F)+2*b*c*f^2*F^(b*c*x+a*c)*cos(e*x+d)*ln(F)/(e^2+b
^2*c^2*ln(F)^2)+2*e^2*f^2*F^(b*c*x+a*c)/b/c/ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+b
*c*f^2*F^(b*c*x+a*c)*cos(e*x+d)^2*ln(F)/(4*e^2+b^2*c^2*ln(F)^2)+2*e*f^2*F^(
b*c*x+a*c)*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)+2*e*f^2*F^(b*c*x+a*c)*cos(e*x+d
)*sin(e*x+d)/(4*e^2+b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6873, 12, 6874, 2225, 4518, 4520}

$$\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx=\frac{2ef^2\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bcf^2\log(F)\cos^2(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2bcf^2\log(F)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{2ef^2\sin(d+ex)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}+\frac{f^2F^{ac+bcx}}{bc\log(F)}$$

```
[In] Int[F^(c*(a + b*x))*(f + f*Cos[d + e*x])^2,x]
```

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) + (2*b*c*f^2*F^(a*c + b*c*x)*Cos[d + e*x
]*Log[F])/(e^2 + b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[F
]*(4*e^2 + b^2*c^2*Log[F]^2)) + (b*c*f^2*F^(a*c + b*c*x)*Cos[d + e*x]^2*Log
[F])/(4*e^2 + b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Sin[d + e*x])/(e
^2 + b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Cos[d + e*x]*Sin[d + e*x]
)/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4520

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]^m/(e^2*m^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[e*m*F^(c*(a + b*x))*Sin[d +
 e*x]*(Cos[d + e*x]^(m - 1)/(e^2*m^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int f^2 F^{ac+bcx}(1+\cos(d+ex))^2\,dx\\
&= f^2\int F^{ac+bcx}(1+\cos(d+ex))^2\,dx\\
&= f^2\int \left(F^{ac+bcx}+2F^{ac+bcx}\cos(d+ex)+F^{ac+bcx}\cos^2(d+ex)\right)\,dx\\
&= f^2\int F^{ac+bcx}\,dx+f^2\int F^{ac+bcx}\cos^2(d+ex)\,dx+\left(2f^2\right)\int F^{ac+bcx}\cos(d+ex)\,dx\\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2bcf^2F^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{bcf^2F^{ac+bcx}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)}\\
&\quad+\frac{2ef^2F^{ac+bcx}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{2ef^2F^{ac+bcx}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}+\frac{(2e^2f^2)\int F^{ac+bcx}\,dx}{4e^2+b^2c^2\log^2(F)}\\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2bcf^2F^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}\\
&\quad+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}+\frac{bcf^2F^{ac+bcx}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)}\\
&\quad+\frac{2ef^2F^{ac+bcx}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{2ef^2F^{ac+bcx}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.93

$$\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx$$
$$= \frac{f^2F^{c(a+bx)}\left(12e^4+15b^2c^2e^2\log^2(F)+3b^4c^4\log^4(F)+b^2c^2\cos(2(d+ex))\log^2(F)\left(e^2+b^2c^2\log^2(F)\right)+4b^2 \cdots\right.}{\cdots}$$

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Cos[d + e*x])^2,x]
```

```
[Out] (f^2*F^(c*(a + b*x))*(12*e^4 + 15*b^2*c^2*e^2*Log[F]^2 + 3*b^4*c^4*Log[F]^4
 + b^2*c^2*Cos[2*(d + e*x)]*Log[F]^2*(e^2 + b^2*c^2*Log[F]^2) + 4*b^2*c^2*C
os[d + e*x]*Log[F]^2*(4*e^2 + b^2*c^2*Log[F]^2) + 16*b*c*e^3*Log[F]*Sin[d +
 e*x] + 4*b^3*c^3*e*Log[F]^3*Sin[d + e*x] + 2*b*c*e^3*Log[F]*Sin[2*(d + e*x
)] + 2*b^3*c^3*e*Log[F]^3*Sin[2*(d + e*x)]))/(2*(4*b*c*e^4*Log[F] + 5*b^3*c
^3*e^2*Log[F]^3 + b^5*c^5*Log[F]^5))
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.79

method	result
risch	$\frac{3f^2F^{c(xb+a)}}{2bc\ln(F)}+\frac{2\ln(F)cb\,f^2F^{c(xb+a)}\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{2F^{c(xb+a)}e\,f^2\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{\ln(F)cb\,f^2F^{c(xb+a)}\cos(2ex+2d)}{2b^2c^2\ln(F)^2+8e^2}+\frac{e\,f^2F \cdots}{4 \cdots}$
parallelrisch	$\frac{2F^{c(xb+a)}f^2\left(\frac{b^2c^2\ln(F)^2\left(e^2+b^2c^2\ln(F)^2\right)\cos(2ex+2d)}{4}+\frac{cbe\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\sin(2ex+2d)}{2}+\left(4e^2+b^2c^2\ln(F)^2\right)\left(\cos(ex+d)b^2 \cdots\right)\right.}{bc\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)\left(4e^2+b^2c^2\ln(F)^2\right)}$
norman	$\frac{\frac{12e^3f^2e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2+4e^4}+\frac{4\left(2b^2c^2\ln(F)^2+5e^2\right)e\,f^2e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2+4e^4}+\frac{2f^2\left(2b^4c^4\ln(F)^4+8b^2c^2e^2\ln(F)^2+3e^4\right)e^{c(x \cdots}}{bc\ln(F)\left(b^4c^4\ln(F)^4+5b^2c^2e^2\ln(F)^2+ \cdots\right.}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}$
default	$F^{ac}f^2\left(\frac{2F^{bcx}}{bc\ln(F)}+\frac{\frac{8e\,e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}+\frac{4bc\ln(F)e^{bcx\ln(F)}}{e^2+b^2c^2\ln(F)^2}-\frac{4bc\ln(F)e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}+\frac{\frac{8e\,e^{bcx\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4e^2+b^2c^2\ln(F)^2}-\frac{8e\,e^{bc \cdots}}{\cdots}}{\cdots}\right.$
parts	$\frac{f^2F^{c(xb+a)}}{bc\ln(F)}+\frac{\frac{\left(b^2c^2\ln(F)^2+2e^2\right)f^2e^{c(xb+a)\ln(F)}}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{\left(b^2c^2\ln(F)^2+2e^2\right)f^2e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{bc\ln(F)\left(4e^2+b^2c^2\ln(F)^2\right)}+\frac{4e\,f^2e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2} \cdots\right.}{4e^2+b^2c^2\ln(F)^2}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}$

```
[In] int(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2/b/c/ln(F)*f^2*F^(c*(b*x+a))+2*ln(F)*c*b*f^2*F^(c*(b*x+a))/(e^2+b^2*c^2*
ln(F)^2)*cos(e*x+d)+2*F^(c*(b*x+a))*e*f^2/(e^2+b^2*c^2*ln(F)^2)*sin(e*x+d)+
```

```
1/2/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*c*b*f^2*F^(c*(b*x+a))*cos(2*e*x+2*d)+e*f^
2*F^(c*(b*x+a))/(4*e^2+b^2*c^2*ln(F)^2)*sin(2*e*x+2*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.99

$$\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx = \frac{\left(6\,e^4f^2 + \left(b^4c^4f^2\cos\left(ex+d\right)^2 + 2\,b^4c^4f^2\cos\left(ex+d\right) + b^4c^4f^2\right)\log\left(F\right)^4 + \left(b^2c^2e^2f^2\cos\left(ex+d\right)^2 + 8\,b^2\right.}{b}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] (6*e^4*f^2 + (b^4*c^4*f^2*cos(e*x + d)^2 + 2*b^4*c^4*f^2*cos(e*x + d) + b^4
*c^4*f^2)*log(F)^4 + (b^2*c^2*e^2*f^2*cos(e*x + d)^2 + 8*b^2*c^2*e^2*f^2*co
s(e*x + d) + 7*b^2*c^2*e^2*f^2)*log(F)^2 + 2*((b^3*c^3*e*f^2*cos(e*x + d) +
 b^3*c^3*e*f^2)*log(F)^3 + (b*c*e^3*f^2*cos(e*x + d) + 4*b*c*e^3*f^2)*log(F
))*sin(e*x + d))*F^(b*c*x + a*c)/(b^5*c^5*log(F)^5 + 5*b^3*c^3*e^2*log(F)^3
 + 4*b*c*e^4*log(F))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 2394, normalized size of antiderivative = 9.77

$$\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx = \text{Too large to display}$$

```
[In] integrate(F**(c*(b*x+a))*(f+f*cos(e*x+d))**2,x)
```

```
[Out] Piecewise((x*(f*cos(d) + f)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)),
 (f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x + f**2*sin(d
 + e*x)*cos(d + e*x)/(2*e) + 2*f**2*sin(d + e*x)/e, Eq(F, 1)), (F**(a*c)*(f
**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x + f**2*sin(d +
e*x)*cos(d + e*x)/(2*e) + 2*f**2*sin(d + e*x)/e), Eq(b, 0)), (f**2*x*sin(d
+ e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x + f**2*sin(d + e*x)*cos(d +
 e*x)/(2*e) + 2*f**2*sin(d + e*x)/e, Eq(c, 0)), (I*F**(a*c + b*c*x)*f**2*x*
sin(I*b*c*x*log(F) - d) + F**(a*c + b*c*x)*f**2*x*cos(I*b*c*x*log(F) - d) +
 2*F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) - d)**2/(3*b*c*log(F)) - 2*I*F*
*(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) - d)*cos(I*b*c*x*log(F) - d)/(3*b*c*
log(F)) - I*F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) - d)/(b*c*log(F)) + F*
*(a*c + b*c*x)*f**2*cos(I*b*c*x*log(F) - d)**2/(3*b*c*log(F)) + F**(a*c + b
```

```
*c*x)*f**2/(b*c*log(F)), Eq(e, -I*b*c*log(F))), (-F**(a*c + b*c*x)*f**2*x*s
in(I*b*c*x*log(F)/2 - d)**2/4 + I*F**(a*c + b*c*x)*f**2*x*sin(I*b*c*x*log(F
)/2 - d)*cos(I*b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*f**2*x*cos(I*b*c*x*
log(F)/2 - d)**2/4 + I*F**(a*c + b*c*x)*f**2*sin(I*b*c*x*log(F)/2 - d)*cos(
I*b*c*x*log(F)/2 - d)/(2*b*c*log(F)) + 4*I*F**(a*c + b*c*x)*f**2*sin(I*b*c*
x*log(F)/2 - d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2*cos(I*b*c*x*log(F)/2
 - d)**2/(b*c*log(F)) + 8*F**(a*c + b*c*x)*f**2*cos(I*b*c*x*log(F)/2 - d)/(
3*b*c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*log(F)), Eq(e, -I*b*c*log(F)/2))
, (-F**(a*c + b*c*x)*f**2*x*sin(I*b*c*x*log(F)/2 + d)**2/4 + I*F**(a*c + b*
c*x)*f**2*x*sin(I*b*c*x*log(F)/2 + d)*cos(I*b*c*x*log(F)/2 + d)/2 + F**(a*c
 + b*c*x)*f**2*x*cos(I*b*c*x*log(F)/2 + d)**2/4 + F**(a*c + b*c*x)*f**2*sin
(I*b*c*x*log(F)/2 + d)**2/(b*c*log(F)) - 3*I*F**(a*c + b*c*x)*f**2*sin(I*b*
c*x*log(F)/2 + d)*cos(I*b*c*x*log(F)/2 + d)/(2*b*c*log(F)) + 4*I*F**(a*c +
b*c*x)*f**2*sin(I*b*c*x*log(F)/2 + d)/(3*b*c*log(F)) + 8*F**(a*c + b*c*x)*f
**2*cos(I*b*c*x*log(F)/2 + d)/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*l
og(F)), Eq(e, I*b*c*log(F)/2)), (I*F**(a*c + b*c*x)*f**2*x*sin(I*b*c*x*log(
F) + d) + F**(a*c + b*c*x)*f**2*x*cos(I*b*c*x*log(F) + d) + 2*F**(a*c + b*c
*x)*f**2*sin(I*b*c*x*log(F) + d)**2/(3*b*c*log(F)) - 2*I*F**(a*c + b*c*x)*f
**2*sin(I*b*c*x*log(F) + d)*cos(I*b*c*x*log(F) + d)/(3*b*c*log(F)) - I*F**(
a*c + b*c*x)*f**2*sin(I*b*c*x*log(F) + d)/(b*c*log(F)) + F**(a*c + b*c*x)*f
**2*cos(I*b*c*x*log(F) + d)**2/(3*b*c*log(F)) + F**(a*c + b*c*x)*f**2/(b*c*
log(F)), Eq(e, I*b*c*log(F))), (F**(a*c + b*c*x)*b**4*c**4*f**2*log(F)**4*c
os(d + e*x)**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e*
*4*log(F)) + 2*F**(a*c + b*c*x)*b**4*c**4*f**2*log(F)**4*cos(d + e*x)/(b**5
*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + F**(a*c
 + b*c*x)*b**4*c**4*f**2*log(F)**4/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*
log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c + b*c*x)*b**3*c**3*e*f**2*log(F)
**3*sin(d + e*x)*cos(d + e*x)/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F
)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c + b*c*x)*b**3*c**3*e*f**2*log(F)**3*s
in(d + e*x)/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*
log(F)) + 2*F**(a*c + b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*sin(d + e*x)**2/
(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 3*
F**(a*c + b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*cos(d + e*x)**2/(b**5*c**5*l
og(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 8*F**(a*c + b*
c*x)*b**2*c**2*e**2*f**2*log(F)**2*cos(d + e*x)/(b**5*c**5*log(F)**5 + 5*b*
*3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 5*F**(a*c + b*c*x)*b**2*c**2*
e**2*f**2*log(F)**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b
*c*e**4*log(F)) + 2*F**(a*c + b*c*x)*b*c*e**3*f**2*log(F)*sin(d + e*x)*cos(
d + e*x)/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log
(F)) + 8*F**(a*c + b*c*x)*b*c*e**3*f**2*log(F)*sin(d + e*x)/(b**5*c**5*log(
F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c + b*c*x
)*e**4*f**2*sin(d + e*x)**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)*
*3 + 4*b*c*e**4*log(F)) + 2*F**(a*c + b*c*x)*e**4*f**2*cos(d + e*x)**2/(b**
5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 4*F**(
a*c + b*c*x)*e**4*f**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 +
```

```
4*b*c*e**4*log(F)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(245) = 490$.

Time = 0.24 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.36

$$\int F^{c(a+bx)}(f + f\cos(d + ex))^2\,dx$$
$$= \frac{\left(\left(F^{ac}b^2c^2\cos(2\,d)\log(F)^2 + 2\,F^{ac}bce\log(F)\sin(2\,d)\right)F^{bcx}\cos(2\,ex) + \left(F^{ac}b^2c^2\cos(2\,d)\log(F)^2 - 2\,F^{a}\right.}{}$$
$$+ \frac{\left((F^{ac}bc\cos(d)\log(F) - F^{ac}e\sin(d))F^{bcx}\cos(ex + 2\,d) + (F^{ac}bc\cos(d)\log(F) + F^{ac}e\sin(d))F^{bcx}\cos\right.}{b^2c^2\cos(d)^2\log(F)^2 + b^2c^2}$$
$$+ \frac{F^{bcx+ac}f^2}{bc\log(F)}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))*
F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c*e
*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2*si
n(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a*c)*b
^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2
*e*x + 4*d) + 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c^2*log(
F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*F^(b*c*x
))*f^2/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 + 4*(b*c*
cos(2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2) + ((F^(a*c)*b*c*cos(d)*log(
F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x + 2*d) + (F^(a*c)*b*c*cos(d)*log(F
) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x) + (F^(a*c)*b*c*log(F)*sin(d) + F^(
a*c)*e*cos(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a
*c)*e*cos(d))*F^(b*c*x)*sin(e*x))*f^2/(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*
log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2) + F^(b*c*x + a*c)*f^2/(b*c*l
og(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 1736, normalized size of antiderivative = 7.09

$$\int F^{c(a+bx)}(f + f\cos(d + ex))^2\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) -
 1/2*pi*a*c + 2*e*x + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*s
gn(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*f^2*sin(1/2*pi*b*
c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*e*x + 2*d)/(
4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(a
bs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b
*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log
(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c +
2*e)*f^2*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*p
i*a*c + e*x + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^
2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f^2*cos(1/2*pi*b*c*x
*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)*log(abs(
F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*
sgn(F) - pi*b*c - 2*e)*f^2*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*
a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/2*(2*b*c*
f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c
 - 2*e*x - 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 4*e)*f^2*sin(1/2*pi*b*c*x*sgn(F)
- 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*e*x - 2*d)/(4*b^2*c^2*l
og(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2))*e^(b*c*x*log(abs(F)) + a*
c*log(abs(F))) + 3*(2*b*c*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2
*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c
*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F
) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2
 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I
*(I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1
/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(
abs(F)) + 16*I*e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*
I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*e*x - 2*I*d)/(-4*I*pi*b*c*sgn(F) + 4*I
*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F
))) + I*(I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn
(F) - 1/2*I*pi*a*c + I*e*x + I*d)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(a
bs(F)) + 2*I*e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*
pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-I*pi*b*c*sgn(F) + I*pi*b*c +
2*b*c*log(abs(F)) - 2*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*
f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I
*pi*a*c - I*e*x - I*d)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) - 2*
I*e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(
F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(a
bs(F)) + 2*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f^2*e^(1/2*
I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - 2
*I*e*x - 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*
e) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F)
 + 1/2*I*pi*a*c + 2*I*e*x + 2*I*d)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c
```

```
*log(abs(F)) + 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f^2*
e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*
a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*f^2*e^(-1/2*I
*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2
*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) +
a*c*log(abs(F))) + I*(I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2
*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(ab
s(F))) - I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(
b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 27.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx$$
$$=\frac{F^{a\,c+b\,c\,x}\,f^2\,\left(6\,e^4+\frac{3\,b^4\,c^4\,\ln(F)^4}{2}+2\,b^4\,c^4\,\cos\left(d+e\,x\right)\,\ln\left(F\right)^4+\frac{b^4\,c^4\,\ln(F)^4\,\cos(2\,d+2\,e\,x)}{2}+\frac{15\,b^2\,c^2\,e^2\,\ln(F)^2}{2}+8\,b\,c\right.}{}$$

```
[In] int(F^(c*(a + b*x))*(f + f*cos(d + e*x))^2,x)
```

```
[Out] (F^(a*c + b*c*x)*f^2*(6*e^4 + (3*b^4*c^4*log(F)^4)/2 + 2*b^4*c^4*cos(d + e*
x)*log(F)^4 + (b^4*c^4*log(F)^4*cos(2*d + 2*e*x))/2 + (15*b^2*c^2*e^2*log(F
)^2)/2 + 8*b*c*e^3*sin(d + e*x)*log(F) + 8*b^2*c^2*e^2*cos(d + e*x)*log(F)^
2 + b^3*c^3*e*log(F)^3*sin(2*d + 2*e*x) + b*c*e^3*log(F)*sin(2*d + 2*e*x) +
 (b^2*c^2*e^2*log(F)^2*cos(2*d + 2*e*x))/2 + 2*b^3*c^3*e*sin(d + e*x)*log(F
)^3))/(b*c*log(F)*(4*e^4 + b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2))
```

3.140 $\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx=\frac{fF^{ac+bcx}}{bc\log(F)}+\frac{bcfF^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{efF^{ac+bcx}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

```
[Out] f*F^(b*c*x+a*c)/b/c/ln(F)+b*c*f*F^(b*c*x+a*c)*cos(e*x+d)*ln(F)/(e^2+b^2*c^2
*ln(F)^2)+e*f*F^(b*c*x+a*c)*sin(e*x+d)/(e^2+b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6873, 12, 6874, 2225, 4518}

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx=\frac{ef\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bcf\log(F)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

```
[In] Int[F^(c*(a + b*x))*(f + f*Cos[d + e*x]),x]
```

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) + (b*c*f*F^(a*c + b*c*x)*Cos[d + e*x]*Log[
F])/(e^2 + b^2*c^2*Log[F]^2) + (e*f*F^(a*c + b*c*x)*Sin[d + e*x])/(e^2 + b^
2*c^2*Log[F]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int fF^{ac+bcx}(1+\cos(d+ex))\,dx \\
&= f\int F^{ac+bcx}(1+\cos(d+ex))\,dx \\
&= f\int \left(F^{ac+bcx}+F^{ac+bcx}\cos(d+ex)\right)\,dx \\
&= f\int F^{ac+bcx}\,dx + f\int F^{ac+bcx}\cos(d+ex)\,dx \\
&= \frac{fF^{ac+bcx}}{bc\log(F)} + \frac{bcfF^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)} + \frac{efF^{ac+bcx}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx = \frac{fF^{c(a+bx)}\left(e^2+b^2c^2\log^2(F)+b^2c^2\cos(d+ex)\log^2(F)+bce\log(F)\sin(d+ex)\right)}{bc\log(F)\left(e^2+b^2c^2\log^2(F)\right)}$$

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Cos[d + e*x]),x]
[Out] (f*F^(c*(a + b*x))*(e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cos[d + e*x]*Log[F]^2
+ b*c*e*Log[F]*Sin[d + e*x]))/(b*c*Log[F]*(e^2 + b^2*c^2*Log[F]^2))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{f F^{c(xb+a)}\left(\cos(ex+d)b^2c^2\ln(F)^2+b^2c^2\ln(F)^2+e\sin(ex+d)\ln(F)bc+e^2\right)}{bc\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)}$	83
risch	$\frac{f F^{c(xb+a)}}{bc\ln(F)}+\frac{\ln(F)cbf F^{c(xb+a)}\cos(ex+d)}{e^2+b^2c^2\ln(F)^2}+\frac{ef F^{c(xb+a)}\sin(ex+d)}{e^2+b^2c^2\ln(F)^2}$	96
parts	$\frac{f F^{c(xb+a)}}{bc\ln(F)}+\frac{\frac{fbc\ln(F)e^{c(xb+a)\ln(F)}}{e^2+b^2c^2\ln(F)^2}+\frac{2ef\,e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}-\frac{fbc\ln(F)e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$	158
norman	$\frac{\frac{\left(2b^2c^2\ln(F)^2+e^2\right)f\,e^{c(xb+a)\ln(F)}}{bc\ln(F)\left(e^2+b^2c^2\ln(F)^2\right)}+\frac{e^2f\,e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{\left(e^2+b^2c^2\ln(F)^2\right)bc\ln(F)}+\frac{2ef\,e^{c(xb+a)\ln(F)}\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e^2+b^2c^2\ln(F)^2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$	166

```
[In] int(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x,method=_RETURNVERBOSE)
[Out] f*F^(c*(b*x+a))*(cos(e*x+d)*b^2*c^2*ln(F)^2+b^2*c^2*ln(F)^2+e*sin(e*x+d)*ln
(F)*b*c+e^2)/b/c/ln(F)/(e^2+b^2*c^2*ln(F)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx = \frac{\left(bcef\log(F)\sin(ex+d)+e^2f+\left(b^2c^2f\cos(ex+d)+b^2c^2f\right)\log(F)^2\right)F^{bcx+ac}}{b^3c^3\log(F)^3+bce^2\log(F)}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x, algorithm="fricas")
[Out] (b*c*e*f*log(F)*sin(e*x + d) + e^2*f + (b^2*c^2*f*cos(e*x + d) + b^2*c^2*f)
*log(F)^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + b*c*e^2*log(F))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 478, normalized size of antiderivative = 4.88

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx = \begin{cases} x(f\cos(d)+f) & \text{for } \\ fx+\frac{f\sin(d+ex)}{e} & \text{for } \\ F^{ac}\left(fx+\frac{f\sin(d+ex)}{e}\right) & \text{for } \\ fx+\frac{f\sin(d+ex)}{e} & \text{for } \\ \frac{iF^{ac+bcx}fx\sin(ibcx\log(F)-d)}{2}+\frac{F^{ac+bcx}fx\cos(ibcx\log(F)-d)}{2}-\frac{iF^{ac+bcx}f\sin(ibcx\log(F)-d)}{2bc\log(F)}+\frac{F^{ac+bcx}f}{bc\log(F)} & \text{for } \\ \frac{iF^{ac+bcx}fx\sin(ibcx\log(F)+d)}{2}+\frac{F^{ac+bcx}fx\cos(ibcx\log(F)+d)}{2}-\frac{iF^{ac+bcx}f\sin(ibcx\log(F)+d)}{2bc\log(F)}+\frac{F^{ac+bcx}f}{bc\log(F)} & \text{for } \\ \frac{F^{ac+bcx}b^2c^2f\log(F)^2\cos(d+ex)}{b^3c^3\log(F)^3+bce^2\log(F)}+\frac{F^{ac+bcx}b^2c^2f\log(F)^2}{b^3c^3\log(F)^3+bce^2\log(F)}+\frac{F^{ac+bcx}bcef\log(F)\sin(d+ex)}{b^3c^3\log(F)^3+bce^2\log(F)}+\frac{F^{ac+bcx}e^2f}{b^3c^3\log(F)^3+bce^2\log(F)} & \text{othe} \end{cases}$$

```
[In] integrate(F**(c*(b*x+a))*(f+f*cos(e*x+d)),x)
[Out] Piecewise((x*(f*cos(d) + f), Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (f
*x + f*sin(d + e*x)/e, Eq(F, 1)), (F**(a*c)*(f*x + f*sin(d + e*x)/e), Eq(b,
 0)), (f*x + f*sin(d + e*x)/e, Eq(c, 0)), (I*F**(a*c + b*c*x)*f*x*sin(I*b*c
*x*log(F) - d)/2 + F**(a*c + b*c*x)*f*x*cos(I*b*c*x*log(F) - d)/2 - I*F**(a
*c + b*c*x)*f*sin(I*b*c*x*log(F) - d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*f/(
b*c*log(F)), Eq(e, -I*b*c*log(F))), (I*F**(a*c + b*c*x)*f*x*sin(I*b*c*x*log
(F) + d)/2 + F**(a*c + b*c*x)*f*x*cos(I*b*c*x*log(F) + d)/2 - I*F**(a*c + b
*c*x)*f*sin(I*b*c*x*log(F) + d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*f/(b*c*lo
```

```
g(F)), Eq(e, I*b*c*log(F))), (F**(a*c + b*c*x)*b**2*c**2*f*log(F)**2*cos(d
+ e*x)/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c + b*c*x)*b**2*c**2
*f*log(F)**2/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c + b*c*x)*b*c
*e*f*log(F)*sin(d + e*x)/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c
+ b*c*x)*e**2*f/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(98) = 196.

Time = 0.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.20

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx = \frac{((F^{ac}bc\cos{(d)}\log{(F)}-F^{ac}e\sin{(d)})F^{bcx}\cos{(ex+2\,d)}+(F^{ac}bc\cos{(d)}\log{(F)}+F^{ac}e\sin{(d)})F^{bcx}\cos{(ex}}{2\left(b^2c^2\cos{(d)}^2\log{(F)}^2+b^2c^2\log} + \frac{F^{bcx+ac}f}{bc\log{(F)}}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x + 2*d
) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x) + (F^
(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^(
a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x))*f/(b^2*c^2*c
os(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2) +
 F^(b*c*x + a*c)*f/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 920, normalized size of antiderivative = 9.39

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x, algorithm="giac")
```

```
[Out] (2*b*c*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*p
i*a*c + e*x + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi
*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*f*sin(1/2*pi*b*c*x*sgn(F) -
 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + e*x + d)/(4*b^2*c^2*log(ab
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log
(abs(F))) + (2*b*c*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sg
n(F) - 1/2*pi*a*c - e*x - d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c
```

```
*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 2*e)*f*sin(1/2*pi*b*
c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - e*x - d)/(4*b^
2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F
)) + a*c*log(abs(F))) + 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (p
i*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sg
n(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F)
)^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
+ I*(I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) -
1/2*I*pi*a*c + I*e*x + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs
(F)) + 4*I*e) - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a
*c*sgn(F) + 1/2*I*pi*a*c - I*e*x - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c +
4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*
f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*p
i*a*c - I*e*x - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) -
4*I*e) - I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(
F) + 1/2*I*pi*a*c + I*e*x + I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*l
og(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*f*e^(1/
2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/
(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) - I*f*e^(-1/2*I*pi*b*c*x*s
gn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn
(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))
)
```

Mupad [B] (verification not implemented)

Time = 26.70 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx = \frac{F^{a\,c+b\,c\,x}\,f\,\left(e^2+b^2\,c^2\ln\left(F\right)^2+b^2\,c^2\,\cos\left(d+e\,x\right)\,\ln\left(F\right)^2+b\,c\,e\,\sin\left(d+e\,x\right)\,\ln\left(F\right)\right)}{b\,c\,\ln\left(F\right)\,\left(b^2\,c^2\ln\left(F\right)^2+e^2\right)}$$

```
[In] int(F^(c*(a + b*x))*(f + f*cos(d + e*x)),x)
```

```
[Out] (F^(a*c + b*c*x)*f*(e^2 + b^2*c^2*log(F)^2 + b^2*c^2*cos(d + e*x)*log(F)^2
+ b*c*e*sin(d + e*x)*log(F)))/(b*c*log(F)*(e^2 + b^2*c^2*log(F)^2))
```

3.141 $\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = \frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)}{f(ie+bc\log(F))}$$

```
[Out] 2*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-I*b*c*ln(F)/e],[2-I*b*c*ln(F
)/e],-exp(I*(e*x+d)))/f/(b*c*ln(F)+I*e)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4542, 4536}

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = \frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)}{f(bc\log(F)+ie)}$$

```
[In] Int[F^(c*(a + b*x))/(f + f*Cos[d + e*x]),x]
```

```
[Out] (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])/
e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))])/(f*(I*e + b*c*Log[F]))
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 4542

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + e*(x/2)]^
(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && ILt
Q[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int F^{c(a+bx)} \sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)\, dx}{2f}$$
$$= \frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2, 1-\frac{ibc\log(F)}{e}, 2-\frac{ibc\log(F)}{e}, -e^{i(d+ex)}\right)}{f(ie+bc\log(F))}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\, dx = -\frac{2ie^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2, 1-\frac{ibc\log(F)}{e}, 2-\frac{ibc\log(F)}{e}, -e^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Cos[d + e*x]),x]
[Out] ((-2*I)*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log
[F])/e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))])/(f*(e - I*b*c*Log[F]))
```

Maple [F]

$$\int \frac{F^{c(xb+a)}}{f+f\cos(ex+d)}dx$$

```
[In] int(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x)
[Out] int(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x)
```

Fricas [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\cos(ex+d)+f}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)/(f*cos(e*x + d) + f), x)
```

Sympy [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = \frac{\int \frac{F^{ac+bcx}}{\cos(d+ex)+1}\,dx}{f}$$

```
[In] integrate(F**(c*(b*x+a))/(f+f*cos(e*x+d)),x)
[Out] Integral(F**(a*c + b*c*x)/(cos(d + e*x) + 1), x)/f
```

Maxima [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\cos(ex+d)+f}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x, algorithm="maxima")
[Out] 2*(6*F^(b*c*x)*F^(a*c)*b*c*e^2*log(F) + 2*(F^(a*c)*b^3*c^3*log(F)^3 + 4*F^(
a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(e*x + d)^2 + 2*(F^(a*c)*b^3*c^3*log(F)^3
 + 4*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*sin(e*x + d)^2 + (F^(a*c)*b^3*c^3*lo
g(F)^3 + 16*F^(a*c)*b*c*e^2*log(F))*F^(b*c*x)*cos(e*x + d) - (5*F^(a*c)*b^2
*c^2*e*log(F)^2 - 4*F^(a*c)*e^3)*F^(b*c*x)*sin(e*x + d) + (6*F^(b*c*x)*F^(a
*c)*b*c*e^2*log(F) + (F^(a*c)*b^3*c^3*log(F)^3 + 4*F^(a*c)*b*c*e^2*log(F))*
F^(b*c*x)*cos(e*x + d) - (F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*e^3)*F^(b*
c*x)*sin(e*x + d))*cos(2*e*x + 2*d) - 2*((F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^
(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*cos(2*e*x + 2*d)^2
 + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*
c)*b*c*e^5*log(F))*f*cos(e*x + d)^2 + (F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*
c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*sin(2*e*x + 2*d)^2 +
4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*
b*c*e^5*log(F))*f*sin(2*e*x + 2*d)*sin(e*x + d) + 4*(F^(a*c)*b^5*c^5*e*log(
F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f*sin(e*x
 + d)^2 + 4*(F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 +
```

```
4*F^(a*c)*b*c*e^5*log(F))*f*cos(e*x + d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 5*
F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f + 2*(2*(F^(a*c)*
b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*log(F)^3 + 4*F^(a*c)*b*c*e^5*log
(F))*f*cos(e*x + d) + (F^(a*c)*b^5*c^5*e*log(F)^5 + 5*F^(a*c)*b^3*c^3*e^3*l
og(F)^3 + 4*F^(a*c)*b*c*e^5*log(F))*f)*cos(2*e*x + 2*d))*integrate((3*F^(b*
c*x)*b*c*e*cos(3*e*x + 3*d)*log(F) + 9*F^(b*c*x)*b*c*e*cos(2*e*x + 2*d)*log
(F) + 9*F^(b*c*x)*b*c*e*cos(e*x + d)*log(F) + 3*F^(b*c*x)*b*c*e*log(F) - (b
^2*c^2*log(F)^2 - 2*e^2)*F^(b*c*x)*sin(3*e*x + 3*d) - 3*(b^2*c^2*log(F)^2 -
 2*e^2)*F^(b*c*x)*sin(2*e*x + 2*d) - 3*(b^2*c^2*log(F)^2 - 2*e^2)*F^(b*c*x)
*sin(e*x + d))/((b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(3
*e*x + 3*d)^2 + 9*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos
(2*e*x + 2*d)^2 + 9*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*c
os(e*x + d)^2 + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(3
*e*x + 3*d)^2 + 9*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin
(2*e*x + 2*d)^2 + 18*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*
sin(2*e*x + 2*d)*sin(e*x + d) + 9*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^
2 + 4*e^4)*f*sin(e*x + d)^2 + 6*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2
+ 4*e^4)*f*cos(e*x + d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^
4)*f + 2*(3*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(2*e*x
 + 2*d) + 3*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(e*x +
 d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f)*cos(3*e*x + 3*
d) + 6*(3*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(e*x + d
) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f)*cos(2*e*x + 2*d)
 + 6*((b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(2*e*x + 2*d
) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*sin(e*x + d))*sin
(3*e*x + 3*d)), x) + ((F^(a*c)*b^2*c^2*e*log(F)^2 + 4*F^(a*c)*e^3)*F^(b*c*x
)*cos(e*x + d) + (F^(a*c)*b^3*c^3*log(F)^3 + 4*F^(a*c)*b*c*e^2*log(F))*F^(b
*c*x)*sin(e*x + d) - 2*(F^(a*c)*b^2*c^2*e*log(F)^2 - 2*F^(a*c)*e^3)*F^(b*c*
x))*sin(2*e*x + 2*d))/((b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*
f*cos(2*e*x + 2*d)^2 + 4*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4
)*f*cos(e*x + d)^2 + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*
sin(2*e*x + 2*d)^2 + 4*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*
f*sin(2*e*x + 2*d)*sin(e*x + d) + 4*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F
)^2 + 4*e^4)*f*sin(e*x + d)^2 + 4*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^
2 + 4*e^4)*f*cos(e*x + d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*
e^4)*f + 2*(2*(b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f*cos(e*x
 + d) + (b^4*c^4*log(F)^4 + 5*b^2*c^2*e^2*log(F)^2 + 4*e^4)*f)*cos(2*e*x +
2*d))
```

Giac [F]

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = \int \frac{F^{(bx+a)c}}{f\cos(ex+d)+f}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)/(f*cos(e*x + d) + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = \int \frac{F^{c\,(a+b\,x)}}{f+f\,\cos(d+e\,x)}\,dx$$

```
[In] int(F^(c*(a + b*x))/(f + f*cos(d + e*x)),x)
[Out] int(F^(c*(a + b*x))/(f + f*cos(d + e*x)), x)
```

3.142 $\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 169

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)(ie-bc\log(F))}{3e^2f^2} - \frac{bcF^{c(a+bx)}\log(F)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{6ef^2}$$

```
[Out] -2/3*exp(I*(e*x+d))*F^(c*(b*x+a))*hypergeom([2, 1-I*b*c*ln(F)/e],[2-I*b*c*l
n(F)/e],-exp(I*(e*x+d)))*(I*e-b*c*ln(F))/e^2/f^2-1/6*b*c*F^(c*(b*x+a))*ln(F
)*sec(1/2*e*x+1/2*d)^2/e^2/f^2+1/6*F^(c*(b*x+a))*sec(1/2*e*x+1/2*d)^2*tan(1
/2*e*x+1/2*d)/e/f^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4542, 4533, 4536}

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = -\frac{2e^{i(d+ex)}F^{c(a+bx)}(-bc\log(F)+ie)\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)}{3e^2f^2} - \frac{bc\log(F)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{6e^2f^2} + \frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{6ef^2}$$

```
[In] Int[F^(c*(a + b*x))/(f + f*Cos[d + e*x])^2,x]
```

```
[Out] (-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])
/e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))]*(I*e - b*c*Log[F]))/(3*e^2*f^2)
 - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d/2 + (e*x)/2]^2)/(6*e^2*f^2) + (F^(c*(a
 + b*x))*Sec[d/2 + (e*x)/2]^2*Tan[d/2 + (e*x)/2])/(6*e*f^2)
```

Rule 4533

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sec[d + e*x]^(n - 2)/(e^2*(n - 1)
*(n - 2))), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
 2)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sec[d + e*x]^(n - 1)*(Sin[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b,
c, d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && Ne
Q[n, 2]
```

Rule 4536

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[2^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 - I*b*c*(Log[F]/(2*e)), 1 + n/2 - I*b*c*(Log[F]/(2*e
)), -E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rule 4542

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + e*(x/2)]^
(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && ILt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int F^{c(a+bx)} \sec^4\left(\frac{d}{2}+\frac{ex}{2}\right)\,dx}{4f^2} \\
&= -\frac{bcF^{c(a+bx)}\log(F)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{6ef^2} \\
&\quad + \frac{\left(1+\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)\,dx}{6f^2} \\
&= -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)(ie-bc\log(F))}{3e^2f^2} \\
&\quad - \frac{bcF^{c(a+bx)}\log(F)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{6ef^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = \frac{2F^{c(a+bx)}\cos\left(\frac{1}{2}(d+ex)\right)\left(-bc\cos\left(\frac{1}{2}(d+ex)\right)\log(F)+4e^{i(d+ex)}\cos^3\left(\frac{1}{2}(d+ex)\right)\text{Hypergeometric2F1}\left(2\right.\right.}{3e^2f^2(1+\cos(d+ex))^2}$$

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Cos[d + e*x])^2,x]
```

```
[Out] (2*F^(c*(a + b*x))*Cos[(d + e*x)/2]*(-(b*c*Cos[(d + e*x)/2]*Log[F]) + 4*E^(
I*(d + e*x))*Cos[(d + e*x)/2]^3*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])/e,
2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))]*((-I)*e + b*c*Log[F]) + e*Sin[(d +
e*x)/2]))/(3*e^2*f^2*(1 + Cos[d + e*x])^2)
```

Maple [F]

$$\int \frac{F^{c(xb+a)}}{(f+f\cos(ex+d))^2}dx$$

```
[In] int(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x)
```

Fricas [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\cos(ex+d)+f)^2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)/(f^2*cos(e*x + d)^2 + 2*f^2*cos(e*x + d) + f^2), x
)
```

Sympy [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = \frac{\int \frac{F^{ac+bcx}}{\cos^2(d+ex)+2\cos(d+ex)+1}\,dx}{f^2}$$

[In] integrate(F**(c*(b*x+a))/(f+f*cos(e*x+d))**2,x)

[Out] Integral(F**(a*c + b*c*x)/(cos(d + e*x)**2 + 2*cos(d + e*x) + 1), x)/f**2

Maxima [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{(f\cos(ex+d)+f)^2}\,dx$$

[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x, algorithm="maxima")

[Out] 4*(6*(F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d)^2 + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d)^2 + 6*(F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*e*x + 2*d)^2 + 80*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(e*x + d)^2 - 140*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 8*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d) + 20*(F^(a*c)*b^4*c^4*e*log(F)^4 - 26*F^(a*c)*b^2*c^2*e^3*log(F)^2)*F^(b*c*x)*sin(e*x + d) - 40*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x) + ((F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 20*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d) - 2*(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 144*F^(a*c)*e^5)*F^(b*c*x)*sin(2*e*x + 2*d) + 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 10*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x)*sin(e*x + d) - 40*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(4*e*x + 4*d) + 4*((F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 20*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d) - 2*(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 144*F^(a*c)*e^5)*F^(b*c*x)*sin(2*e*x + 2*d) + 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 10*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x)*sin(e*x + d) - 40*(F^(a*c)*b^3*c^3*e^2*log(F)^3 - 5*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x))*cos(3*e*x + 3*d) + (4*(F^(a*c)*b^5*c^5*log(F)^5 + 55*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 624*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*cos(e*x + d) + 8*(4*F^(a*c)*b^4*c^4*e*log(F)^4 + 55*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 144*F^(a*c)*e^5)*F^(b*c*x)*sin(e*x + d) + (F^(a*c)

```
*b^5*c^5*log(F)^5 - 215*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 1344*F^(a*c)*b*c*e^4
*log(F))*F^(b*c*x))*cos(2*e*x + 2*d) - 4*((F^(a*c)*b^7*c^7*e^2*log(F)^7 + 2
9*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(
a*c)*b*c*e^8*log(F))*f^2*cos(4*e*x + 4*d)^2 + 16*(F^(a*c)*b^7*c^7*e^2*log(F
)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 +
576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(3*e*x + 3*d)^2 + 36*(F^(a*c)*b^7*c^7*e^
2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(
F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(2*e*x + 2*d)^2 + 16*(F^(a*c)*b^7
*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e
^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*cos(e*x + d)^2 + (F^(a*c)*b^7
*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e
^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(4*e*x + 4*d)^2 + 16*(F^(a
*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^
3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(3*e*x + 3*d)^2 + 3
6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(
a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(2*e*x + 2*d
)^2 + 48*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 +
244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(2*e*
x + 2*d)*sin(e*x + d) + 16*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c
^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*lo
g(F))*f^2*sin(e*x + d)^2 + 8*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5
*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*
log(F))*f^2*cos(e*x + d) + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c
^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*lo
g(F))*f^2 + 2*(4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log
(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*
cos(3*e*x + 3*d) + 6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4
*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*
f^2*cos(2*e*x + 2*d) + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5
*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(
F))*f^2*cos(e*x + d) + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e
^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F)
)*f^2)*cos(4*e*x + 4*d) + 8*(6*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b
^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^
8*log(F))*f^2*cos(2*e*x + 2*d) + 4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*
c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*
c*e^8*log(F))*f^2*cos(e*x + d) + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)
*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*
e^8*log(F))*f^2)*cos(3*e*x + 3*d) + 12*(4*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 2
9*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(
a*c)*b*c*e^8*log(F))*f^2*cos(e*x + d) + (F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*
F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*
c)*b*c*e^8*log(F))*f^2)*cos(2*e*x + 2*d) + 4*(2*(F^(a*c)*b^7*c^7*e^2*log(F)
^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 5
76*F^(a*c)*b*c*e^8*log(F))*f^2*sin(3*e*x + 3*d) + 3*(F^(a*c)*b^7*c^7*e^2*lo
```

g(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(2*e*x + 2*d) + 2*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(e*x + d))*sin(4*e*x + 4*d) + 16*(3*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(2*e*x + 2*d) + 2*(F^(a*c)*b^7*c^7*e^2*log(F)^7 + 29*F^(a*c)*b^5*c^5*e^4*log(F)^5 + 244*F^(a*c)*b^3*c^3*e^6*log(F)^3 + 576*F^(a*c)*b*c*e^8*log(F))*f^2*sin(e*x + d))*sin(3*e*x + 3*d))*integrate(-((b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*cos(5*e*x + 5*d) + 5*(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*cos(4*e*x + 4*d) + 10*(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*cos(3*e*x + 3*d) + 10*(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*cos(2*e*x + 2*d) + 5*(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*cos(e*x + d) + 3*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*sin(5*e*x + 5*d) + 15*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*sin(4*e*x + 4*d) + 30*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*sin(3*e*x + 3*d) + 30*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*sin(2*e*x + 2*d) + 15*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*sin(e*x + d) + (b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x))/((b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(5*e*x + 5*d)^2 + 25*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(4*e*x + 4*d)^2 + 100*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(3*e*x + 3*d)^2 + 100*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d)^2 + 25*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d)^2 + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(5*e*x + 5*d)^2 + 25*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(4*e*x + 4*d)^2 + 100*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d)^2 + 100*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*x + 2*d)^2 + 100*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*x + 2*d)*sin(e*x + d) + 25*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d)^2 + 10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2 + 2*(5*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(4*e*x + 4*d) + 10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(3*e*x + 3*d) + 10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d) + 5*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2)*cos(5*e*x + 5*d) + 10*(10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*co

s(3*e*x + 3*d) + 10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c
^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d) + 5*(b^6*c^6*log(F)^6 + 29*
b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d)
 + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 +
 576*e^6)*f^2)*cos(4*e*x + 4*d) + 20*(10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2
*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d) + 5*(b
^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*
e^6)*f^2*cos(e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b
^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2)*cos(3*e*x + 3*d) + 20*(5*(b^6*c^6*log(F
)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos
(e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*l
og(F)^2 + 576*e^6)*f^2)*cos(2*e*x + 2*d) + 10*((b^6*c^6*log(F)^6 + 29*b^4*c
^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(4*e*x + 4*d)
+ 2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2
+ 576*e^6)*f^2*sin(3*e*x + 3*d) + 2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(
F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*x + 2*d) + (b^6*c^6*
log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^
2*sin(e*x + d))*sin(5*e*x + 5*d) + 50*(2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2
*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d) + 2*(b
^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*
e^6)*f^2*sin(2*e*x + 2*d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 2
44*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d))*sin(4*e*x + 4*d) + 100
(2(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2
+ 576*e^6)*f^2*sin(2*e*x + 2*d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)
^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d))*sin(3*e*x + 3*d)
), x) + 4*((F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 +
244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*co
s(4*e*x + 4*d)^2 + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*
log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F
)^2)*f^2*cos(3*e*x + 3*d)^2 + 36*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b
^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^
2*e^7*log(F)^2)*f^2*cos(2*e*x + 2*d)^2 + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 2
9*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(
a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(e*x + d)^2 + (F^(a*c)*b^8*c^8*e*log(F)^8
 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576
*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(4*e*x + 4*d)^2 + 16*(F^(a*c)*b^8*c^8
*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log
(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(3*e*x + 3*d)^2 + 36*(F^(a
*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*
c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(2*e*x + 2*d)^2
 + 48*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F
^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(2*e
*x + 2*d)*sin(e*x + d) + 16*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^
6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7
*log(F)^2)*f^2*sin(e*x + d)^2 + 8*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*

```
b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c
^2*e^7*log(F)^2)*f^2*cos(e*x + d) + (F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c
)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2
*c^2*e^7*log(F)^2)*f^2 + 2*(4*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*
c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e
^7*log(F)^2)*f^2*cos(3*e*x + 3*d) + 6*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a
*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b
^2*c^2*e^7*log(F)^2)*f^2*cos(2*e*x + 2*d) + 4*(F^(a*c)*b^8*c^8*e*log(F)^8 +
 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F
^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(e*x + d) + (F^(a*c)*b^8*c^8*e*log(F)^8
 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576
*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2)*cos(4*e*x + 4*d) + 8*(6*(F^(a*c)*b^8*c^
8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*lo
g(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(2*e*x + 2*d) + 4*(F^(a*c
)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^
4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(e*x + d) + (F^(a
*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*
c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2)*cos(3*e*x + 3*d)
+ 12*(4*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244
*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*cos(e
*x + d) + (F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 2
44*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2)*co
s(2*e*x + 2*d) + 4*(2*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*
log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F
)^2)*f^2*sin(3*e*x + 3*d) + 3*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*
c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e
^7*log(F)^2)*f^2*sin(2*e*x + 2*d) + 2*(F^(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a
*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b
^2*c^2*e^7*log(F)^2)*f^2*sin(e*x + d))*sin(4*e*x + 4*d) + 16*(3*(F^(a*c)*b^
8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^4*c^4*e^
5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(2*e*x + 2*d) + 2*(F^
(a*c)*b^8*c^8*e*log(F)^8 + 29*F^(a*c)*b^6*c^6*e^3*log(F)^6 + 244*F^(a*c)*b^
4*c^4*e^5*log(F)^4 + 576*F^(a*c)*b^2*c^2*e^7*log(F)^2)*f^2*sin(e*x + d))*si
n(3*e*x + 3*d))*integrate((3*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*cos(5
*e*x + 5*d) + 15*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*cos(4*e*x + 4*d)
+ 30*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*cos(3*e*x + 3*d) + 30*(3*b^2*
c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x)*cos(2*e*x + 2*d) + 15*(3*b^2*c^2*e*log(F)
^2 - 8*e^3)*F^(b*c*x)*cos(e*x + d) - (b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))
*F^(b*c*x)*sin(5*e*x + 5*d) - 5*(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b
*c*x)*sin(4*e*x + 4*d) - 10*(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x
)*sin(3*e*x + 3*d) - 10*(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*si
n(2*e*x + 2*d) - 5*(b^3*c^3*log(F)^3 - 26*b*c*e^2*log(F))*F^(b*c*x)*sin(e*x
 + d) + 3*(3*b^2*c^2*e*log(F)^2 - 8*e^3)*F^(b*c*x))/((b^6*c^6*log(F)^6 + 29
*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(5*e*x +
 5*d)^2 + 25*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*
```

log(F)^2 + 576*e^6)*f^2*cos(4*e*x + 4*d)^2 + 100*(b^6*c^6*log(F)^6 + 29*b^4
*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(3*e*x + 3*d
)^2 + 100*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log
(F)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d)^2 + 25*(b^6*c^6*log(F)^6 + 29*b^4*c^4
*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d)^2 + (b
^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*
e^6)*f^2*sin(5*e*x + 5*d)^2 + 25*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^
4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(4*e*x + 4*d)^2 + 100*(b^6*c
^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)
*f^2*sin(3*e*x + 3*d)^2 + 100*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 +
 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*x + 2*d)^2 + 100*(b^6*c^6*
log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^
2*sin(2*e*x + 2*d)*sin(e*x + d) + 25*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log
(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d)^2 + 10*(b^6*c^
6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*
f^2*cos(e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^
2*e^4*log(F)^2 + 576*e^6)*f^2 + 2*(5*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log
(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(4*e*x + 4*d) + 10*(b^6*
c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6
)*f^2*cos(3*e*x + 3*d) + 10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 2
44*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d) + 5*(b^6*c^6*log(F)
^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(
e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*lo
g(F)^2 + 576*e^6)*f^2)*cos(5*e*x + 5*d) + 10*(10*(b^6*c^6*log(F)^6 + 29*b^4
*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(3*e*x + 3*d
) + 10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)
^2 + 576*e^6)*f^2*cos(2*e*x + 2*d) + 5*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*l
og(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d) + (b^6*c^6*l
og(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2
)*cos(4*e*x + 4*d) + 20*(10*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 2
44*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d) + 5*(b^6*c^6*log(F)
^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(
e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*lo
g(F)^2 + 576*e^6)*f^2)*cos(3*e*x + 3*d) + 20*(5*(b^6*c^6*log(F)^6 + 29*b^4*
c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d) + (
b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576
*e^6)*f^2)*cos(2*e*x + 2*d) + 10*((b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)
^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(4*e*x + 4*d) + 2*(b^6*c^6*
log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^
2*sin(3*e*x + 3*d) + 2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^
2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*x + 2*d) + (b^6*c^6*log(F)^6 + 29
*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d
))*sin(5*e*x + 5*d) + 50*(2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 2
44*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d) + 2*(b^6*c^6*log(F)
^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(

2*e*x + 2*d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^
4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d))*sin(4*e*x + 4*d) + 100*(2*(b^6*c^6*
log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^
2*sin(2*e*x + 2*d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*
c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d))*sin(3*e*x + 3*d)), x) + (2*(F
^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 144*F^(a*c)*e
^5)*F^(b*c*x)*cos(2*e*x + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log(F)^4 + 10*F^(a*c)
*b^2*c^2*e^3*log(F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x)*cos(e*x + d) + (F^(a*c)*b
^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a*c)*b*c*e^4*log
(F))*F^(b*c*x)*sin(2*e*x + 2*d) + 20*(F^(a*c)*b^3*c^3*e^2*log(F)^3 + 16*F^(
a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(e*x + d) + 4*(F^(a*c)*b^4*c^4*e*log(F)^4
 - 35*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 24*F^(a*c)*e^5)*F^(b*c*x))*sin(4*e*x +
 4*d) + 4*(2*(F^(a*c)*b^4*c^4*e*log(F)^4 + 25*F^(a*c)*b^2*c^2*e^3*log(F)^2
+ 144*F^(a*c)*e^5)*F^(b*c*x)*cos(2*e*x + 2*d) - 4*(F^(a*c)*b^4*c^4*e*log(F)
^4 + 10*F^(a*c)*b^2*c^2*e^3*log(F)^2 - 96*F^(a*c)*e^5)*F^(b*c*x)*cos(e*x +
d) + (F^(a*c)*b^5*c^5*log(F)^5 + 25*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 144*F^(a
*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(2*e*x + 2*d) + 20*(F^(a*c)*b^3*c^3*e^2*lo
g(F)^3 + 16*F^(a*c)*b*c*e^4*log(F))*F^(b*c*x)*sin(e*x + d) + 4*(F^(a*c)*b^4
*c^4*e*log(F)^4 - 35*F^(a*c)*b^2*c^2*e^3*log(F)^2 + 24*F^(a*c)*e^5)*F^(b*c*
x))*sin(3*e*x + 3*d) - 2*(4*(4*F^(a*c)*b^4*c^4*e*log(F)^4 + 55*F^(a*c)*b^2*
c^2*e^3*log(F)^2 - 144*F^(a*c)*e^5)*F^(b*c*x)*cos(e*x + d) - 2*(F^(a*c)*b^5
*c^5*log(F)^5 + 55*F^(a*c)*b^3*c^3*e^2*log(F)^3 + 624*F^(a*c)*b*c*e^4*log(F
))*F^(b*c*x)*sin(e*x + d) - (11*F^(a*c)*b^4*c^4*e*log(F)^4 - 445*F^(a*c)*b^
2*c^2*e^3*log(F)^2 + 144*F^(a*c)*e^5)*F^(b*c*x))*sin(2*e*x + 2*d))/((b^6*c^
6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*
f^2*cos(4*e*x + 4*d)^2 + 16*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 2
44*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(3*e*x + 3*d)^2 + 36*(b^6*c^6*log
(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*c
os(2*e*x + 2*d)^2 + 16*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^
2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d)^2 + (b^6*c^6*log(F)^6 + 29*b
^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(4*e*x + 4
d)^2 + 16(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*lo
g(F)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d)^2 + 36*(b^6*c^6*log(F)^6 + 29*b^4*c^
4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(2*e*x + 2*d)^2
 + 48*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^
2 + 576*e^6)*f^2*sin(2*e*x + 2*d)*sin(e*x + d) + 16*(b^6*c^6*log(F)^6 + 29*
b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d)
^2 + 8*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)
^2 + 576*e^6)*f^2*cos(e*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^
4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2 + 2*(4*(b^6*c^6*log(F)^6 + 29*b
^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(3*e*x + 3
d) + 6(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F
)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d) + 4*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*
log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d) + (b^6*c^6*
log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^

```
2)*cos(4*e*x + 4*d) + 8*(6*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 24
4*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(2*e*x + 2*d) + 4*(b^6*c^6*log(F)^
6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e
*x + d) + (b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log
(F)^2 + 576*e^6)*f^2)*cos(3*e*x + 3*d) + 12*(4*(b^6*c^6*log(F)^6 + 29*b^4*c
^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*cos(e*x + d) + (b
^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*
e^6)*f^2)*cos(2*e*x + 2*d) + 4*(2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)
^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(3*e*x + 3*d) + 3*(b^6*c^6*
log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^
2*sin(2*e*x + 2*d) + 2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^
2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d))*sin(4*e*x + 4*d) + 16*(3*(b
^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 + 244*b^2*c^2*e^4*log(F)^2 + 576*
e^6)*f^2*sin(2*e*x + 2*d) + 2*(b^6*c^6*log(F)^6 + 29*b^4*c^4*e^2*log(F)^4 +
 244*b^2*c^2*e^4*log(F)^2 + 576*e^6)*f^2*sin(e*x + d))*sin(3*e*x + 3*d))
```

Giac [F]

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = \int \frac{F^{(bx+a)c}}{\left(f\cos\left(ex+d\right)+f\right)^2}\,dx$$

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)/(f*cos(e*x + d) + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = \int \frac{F^{c\,(a+b\,x)}}{\left(f+f\,\cos\left(d+e\,x\right)\right)^2}\,dx$$

```
[In] int(F^(c*(a + b*x))/(f + f*cos(d + e*x))^2,x)
[Out] int(F^(c*(a + b*x))/(f + f*cos(d + e*x))^2, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                    end if
                else #result contains complex but optimal is not
                    if debug then
                            print("result contains complex but optimal is not");
                    fi;
                    return "C","Result contains complex when optimal does not.";
                fi;
            else # result do not contain complex
                 # this assumes optimal do not as well. No check is needed here.
                if debug then
                       print("result do not contain complex, this assumes optimal do not as well")
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    if debug then
                        print("leaf_count_result<=2*leaf_count_optimal");
                    fi;
                    return "A"," ";
                else
                    if debug then
                        print("leaf_count_result>2*leaf_count_optimal");
                    fi;
                    return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
                fi;
            fi;
        else #ExpnType(result) > ExpnType(optimal)
            if debug then
                print("ExpnType(result) > ExpnType(optimal)");
            fi;
            return "C",cat("Result contains higher order function than in optimal. Order ",
                           convert(ExpnType_result,string)," vs. order ",
                           convert(ExpnType_optimal,string),".");
        fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```